Folded Cookies for a Skeptic

G4GX
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This short note is meant to accompany my physical gift of the same name. That gift was a box of fortune cookies. Each cookie contained a quotation from Martin Gardner vis-à-vis his skeptical inquiries into pseudoscience and other such things.[1]

As a G4G reader will know, it was paper folding that brought Gardner back to math. Here first I recount the fortune cookie's history (thus debunking conventional wisdom) before turning to the main aim of this short note, namely, the folding itself of the fortune cookie.

Can one be folded from paper? What sort of question is that? How shall the creases be represented? What kind of shape is the folded cookie? We vary the crease pattern, the initial shape of the paper, and the proportions of the bounding rectangle. The superellipse seems promising (as a generalized solution).

We follow where this leads, namely, to the tortellino (singular of "tortellini"), the deep-fried wonton (apparently a special case among wontons), and the conventional *jiaozi* 饺子 (dumpling).[2]

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Until recently, fortune cookies could not be found in China. There was not even a Chinese word for them.[3]

In America they were first produced approximately one hundred years ago in southern California by Japanese immigrants who also owned and operated the so-called "chop suey" Chinese restaurants in which the American style of Chinese food was created.

Their model appears to have been an obscure cracker (Figure 1) made by the bakeries that clustered around the Fushimi Inari Taisha 伏見稲荷大社 Shinto shrine in Kyoto.

This cracker had several names: *tsujiura senbei* 辻占煎餅 (fortune crackers), *omikuji senbei* 御神籤煎餅 (written fortune crackers), and *suzu senbei* 鈴煎餅 (bell crackers).

Figure 1

The crackers were baked in a round mold over an open grill. While still warm and pliable they were folded around fortunes, and then placed for cooling in trays that had receptacles for them (Figure 2).

Figure 2

Fortune cookies are still folded in the same way (Figures 3): The east and west sides of the round cookie are pulled in and forward around the fortune. The north and south points are folded backward until they meet, which compels the east and west edges to meet.

Figure 3

Now Chinese restaurants in Britain, Mexico, Italy, France, Brazil, India and elsewhere also hand out the cookies. In America, they are now made by Chinese-American enterprises. And in Japan, because people were accidentally eating the

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[1] E.g., "Are astrology books and newspaper columns as harmless as fortune cookies?" This quote is from Martin Gardner, *From the Wandering Jew to William F. Buckley Jr.* (Amherst, N.Y.: Prometheus Books, 2000), 126.

[2] For Chinese, modern standard pronunciation and orthography (simplified characters) will be used here.

[3] Most of the information in this section was uncovered by Yasuko Nakamachi. Her research is described in "Solving a Riddle Wrapped in a Mystery inside a Cookie," *New York Times* January 16, 2008. The first two photos are from that article.

fortunes, bakeries no longer put them inside the cracker (as in Figure 1).

All the same, we cannot rule out the possibility that these crackers once upon a time did indeed make their way from China to Japan, as so much else did, perhaps a millennia or more ago, though I am not aware of any evidence for this.

One way to rationalize the fortune cookie shape is to replace the curved edges, curved creases, curved faces and soft folds with, respectively, straight edges (i.e., square or rectangular paper shape), straight creases, flat faces, and hard folds, as in Figures 4 and 5 where each lobe of the cookie is a tetrahedron.

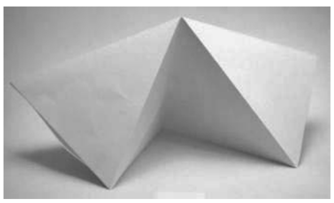

Figure 4

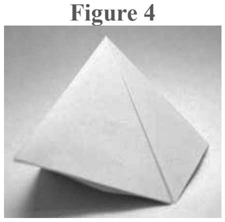

Figure 5

Various less complete rationalizations are also possible. For instance, the creases, mostly soft in the original cookie, could be converted to firm straight creases, while the curved edges (i.e., the circular or elliptical paper shape) and curved faces could be permitted to remain (Figures 6, 7, 8).

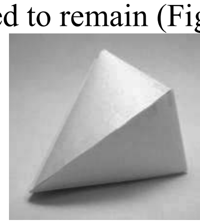

Figure 6

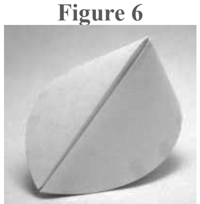

Figure 7

Figure 8

These two realizations can be folded as in Figures 9, 10, 11 and 12, where valley folds are indicated by dashed lines, and mountain folds, dash-dot-dashed. For both circle and square, the distance AB is half the horizontal width of the paper. First, make the creases as in Figures 9 and 11.

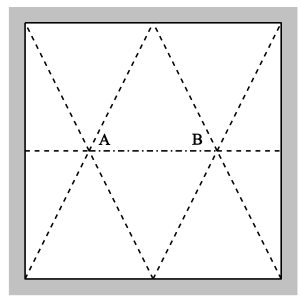

Figure 9

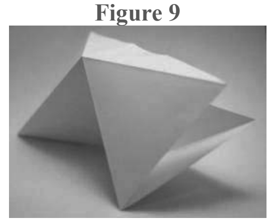

Figure 10

To fold the cookie from a square, pull the east and west edges forward and push the north and south edges back (Figure 10). The same steps can be used for the circle (Figure 12) but unless you do something like the following it will not be obvious that the shape is (or might be) cohesive.

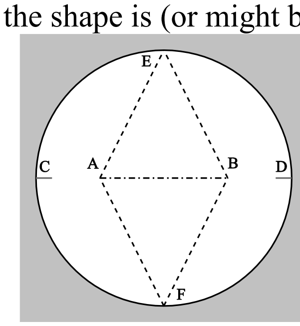

Figure 11

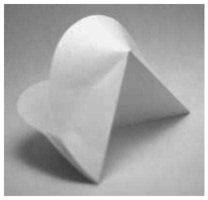

Figure 12

For the circle, after precreasing, lightly fold point C to D. Use small pieces of tape to attach the top layer to the bottom, wrapping the tape around the cut edge at regular intervals. Fold the figure back along AB until E meets F, permitting the paper to use only the indicated creases. It will probably be necessary to insert something long and skinny (a chopstick?) into the interior in order to push it into shape.

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"Ideal" paper cannot be stretched, compressed, molded, etc. It can, however, be made to curve, as a developable ruled surface with Gaussian curvature $k = 0$ everywhere.[4]

Each point on such a surface lies on at least one straight line, and the surface itself could be swept out by moving a straight line through space.

In Figure 13 these curves and lines are indicated according to the following scheme, which is a simplified black and white version of Andrew Hudson's recent proposal:[5]

(a) Thin and light solid or dotted lines represent, respectively, mountain and valley curvature of an elastic reversible nature (that is, they are examples of the straight lines that are contained in the curved developable ruled surface).

(b) Thick and dark solid or dashed lines represent, respectively, conventional mountain and valley creases of a plastic inelastic irreversible nature (that is, forming a crisp angle less than 180° in the desired orientation).

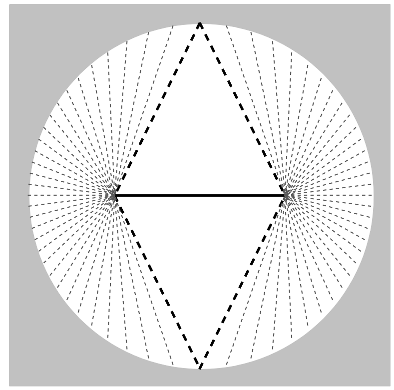

Figure 13

○

For the paper cookie made from circular paper, one generalized nappe is reflected mirror-wise base-to-base about a plane of symmetry. It is thus a generalized bicone. (A second plane of symmetry is orthogonal to the first.)

If the AB-to-width ratio is ½, a single line connects the vertex (apex) of one nappe to that of the other. This line is orthogonal to the first plane of symmetry.

The flat shape of each unfurled nappe is visible on the crease pattern (the darkest regions of Figure 14).

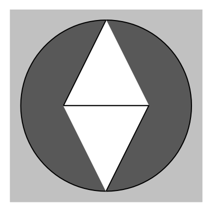

Figure 14

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There are many ways to produce and vary "squareness." Two of the more interesting are (a) the superellipse (Figure 16) and (b) the Guasti method (Figure 15).

The notion of squareness holds promise not only for cookie or paper folding but also for such things as LCD pixels which are only squarish but not square in shape.

The rationalized paper cookie is folded from a square. The incompletely rationalized one is made from a circle. Can they be folded from figures that are in between?

For the Guasti method, not used here, a single parameter s specifies for squareness.[6] The value 0

[4] Recently several interesting projects have focused on producing equations for pasta that is variously extruded, molded or otherwise not folded, e.g., George Liaropoulos-Legendre, *Pasta by Design* (New York: Thames & Hudson, 2011), which includes an equation but not, so to speak, a folding for a tortellino—a folding would be many times more economical.

[5] See Hudson, Andrew. "Diagramming: Curves and Crease Patterns," *The Fold* 5(July-August 2011).

[6] See Manuel Fernandez Guasti, "Analytic Geometry of Some Rectilinear Figures," *International Journal of Mathematical Education in Science and Technology* 23.6 (1992), 895-913.

results in an ellipse, 1, the circumscribed rectangle, and values between 0 and 1, figures that are between an ellipse and rectangle:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{s^2x^2y^2}{a^2b^2}=1 \qquad (1)$$

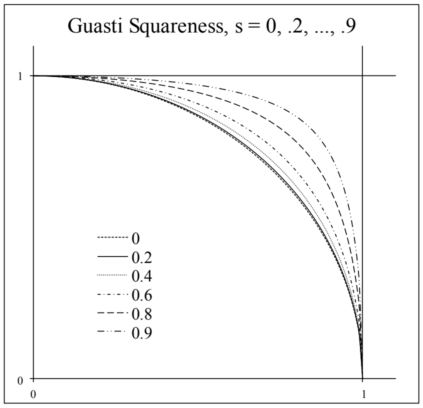

Figure 15

The superellipse, which is used here, is an ellipse for which the exponent of 2 has been replaced with a larger (or smaller) number: [7]

$$\frac{x^n}{a^n}+\frac{y^n}{b^n}=1 \qquad (2)$$

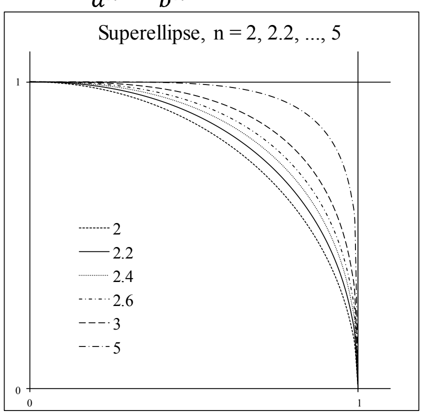

Figure 16

Guasti's figures or another shape might work as well or better. However the superellipse has long been associated with Gardner and especially Piet Hein. It is the first and only interim shape that I tested (that is, made paper models of). Such models can, I believe, contribute to the generation of a hypothesis.

Three parameters (the first two more interesting than the third) suffice nicely to determine and generalize the shape of the paper folded cookie:

(a) ratio of the length AB (above) to that of the width of the figure,
(b) superellipse exponent n (i.e., squareness), and
(c) ratio of height to width of circumscribed rectangle.

Figures 17 and 18 illustrate AB-to-width ratios that are less than ½.

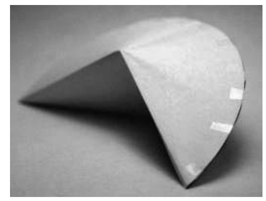

Figure 17

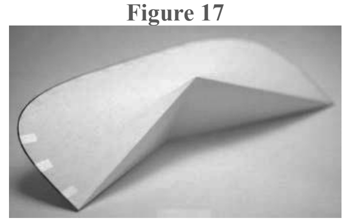

Figure 18

Figure 19 is a cookie folded from an n = 5 superellipse (with lower n-value superellipses indicated on the surface).

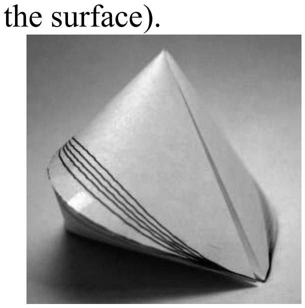

Figure 19

Figure 20 was folded from an ellipse that was inscribed in a 1:√2 rectangle.

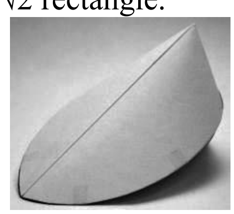

Figure 20

The original fortune cookie has an AB-to-width ratio of ½, a superellipse exponent of 2, and a height-to-width ratio of 1.

The lobes of the cookie touch only if the AB-to-width ratio is ½. As the superellipse exponent increases, the paper cookie's shape approaches the tetrahedral form (though it is not enveloped in it).

[7] For an overview, see Martin Gardner, *Mathematical Carnival* (Washington, D.C.: The Mathematical Association of America, 1989), 240-254. Other terms have been used for the superellipse, including: squircle, rectellipse, Lamé Curve, hyperellipse, k^{th} order ellipse, astroid, and superformula.

A larger height-to-width ratio makes a longer cookie.

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I have put together a PDF workbook of paper shapes for many pertinent figures including those described here. If time allows, I will put together an unabridged version of this note. If you are interested, please contact me.

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In the meantime, the reader might wish to ponder the following:

Could a cookie be folded based on a generalization of the superellipse (as in Figure 21)?[8]

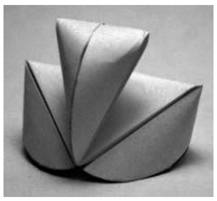

Figure 21

Is a sphericon-type modification possible (as in Figures 22 and 23)?

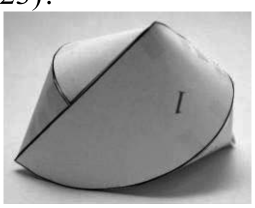

Figure 22

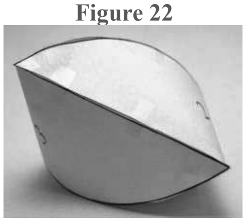

Figure 23

Are asymmetric versions possible (as in Figure 24)?

Figure 24

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To conclude, note that the fortune cookie is, in form and folding, closely related to several old folded foods of which two are Chinese. No discussion of fortune cookie history that I have read mentions this. And that is why my name too shall appear in a very small font size at the fortune cookie history hall of fame.

Wontons (a Cantonese pronunciation for which the Mandarin equivalent is *huntun* 馄饨) are made from square wrappers. Usually they are folded corner to corner. However, in Figure 25 below, the two pairs of corner flaps tell us that this deep-fried version was folded like a fortune cookie (the circle has been creased not cut).

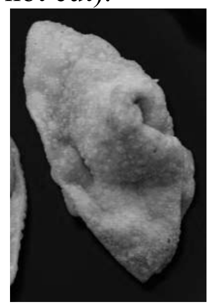

Figure 25

A tortellino (Figure 26) is made from a square wrapper that is folded corner to corner. The characteristic shape is obtained by wrapping the resulting right triangle backward at its hypotenuse around your finger. The degree of curvature confirms an AB-to-width ratio of less than ½.

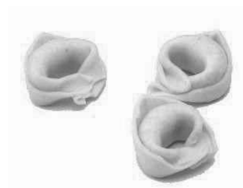

Figure 26

J*iaozi*[9] (Figure 27) are made from circular wrappers. The ratio of AB to the full width is significantly less than ½. Moreover, note the pleats. Both greatly reduce the curvature of the base. Without both, a *jiaozi* would look just like a fortune cookie.

Figure 27

[8] See, for example, Johan Gielis, "A Generic Geometric Transformation that Unifies a Wide Range of Natural and Abstract Shapes." *American Journal of Botany* 90 (March 2003), 333-338.

[9] Boiled *jiaozi* are called *shuijiao* 水饺, steamed, *zhengjiao* 蒸饺, and fried, *guotie* 锅贴 (potsticker). Each region of China (and family, for that matter) has their own way of making *jiaozi*, and that way is the one correct way.