It's Not Music, It's Theory

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Abstract

Can the geometry of a 3D object be represented in music? These experiments in assigning musical notes to the faces of polyhedra and interpreting the result as scales and chords show that the problem has only partial solutions. The study suggests a more general problem in graph theory, similar to coloring.

1 Introduction

The structure of music has some mathematical properties, but do mathematical structures have musical properties? Can geometric symmetry in 3 dimensions be translated to music? These are the questions that motivated this work.

Linear mathematical structures like arithmetic sequences have been investigated by some composers, and the recent popularity of "π day" is perhaps responsible for several interpretations of the digits as musical notes or chords. On the other hand, two or three dimensions seem more difficult to represent.

This work focused on representing two particular Platonic solids with musical tones. The goal was to create something with a faithful interpretation of the geometry and to convey the visual symmetry through some kind of tonal symmetry. The secondary goal was to make a short, playable piece of music that sounded, if not exactly nice, at least not too awful. A tertiary goal was to discover something interesting about music structure.

Of the 5 Platonic solids, the octahedron and the dodecahedron are interesting targets for tonal interpretation because the octahedron has 8 faces and there are 8 tones in the usual musical scale (7 tones plus one an octave higher than the base) and the dodecahedron's 12 faces suggest the 12-tone scale on which most Western music is based.

2 Note = Face, Scale = Adjacent Faces, Chord = Vertex

The strategy we adopted was to assign a note to each face and to interpret the faces musically as the ordered set of tones on adjacent faces. Vertices were interpreted as chords determined by the notes assigned to the faces meeting at the vertex.

The first question we asked was whether or not it was possible to find an assignment of notes that would let each face have a scale with the same intervals. A computer program was set to work searching all assignments of notes to faces, eliminating those that had "bad" chords or "dissimilar" scales. For the octahedron, the scale was C major, for the dodecahedron the scale was the 12-tone.

Scales are similar if they have the same inter-note intervals. The pentatonic scale "C D E G A" has intervals (deltas) "2 2 3 2". There are pentatonic "modes" based on other intervals, particularly the pentatonic minor: "C - Eb - F - G - Bb" (3 3 2 3). Our search criteria did not dictate any particular interval set, but it did look for assignments that results in the same interval set (allowing permutations) for each face. We excluded assignments that had more than one semitone (interval 1), because these sequences were likely to sound dissonant and would not be recognizable as scale sequences. A further criteria was that the vertices (chords) should not have semitone or tritone (5) intervals, because these sound very dissonant.

Face Assignments

a	d	f
e	c	g
c	e	b
c	b	g
d	a	c+
e	g	b
a	f	c+
f	d	c+

Vertex Chords

d	e	c+	b
f	g	c+	b
a	g	e	c+
c	a	f	g
c	d	f	b
c	a	d	e

Figure 1: Octahedron Tones

A (nearly) complete search on the assignment space yielded no results for either the octahedron or the dodecahedron. The octahedron failed on the chord criteria (there are 4 notes in an octahedron chord, but only three in a face scale), and the dodecahedron failed on the scale criteria (the pentatonic scales have 5 notes). We absorbed this disappointment and loosened the criteria to allow more dissonnance.

By allowing one semitone into chords, we were able to get solutions for the octahedron that included some similar scales, illustrated below. The 3-note scales are reasonably musical, but the chords are jarring. The results can be heard via [6] and [5] .

For the dodecahedron, by defining scales as similar if the absolute values of the adjacent tones yielded some identical subsequences, we were able to find an assignment for the dodecahedron that contained some scale similarities. This is illustrated in the table below (we simplified the dodecahedron search by assigning C# to the face opposite the C face). The interval sequence "6 4 6 4 4" occurs twice (faces 1 and 12), the subsequence "3 4 3" occurs 5 times, "7 6 7" twice, and "7 6 3" twice.

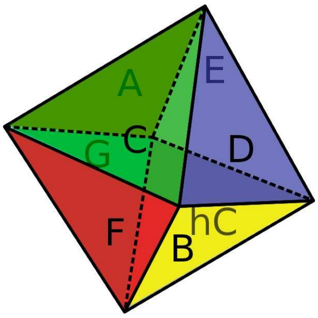

Figure 2: Octahedron

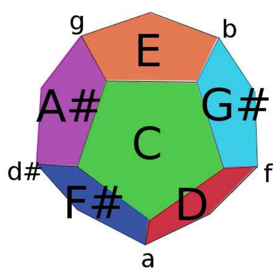

Figure 3: Dodecahedron

Dodecahedron Tones

Face Scales

a#	e	g#	d	f#
c	f#	d#	g	e
c	a#	g	b	g#
c	e	b	f	d
c	g#	f	a	f#
c	d	a	d#	a#
c#	g	a#	f#	a
c#	b	e	a#	d#
c#	f	g#	e	g
c#	a	d	g#	b
c#	d#	f#	d	f
a	f	b	g	d#

Vertex Chords

c#	f	a
c#	f	b
c#	g	b
c#	d#	a
c#	d#	g
d#	f#	a
d	f#	a
d	f	g#
f	g#	b
e	g#	b
e	g	b
e	g#	a#
d#	g#	a#
d#	f#	a#
c	f#	a#
c	d	f#
c	d	g#
c	e	g#
c	e	a#

3 Prior Work

Mappings of number sequences to tones or chords are a popular feature of mathematical shows, and at least one is online [2]. There are a few serious musical pieces based on dodecahedrons, and one, by Joceyln Ho, is available on YouTube [4]; there is also a paper about the composition [3]. An interesting assignment of notes to the faces of a rhombic dodecahedron and pentatonic scale relationship is the subject of part of a book [1].

4 Mathematical Generalization

We might ask if there are other polyhedra that are more amenable to musical assignments, noting that solutions build on the well-known graph coloring problem. We could define a "numbering" for a polyhedron G from a set S by assigning elements of S to faces of G with the conditions that (1) every element of S is assigned to at least one face of G (2) no two adjacent sides have the same member of S, and (3) no vertex has two or more faces with the same member of S. A face scale for face F is the ordered sequence of members of S on faces adjacent to F.

For the musical problem, we wanted all face scales to be similar and all vertex sets to be "musical". The musical constraints limited the set of possible face scales. Which constraints can be satisfied with for a given G and S?

References

[1] David A. Becker. *Incarnations of the Blaring Bluesblinger: A Multimedia Mathmantra on Manifestations of Mutuality in Music, Molecules, and Morphogenesis.* University Readers, 2010.

[2] Michael Blake. What π sounds like. `http://www.youtube.com/watch?v=YOQb_mtkEEE`. YouTube.

[3] Jocelyn Ho. From 3D to 2D: Using geometry and group theory to model motivic structure in musical composition. In Carlos Agon, Emmanuel Amiot, Moreno Andreatta, Gérard Assayag, Jean Bresson, and John Manderau, editors, *Mathematics and Computation in Music: Third International Conference, MCM*, June 2011.

[4] Jocelyn Ho and Avinash Krishnan. 12 variations on a dodecahedron. YouTube. A piano solo where the structure of the music is mapped on the 12 faces of a dodecahedron and can be visualized as two pianists moving across different faces.

[5] itzaloulou. Chromatic dodecahedron, 2012. YouTube.

[6] itzaloulou. Musical octahedron, 2012. YouTube.