Non-Euclidean Board Games

Andrea Hawksley
www.andreahawksley.com
hawksley@gmail.com

Introduction

Most traditional grid-based board games are played in Euclidean space. It is a fun and interesting exercise to try and play them in non-Euclidean spaces. In this paper, we will explore playing the game of Chess in non-Euclidean space, but the concepts used to extend Chess to non-standard geometries can be extended to many other board games. A personal favorite game for these purposes is non-Euclidean Carcassonne.

Boards with Unusual Macrostructure

The simplest way to transform a standard board into a non-Euclidean board is to connect the sides of the board together in some way. Connecting the squares on adjacent sides to each other creates a board with an inherently spherical topology. In contrast, connecting the squares on opposite sides can lead to a board with a toroidal structure. A twist in the way the squares are connected can lead to a board with the topology of a klein bottle.

While these macrostructures are interesting, they are not inherently well suited for Chess. In particular, none of these boards have edges. Thus, it is impossible for a Pawn to reach the edge of the board and become a Queen. In fact, it's not even clear that we weren't playing on boards with these macrostructures all along, and were just playing a subsection of space that was locally equivalent to a subsection of a Euclidean space.

The Chutes and Ladders Connection

How can we create a non-Euclidean board that does not lose the edge constraints of the original Chess board? One way is to connect random squares to each other by the edges. We will refer to these kinds of connections as "chutes" in homage to a classic board game with these kinds of connections - Chutes and Ladders.

We will use the chutes defined by the board in Figure **??** to explain basic movement on a board with chutes, however one can easily add their own chutes to any standardly gridded board to make it non-Euclidean.

Chutes and ladders are functionally the same. The side of the tile going into the "top" of each chute or ladder connects to the side of the tile going out of the "bottom" of that chute or ladder.1 For example, a pawn moving forward two squares out of tile 84 would move out the left side of tile 28 and into tile 29. Note that the pawn is now moving sideways, and can be Queen-ed on its next move onto tile 30. We recommend playing with pieces with some obvious indication of directionality on them, so that you don't lose track of the traveling direction of your pawns.

In order for the board to be connected, we must also map the tiles that come out of the tops and bottoms of each chute and ladder to each other. Thus, the bottom of tile 20 must match the bottom of tile 23 and the left side of tile 18 must match the top of tile 59.
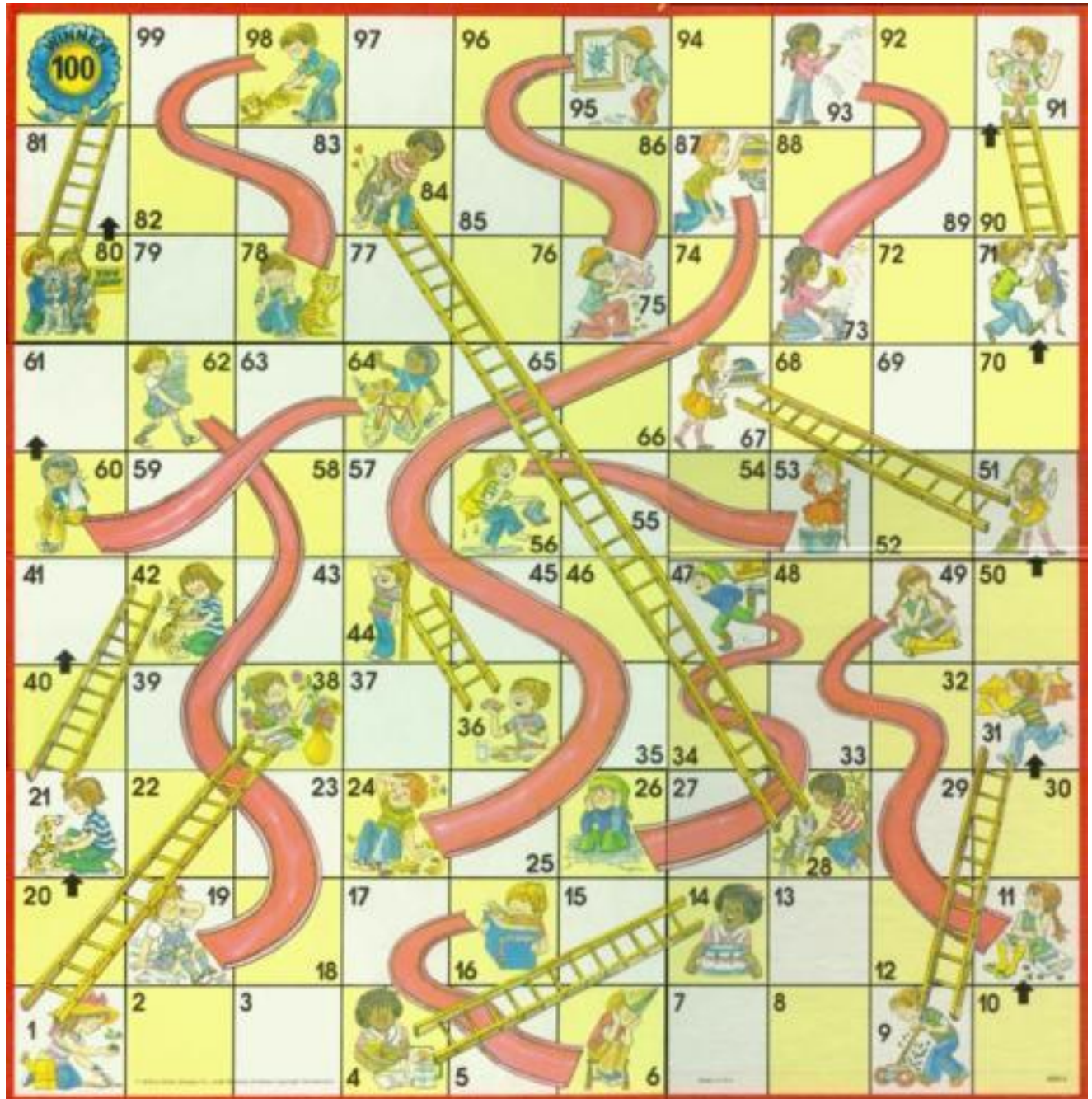
Figure 1 : *A standard Chutes and Ladders board*

The Turn-Left Metric

At this point, it is pretty clear how to travel across the sides of the tiles, but what it means to travel diagonally is less well defined. We have chosen to use a "turn-left" metric to define diagonal moves. Let us imagine a bishop moving diagonally out of the top right corner of square 4. In a standard Euclidean board, it would end up in square 16 after moving one square. This is equivalent to moving into square 5, then turning left and moving into square 16.

On our non-Euclidean board, we find the corner by moving in the direction of square 5, which brings us out square 14 without rotation and turning left to end up in square 27. Since we haven't rotated, if the bishop wants to continue moving along the same diagonal it may. For its second square of movement, it can travel in the direction of square 28 causing it to enter square 77 from the top. This time, when it turns left, it enters square 76. Additionally, it has rotated $-\pi/2$ radians, thus, to continue along the same diagonal it will want to move out of the bottom right corner of square 76, and comparably straight-forwardly into square 66. Following similar logic, the next squares that the bishop can visit are 54, 26, 36, and 37.

Extensions

There are many other fascinating games that you can try playing along similar lines. Players might take turns moving, then adding a new chute to the board. Roice Nelson has suggested playing chess or checkers on a hyperbolic checkerboard. Are there any other interesting boards that you can imagine playing chess on?

A Puzzle

Can you design a chess board with a single chute such that White as a mate in one?