# 3D SUDOKU and 4D SUDOKU

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On a 8 x 8 x 8 grid of cubes, we can consider two SUDOKU-like puzzles.

**Puzzle $A_0$:**　Assign numbers from1 to 64 to an 8 × 8 × 8 grid of cubes so that each 8 × 8-plane (3 × 8 exist) and each 4 × 4 × 4-block (8 exist) contains all 64 numbers.

**Puzzle $B_0$:**　Assign digits from 1 to 8 to an 8 × 8 × 8 grid of cubes so that each 8-sequence (3 × 64 exist) and each 2 × 2 × 2-block (64 exist) contains all 8 digits.

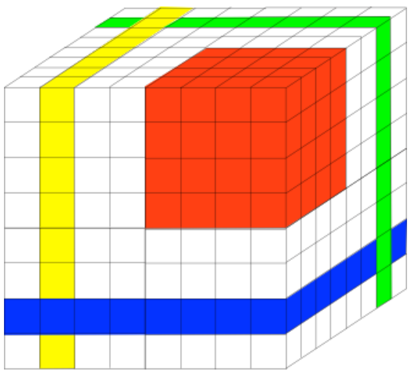

*Figure 1:　The constraints of Puzzle $A_0$.*

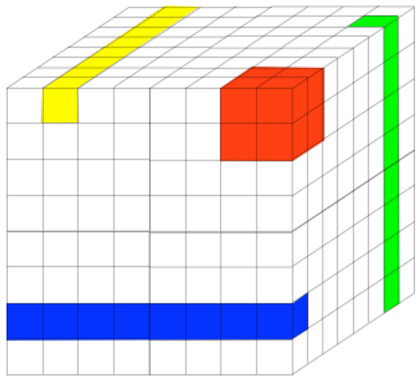

*Figure 2:　The constraints of Puzzle $B_0$.*

Moreover, one can consider more complicated puzzles based on them.　A 16-cell, which is a 4-dim object, has cubic projections in 8 different ways, and the third level approximation of the 16-cell fractal, which is composed of 512 16-cell pieces, is projected to 8 × 8 × 8 grids of cubes in 8 different ways.　Therefore, one can consider two puzzles Puzzle A (resp. Puzzle B) to assign 64 (resp. 8) numbers to the 512 pieces of this object so that all the 8 projections form solutions of Puzzle A (resp. Puzzle B).　In addition, the 8 cubic projections are divided into two sets of 4 orthogonal projections.　Therefore, one can also consider simpler puzzles which only consider a set of 4 orthogonal projections to define Puzzle $A_S$ and Puzzle $B_S$.　In [1], it is shown that puzzle B does not have a solution but all the other five puzzles have solutions, and constructed solutions through algebraic methods. It also calculated the number of solutions for Puzzle $B_S$ to be 1148928.

The purpose of this note is to present that it is possible to enjoy Puzzle $B_0$ and Puzzle $B_S$ as pencil puzzles.　Puzzle A, $A_S$, and $A_0$ require 64 different digits whereas Puzzle $B_0$ and $B_S$, use only eight digits. It is almost impossible to draw a 8 × 8 × 8 grid of cubes on a paper, but one can draw eight 8 × 8 grid of squares instead. The following is an easy to medium level of such a puzzle to fill the blanks to complete a solution of Puzzle $B_0$.

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8123456_ 2______5 4_6_2_3_ _5_7_1_8 __3_1_7_ 3_5_7_1_ _6__8__3 __7_5__2
4__52__1 __4_8_2_ ____4_6_ 2____7_5 7_1_3_5_ 1______2 __8_1__4 82____1_
___6_2_3 _____3__ _8___4__ _2__7_1_ _74_2_3_ 4__73__8 8_2_6_4_ __8_4_6_
____1___ 8__3_6__ 7_1_3_5_ _______2 3_____4_ _8_1__6_ 6__5_3__ 4__7__2_
__5__7_8 _4__1__7 ____5__4 _7__2___ __6_____ _1___5__ 3__8____ 7__5___1
1_3____4 _71__4__ 2__4__1_ ______8_ __7____2 _____2__ 4__1___5 __4__5__
________ ________ ________ 4__2__7_ 1__7___6 8__3__5_ __5____8 5____8_7
________ ________ _5__6___ ___5___1 _4___713 56___3_7 ___7_42_ __1__6_4
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It is not easy to understand the constraint of Puzzle $B_S$. However, it can be restated on a 8 × 8 × 8 grid of cubes that, on all the 24 8 × 8-planes, the squares with the same color in Figure 3 have different digits.

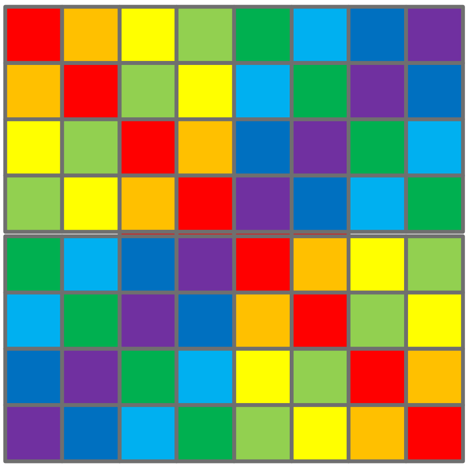

Figure 3: Constraints of Puzzle $A_S$.

Figure 43: A Physical solution of Puzzle $A_0$.

With this constraint, the following puzzle has a unique solution, which is the same as the solution of the $B_0$ puzzle above.

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8123456_ ________ ________ ________ ________ 3_______ ________ ________
4_______ ________ ________ ________ ________ ________ ________ ________
________ ________ ________ ________ ________ ________ ________ ________
________ ________ ________ ________ 3_______ ________ ________ ________
________ ________ ________ ________ ________ ________ ________ ________
________ _7______ ________ ________ ________ ________ ________ ________
________ ________ ________ ________ ________ ________ ________ ________
________ ________ ________ ________ ______1_ 5_______ ________ ________
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[1] Hideki Tsuiki and Yasuyuki Tsukamoto: Sudoku Colorings of a 16-cell Pre-Fractal, in Discrete and Computational Geometry and Graphs, Proceedings of JCDCGG 2015, to appear in LNCS, Springer, 2017.