Equal Product of Reversible Numbers (EPRNs)

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Abstract: *The numbers that can be expressed as the product of a number and its reversal in two different ways, like 144648 = 861 × 168 = 492 × 294, are termed EPRNs (Equal Product of Reversible Numbers). Based on properties and observations, method of computing the EPRNs are discussed. The connection between EPRNs and Par numbers mentioned by Martin Gardner and noted recently makes this class of numbers more interesting.*

EPRNs (Equal Product of Reversible Numbers) are defined as the numbers that can be expressed as the product of a number and its reversal in two different ways [1]. For example,

2520 = 120 × 021 = 210 × 012,

144648 = 861 × 168 = 492 × 294,

9949716 = 2583 × 3852 = 1476 × 6741, and so on.

The sequence of numbers that can be expressed as the product of a number and its reversal in at least two different ways is given below:

2520, 4030, 5740, 7360, 7650, 9760, 10080, 12070, 13000, 14580, 14620, 16120, 17290, 18550, 19440, 22680, 22960, and so on [2].

Properties of EPRNs

Some of the properties of EPRNs are:

- For an EPRN, the sum of digits of any factor in the first set is equal to the sum of digits of any factor in the second set. For example:
  144648 = 861 × 168 = 492 × 294, and 8 + 6 + 1 = 4 + 9 + 2 = 15.
  9949716 = 2583 × 3852 = 1476 × 6741, and 2 + 5 + 8 + 3 = 1 + 4 + 7 + 6 = 18.
- For an EPRN, the sum of squares of digits of any factor in the first set is equal to the sum of squares of digits of any factor in the second set. For example,
  144648 = 861 × 168 = 492 × 294, and $8^2 + 6^2 + 1^2 = 4^2 + 9^2 + 2^2 = 101$.
  9949716 = 2583 × 3852 = 1476 × 6741, and $2^2 + 5^2 + 8^2 + 3^2 = 1^2 + 4^2 + 7^2 + 6^2 = 102$.
- From a known EPRN, a new EPRN can be generated by inserting a zero between each consecutive pair of digits of each factor of each set of numbers.
  For example, EPRN 1256895612 can be generated from EPRN 185472 as shown below:
  185472 = 672 × 276 = 384 × 483,
  60702 × 20706 = 1256895612 = 30804 × 40803.

The above properties can be proved easily [3].

- EPRNs with 1- or 2-digit factors cannot exist.
- The squares of numbers in the sequence 2772, 27972, 279972, 2799972, 27999972, etc., are all EPRNs. So, the number of EPRNs is infinite.
- The digital root (i.e., the ultimate sum of digits until a single digit is obtained) of all EPRNs is always 1, 4, 7, or 9. For example, the digital root of 144648 is 9, as 1 + 4 + 4 + 6 + 4 + 8 = 27, and 2 + 7 = 9. This is obvious, as EPRNs are products of two numbers having the same digital roots, so the digital roots of EPRNs can only be the digital roots of perfect squares.

Some Interesting Observations

Some of the interesting observations about EPRNs are

- Only six numbers, i.e., 2520, 4030, 5740, 7360, 7650, and 9760, are known among the four-digit EPRNs. All other EPRNs consist of five or more digits. For example, some of the five-digit EPRNs are 10080, 12070, 13000, 14580, 14620, 17290, 63504, etc.
- All known four-digit EPRNs are multiples of 10, and all known five-digit EPRNs except 63504 are also multiples of 10. These can be obtained by multiplying the product of two different reversible numbers by 10, with the condition that none of the two reversible numbers be a multiple of 10. For example, 12 × 21 × 10 = 2520, 13 × 31 × 10 = 4030, 14 × 41 × 10 = 5740, etc., give EPRNs, but 10 × 01 × 10 = 100, 20 × 02 × 10 = 400, etc. are not EPRNs, as one of the reversible numbers is 10, 20, etc., which is a multiple of 10. The smallest EPRN that is not a multiple of 10 is 63504, which is equal to 441 × 144 = 252 × 252, where 252 is a palindromic number, so its reverse is the same as the original number. All other numbers in this category (i.e., those that are not multiples of 10) consist of six or more digits. For example, 101556, 144648, 185472, 5955264, 6794424, 7683984, 8856036, 9523696, 9949716, 11093796, 11807208, etc. [1][2]. The EPRNs, which are non-multiples of 10, can be termed as fundamental EPRNs.
- Some of the EPRNs are perfect squares [1][2]. For example, 63504, 7683984, 782432784, etc., as shown below:

  $63504 = 252^2 = 441 \times 144$,

  $7683984 = 2772^2 = 1584 \times 4851$, etc.

  The examples of EPRNs shown above are perfect squares when they are the squares of palindromic numbers. However, there are EPRNs, which are perfect squares but not the square of a palindromic number. For example:

  $435600 = 660^2 = 582 \times 825 = 6600 \times 0066$,

  $6350400 = 2520^2 = 14400 \times 00441 = 25200 \times 00252$, etc.

Computation of EPRNs

PORERОP Pairs: PORERОP stands for **P**roduct **O**f **R**everses **E**quals **R**everse **O**f **P**roduct. A pair of numbers (A, B) where $A \leq B$ can be defined as a porerop pair if the product of the reverses of A and B is equal to the reverse of the product of A and B, i.e., $\overline{A} \times \overline{B} = \overline{A \times B}$, where $\overline{A}$ and $\overline{B}$ denote the numbers obtained from A and B, respectively, by reversing their digits. $\overline{A \times B}$ denotes the reverse of the product of A and B. For a pair (A, B) to be a porerop pair, neither A nor B shall be a palindromic number. For example, the pair (13, 23) is a porerop pair because

$13 \times 23 = 299$ and $31 \times 32 = 992$, so $\overline{A} \times \overline{B} = \overline{A \times B}$. Similarly, the pair (10, 12) is a porerop pair because 10 × 12 = 120 and 01 × 21 = 021, so $\overline{A} \times \overline{B} = \overline{A \times B}$. However, (22, 31) is not a porerop pair as 22 is a palindromic number; though 22 × 31 = 682 and 22 × 13 = 286, satisfying $\overline{A} \times \overline{B} = \overline{A \times B}$.

Porerop pairs are very useful in generating EPRNs [1], as follows:

If a porerop pair (A, B) is such that (A, $\overline{B}$) or ($\overline{A}$, B) is also a different porerop pair, then $A \times B \times \overline{A} \times \overline{B}$ gives an EPRN. For example, (12, 13) is a porerop pair such that (12, 31) is another porerop pair, so 12 × 13 × 21 × 31 gives an EPRN 101556, as shown below:

101556 = (12 × 13) × (21 × 31) = (12 × 31) × (21 × 13), which is 156 × 651 = 372 × 273.

(12, 24) is a porerop pair such that neither (12, 42) nor (21, 24) is a porerop pair, so 12 × 24 × 21 × 42 = 254016 is not an EPRN, as shown below:

254016 = (12 × 24) × (21 × 42) = (12 × 42) × (21 × 24), which is 288 × 882 = 504 × 504, where 504 is not a reversible number.

A porerop pair (A, B) can be termed a fundamental porerop pair if neither A nor B is a multiple of 10. All fundamental porerop pairs where A and B are 2-digit numbers are (12, 12), (12, 13), (12, 14), (12, 21), (12, 23), (12, 24), (12, 31), (12, 32), (12, 41), (13, 13), (13, 21), (13, 23), (14, 21), (21, 21), (21, 23), (21, 31), (21, 32), (21, 41), (21, 42), (31, 31), and (31, 32). Out of these 21 fundamental porerop pairs (A, B), only 15 pairs generate EPRNs. These are (12, 12), (12, 13), (12, 14), (12, 21), (12, 23), (12, 31), (12, 32), (12, 41), (13, 21), (14, 21), (21, 21), (21, 23), (21, 31), (21, 32), and (21, 41). Six pairs, i.e., (12, 24), (13, 13), (13, 23), (21, 42), (31, 31), and (31, 32), cannot generate EPRN numbers because (A, $\overline{B}$) or ($\overline{A}$, B) is not a porerop pair.

The number of fundamental porerop pairs is infinite, as zero can be inserted in any one or both of the numbers of the pair. For example, (102, 12), (102, 103), (102, 104), etc., are all fundamental porerop pairs.

A necessary condition for a porerop pair (A, B) is

The sum of digits of A × The sum of digits of B = The sum of digits of (A × B).

So, if this condition is not satisfied, then the pair (A, B) cannot be a porerop pair. For example, (12, 34), (12, 43), (13, 14), and (13, 24) are not porerop pairs.

There are 1634 fundamental EPRNs below 10^{12} [1]. It shall be noted that all EPRNs cannot be computed from porerop pairs. Can you find the EPRNs that cannot be computed using porerop pairs and devise a methodology to compute all EPRNs?

Par Numbers:

Amos Voil [4] presented a set of "Square Reversals," i.e., numbers such that the squares of their reverses equal the reverses of their squares. For example,

$12^2 = 144$ and $441 = 21^2$
$13^2 = 169$ and $961 = 31^2$

Father Victor Feser [5] reported that the key to such square reversals is that there is no carrying. He reported that there can be no solution containing both 2's and 3's and none containing more than one 3. Martin Gardner [6] termed such numbers as **par numbers** and mentioned the following observations about par numbers:

(i) No par number can contain digits other than 0, 1, 2, 3, and that a number n is par if and only if the sum of its digits is the square root of the sum of the digits of n^2.

(ii) Since numbers in the series 12, 102, 1002, 10002, etc., and the series 13, 103, 1003, 10003, etc., are all pars, the set of pars is infinite.

(iii) A fundamental par is defined as one with no 0's and includes palindromic pars, the set has 55 members, the smallest is 1, and the largest is 111111111. Eliminating the trivial palindromes, the fundamental pars are the following twenty numbers and their reversals:

12, 13, 112, 113, 122, 1112, 1113, 1121, 1122, 1212, 11112, 11113, 11121, 11122, 111112, 111121, 111211, 1111112, 1111121, 1111211.

From the 40 fundamental par numbers mentioned above, 820 pairs (X, Y) can be formed, where X and Y are fundamental par numbers such that $X \leq Y$.

Out of 820 pairs, it is found that 772 pairs are porerop pairs and 48 pairs are non-porerop pairs. For example, (12, 12), (12, 13), (12, 21), (12, 31), (13, 13), (13, 21), (21, 21), (21, 31), (31, 31), … are porerop pairs, and (13, 31), (13, 311), (13, 3111), … are non-porerop pairs.

It is interesting to note that if A = B in a porerop pair (A, B), then A and B are par numbers. For example,

The porerop pairs (12, 12), (13, 13), (21, 21), and (31, 31), where A = B, give par numbers 12, 13, 21, and 31. So, the par numbers defined by Martin Gardner [6] are a special case of porerop pairs.

From the methodology discussed earlier, EPRNs can be generated from the 714 pairs out of 772 porerop pairs found from fundamental par numbers. For example, the pair (12, 12) generates EPRN, whereas the pair (13, 13) cannot generate EPRN, as shown below:

The pair (12, 12) is a porerop pair such that (12, 21) is another porerop pair, so $12 \times 12 \times 21 \times 21$ gives an EPRN 63504 as shown below:
$63504 = (12 \times 12) \times (21 \times 21) = (12 \times 21) \times (21 \times 12)$, which is $144 \times 441 = 252 \times 252$.

Similarly, (13, 13) is a porerop pair such that neither (13, 31) nor (31, 13) is a porerop pair, so $13 \times 13 \times 31 \times 31 = 162409$ is not an EPRN, as shown below:
$162409 = (13 \times 13) \times (31 \times 31) = (13 \times 31) \times (31 \times 13)$, which is $169 \times 961 = 403 \times 403$, where 403 is not a reversible number.
Can you characterize the 58 porerop pairs found from fundamental par numbers, which cannot generate EPRNs?

EPRNs of Higher Degrees

EPRNs, as defined earlier, are basically EPRNs of degree 2 and can be denoted as 2-EPRN. So, 2-EPRNs are the numbers that can be expressed as the product of a number and its reversal in at least two different ways. Let's define an EPRN of degree k, i.e., a k-EPRN, as the number that can be expressed as the product of a number and its reversal in at least k different ways. For example, numbers that can be expressed as the product of a number and its reversal in at least three different ways are 635040, 1015560, 1446480, 1854720, 4356000, 6350400, 10155600, 14464800, 18547200, 43560000, 51665040, 59552640, and so on [1][2].
The smallest 3-EPRN can be expressed as $635040 = 1440 \times 411 = 2520 \times 252 = 4410 \times 144$.

If multiples of 10 are excluded, then the smallest fundamental 3-EPRN, as it can be called, is $1115892288 = 13248 \times 84231 = 23184 \times 48132 = 27504 \times 40572$.

It may be noted that 3-EPRNs are also included in 2-EPRNs, as 3-EPRNs can be expressed as the product of a number and its reversal in two different ways. So, the sequence of k-EPRNs is a subsequence of (k-1)-EPRNs for k > 2.

Similarly, the smallest k-EPRNs for k = 4 to 8 are 1015560, 10119126106147652568, 11158922880, 5918858243794044864, and 1046458990080, respectively [1][2].

Conclusion

The concept of porerop pairs to compute EPRNs has been discussed in the paper. There is a need to characterize EPRNS, which cannot be computed using porerop pairs. There is also a need to devise a suitable method to compute all EPRNs up to a given limit.

It can be interesting to investigate EPRNs of higher degrees and, at the same time, find faster ways to compute all k-EPRNs up to a given limit.

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