

Martin Gardner
in the
Twenty-First Century

edited by

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Preface

Martin Gardner and the MAA share a long history. In 1958, around the time he started his famous “Mathematical Games” column for *Scientific American*, he submitted the first of many problems to *The American Mathematical Monthly*. In 1982, as his column wound down, Gardner’s first MAA article was published in *The Two Year College Mathematics Journal*. He wrote for MAA journals the rest of his life, particularly *The College Mathematics Journal* and *Math Horizons*. Gardner contributed to the latter almost annually from its founding in 1993 until 2005.

Gardner’s prodigious writing activity continued right until his death in 2010. Articles, stories, problems, solutions, Quickies, and other kinds of contributions continued to flow. His last mathematical article to appear in an MAA journal, “L-tromino Tiling of Mutilated Chessboards,” was the centerpiece of a special puzzle issue of *The College Mathematics Journal* in 2009, and it is included here.

Early in 2010, *Math Horizons* editors Steve Abbott and Bruce Torrence were surprised to receive a typescript manuscript. Gardner used a typewriter his whole life, never email. The submission was accompanied by a note, “Is this short story something you can use? I wrote the math column in *Scientific American* for 25 years. If my piece is not right for *Math Horizons*, there is no need to send it back. All best, Martin.” There was not enough time for the editors to thank Martin for his submission [6]. Fittingly, this story, “Superstrings and Thelma,” is the last piece in this collection.

Apart from his own work, Martin Gardner, by enormously expanding the field of recreational mathematics, opened up vast mathematical tracts for exploration by others. This was quite deliberate. In an interview in *The Two-Year College Mathematics Journal* [1], Gardner said, “I’m defining [recreational mathematics] in the very broad sense to include anything that has a spirit of play about it.” Gardner, of course, had a refined and very well-developed sense of play, one quality that made his pieces so enjoyable to read. In almost everything he wrote, Gardner posed problems to challenge his readers, and they responded. He maintained an extensive correspondence with mathematicians, both professional and amateur. Their work fueled his own pieces, but then his correspondents turned around and wrote their own articles.

One consequence was the expansion of recreational mathematics into a major research area (also helped by the development of the computer and the corresponding expanded interest in discrete mathematics) that is such a feature of the current mathematical landscape. The *CMJ* devoted the January 2012 issue to papers on topics that Gardner introduced to the mathematical public. There were so many articles to include that half of the March 2012 issue continued the theme.

Another consequence of the flowering of recreational mathematics is this volume. We have collected MAA journal articles starting from 1999 on topics that Gardner developed. Some are written by Gardner, but most are by others. The tribute articles from the January and March 2012 *CMJ* issues are all here, but they constitute less than half of this collection. All the MAA journals are represented, *Mathematics Magazine* and the *Monthly*, as well as *CMJ* and *Math Horizons*. The limitation to pieces published roughly in the twenty-first century is a practical one. Even so, some puzzle collections, longer articles, and pieces less directly linked to Martin Gardner have been omitted.

The 41 pieces collected here are grouped around common themes, such as geometry, number theory and graph theory, and cards and probability. Flexagons, the topic of Gardner's first *Scientific American* column, are seen to be associated with Catalan numbers and together merit their own section. Geometric tiling and various "magic" number puzzles are all about "Making Things Fit," and there are enough other puzzles and games to fill another section.

Gardner's interests ranged far beyond mathematics. A fan of magicians and magic tricks from childhood ("I waste a lot of time on it" [2]), he wrote several books on magic. He annotated Lewis Carroll's *Alice* books and other classics, and produced two novels of his own. Other topics included philosophy, religion, literature and pseudo-science, leading to some 70 books.

The last section of this volume highlights some of these other facets of Martin Gardner's wide-ranging interests. It includes two short stories by Gardner and several other pieces that demonstrate his support for amateur mathematicians, his love of play (about an April Fool's joke he played on his *Scientific American* readers), and his interest in debunking false science. Also included is Gardner's review of a 2004 novel in which an important character seems to be based on him.

Our hope is that this volume will play a role in perpetuating the memory of Martin Gardner, modest celebrity, larger-than-life character, self-confessed amateur as a mathematician, popularizer of recreational mathematics in the broadest sense, prolific and brilliant writer. Given the lasting impression he made on several generations of mathematics enthusiasts of all backgrounds, we are confident that the MAA and others will be publishing articles inspired by Gardner's work for a long time.

Gardner, like the readers of this book, loved mathematics. We close this preface with Gardner's own words on his background, the community, and why he enjoyed the field so much (from [5], [4], and [3], respectively).

I took no math in college. I'm like a person who loves music but can't hum a tune or play an instrument. My understanding of math does not go beyond a minimal understanding of calculus. I hasten to add that I consider this one reason for the success of my *Scientific American* column. I had to work hard to understand whatever I wrote about, and this made it much easier for me to write in a way that readers could understand.

When I first started the column, I was not in touch with any mathematicians, and gradually mathematicians who were creative in the field found out about the column and began corresponding with me. So my most interesting columns were columns based on material that I got from them, so I owe them a big debt of gratitude.

[I enjoy mathematics] because it has a strange kind of unearthly beauty. There is a strong feeling of pleasure, hard to describe, in thinking through an elegant proof, and even greater pleasure in discovering a proof not previously known.

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