Fair Poker with Marked Cards

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This is a puzzle. How can a number of poker players play a fair game in which each of the players assumes that any of the other players will bring a marked deck to the game?

We will start with a few assumptions.
1) The only cheating method employed is the use of marked cards.
2) The cards are marked so that each player can only read the marks of the deck they supplied, and even throughout the play of many hands, the other players cannot determine the markings on the cards from the other players decks.

Notation:
A - Ace
J - Jack
Q - Queen
K - King
S - Spades
H - Hearts
D - Diamonds
C - Clubs

A - K means the cards are arranged from top down A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K.

K - A means the cards are arranged from the top down in reverse order than A - K (the order is K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2, A).

Decks are numbered 1 to N.

Thus AS represents the Ace of Spades. If a number follows the card designation, such as AS2, that means that the card comes from the deck associated with the suffix digit.

There are many variations of poker, some of which have cards that are dealt face-up so that all players can see their values. Do markings matter in such circumstances? Yes, since there is usually a round of betting prior to the face up card(s) being dealt. If the card is dealt off the top of the deck, the dealer, and perhaps others, could bet with the knowledge of the top card's markings. One could deal by removing cards from the deck center, thereby removing that visible top card advantage, but one still knows the top card is not in play, and that changes the odds.

As shown in the above paragraph, there are many considerations. In this paper we will only address games in which cards are dealt face down and the top card is dealt so that the back cannot be seen in advance to being dealt. (This can be accomplished by placing a Joker face-up on top of the deck. For example, to cut the deck, a face-up Joker is inserted into the deck, the deck is cut at that point, leaving the Joker on the top of the deck. Then by dealing the second cards of the deck, their backs are not shown in advance. This leaves the situation where the visible back of the top card prior to the cut can be read. There are ways to handle this condition, but they will not be discussed in this paper.)

If there is one marked deck and only one player knows the markings, then it is clear that player has the advantage. But what if there is one deck consisting of cards taken from N marked decks when there are N players? For the time being, we will ignore the case of one deck consisting of cards from N decks with M players, where N does not equal M.

The issue of back design needs to be addressed. If all N decks have different back designs and those designs are associated with particular players, then a player will know which of the cards they are holding can be read by which other player. Their best hand of cards will have only the backs associated with that player. A particularly bad hand of cards will have all the backs associated with only one other player, as that other player will know all of the cards in that hand.

How can this be made fairer? What if each player only receives cards from the deck they supplied? Assume that N players supply N decks and each player's deck is shuffled, cut and dealt by other players. Then each player receives all their cards from the deck they supplied. That way no player can read another player's cards. This impacts the game and would require an additional rule, because duplicate hands can now exist. Who wins in a game of five players when all five players hold four aces?

Also certain conditions that cannot exist in a single deck game can now occur. For example, one player holds four aces and another player holds a royal flush. In another example, one player holds a 10 high straight Spades flush and another player holds a 9 high straight Spades flush.

Given the constraint of duplicate hands, using N decks for N players provides a fair outcome and is easy to implement. Note: with this procedure and the previously stated assumptions, there is no advantage to having the cards marked, except if a player can see the top card of their deck and the game includes drawing cards from the top.

Are there other ways to implement a fair game using only one set of 52 cards? If yes, how does one form that set?

Let each player bring to the game their own deck of marked cards. Cards are removed from each deck to form one deck of 52 cards with unique faces. Let the N decks be arranged top down as:
A - K S
A - K H
A - K C
A - K D.

The objective is to prevent any player from gaining more knowledge of another player's hand than the other players have of that player's hand. This forces certain constraints on how the new deck is built.

For example, if N is four and the new deck is made by taking cards in turn from each A-K stack, then the following set is formed:
AS1, AH2, AC3, AD4, 2S1, 2H2, 2C3, 2D4, 3S1, 3H2, 3C3, 3D4,
4S1, 4H2, 4C3, 4D4, 5S1, 5H2, 5C3, 5D4, 6S1, 6H2, 6C3, 6D4, and so on.

A good result from this arrangement is that four of a kind, a full house, three of a kind and a pair cannot be determined. Even a straight cannot be determined, except if it is a straight flush.

One problem with this arrangement is a player could get a flush all from the same deck and the owner of that deck would know that, while the other players would not. This would not be a fair situation.

This problem can be addressed by saying that if one gets a flush, it counts for nothing, and one can remove all of their bets. That works for the holder of the flush, but other players might have placed bets based upon the flush holder's bets. Does one declare that all hands are invalid? The holders of really good hands would not like that, especially if they had been losing prior to the current hand.

In the most equitable scenario each player could only determine the value of at most one card in any other player's hand. There is no way to guarantee this outcome.

Assume five players with five decks assembled into one deck, then it is possible that all five cards in one hand come from different decks. It is also possible for all five cards to come from the same deck.

One way to form the new deck is to take a card from each deck in order of uniqueness to produce the new deck in order A-KS, A-KH, A-KC, A-KD. For the example, the new deck top down will start out as:
AS1, 2S2, 3S3, 4S4, 5S5, 6S1, 7S2, 8S3, 9S4, 10S5, JS1, QS2, KS3,
AH4, 2H5, 3H1, 4H2, 5H3, 6H4, 7H5, 8H1, 9H2, 10H3, JH4, QH5, KH1

An advantage of this arrangement is that there will never be a flush hand with all the cards from the same deck, thus solving the problem with the prior arrangement. Also, any four of a kind will always have cards from four decks. Three of a kind, a pair, two pairs and a full house also must have cards from different decks.

Is the problem solved? Not quite. Five does not divide evenly into 52. There is a remainder of 2, thus two decks will each have one more card in the assembled deck than the other decks will. This will give a slight edge to the owners of the two decks that contributed extra cards to the assembled deck.

So what would be a good number for N? 52 can be prime factored into 2 * 2 * 13. That leaves 2 and 4 as possible values of N. 13 is also a possibility, except that would leave each hand with only 4 cards, thus 13, 26 and 52 are not good values for N.

Would two players with two decks assembled as one deck be fair? With two decks, each suit would consist of either six or seven cards from the same deck. In that case, a flush can be dealt with cards all from the same deck, thus tipping the owner of that deck to the hand of their opponent.

With four decks, each suit would consist of either three or four cards from the same deck, thus a flush cannot exist having all the cards from the same deck. This is much better, but still has an issue. If a player notices that another player has four cards of a flush, then what statistical advantage do they have in determining if there is an actual flush? If not a flush, the only other possibilities are high card (which they will know the highest card of the four cards of the same suit) or one pair. This gives an advantage to the owner of the deck that the cards in the flush came from. Over the course of a night's play that situation might never occur, but it is possible.

There is more to explore regarding this puzzle, but this paper was intended to introduce the problem and offer some considerations to a wider audience. For example, not included in this preliminary paper is any discussions of assembled decks having a number of cards different than 52 (a deck of 48 cards allows for six decks and a deck of 60 cards would allow for five decks).