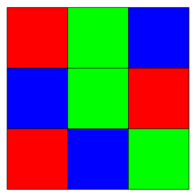
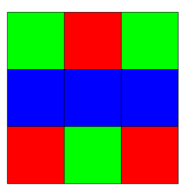
A Sudoku Checking Problem

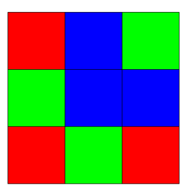
This item is a math problem related to the well-known number puzzle Sudoku. A solved Sudoku puzzle is a 3×3 array of blocks where each block is a 3×3 array of digits 1 through 9. The requirement is that each row, column, and block contain all nine digits. When I finish solving a Sudoku puzzle I like to do a quick check that the requirements are indeed satisfied. Checking rows, columns, and blocks takes more time than I care to spend, and a lot of that work is redundant. I find that checking just the blocks is enough. While it is possible to create examples where the blocks check but rows and/or columns do not, these examples do not arise in practice for me.

How do I check a block quickly and efficiently? I look at the block and find $1, 2$, and 3, noticing their positions within the block. I do the same for $4, 5$, and 6 and finally $7, 8$, and 9. My eye/brain combination seems to be able to find three numbers at a time but not nine. Having done a lot of these I begin to notice patterns in the locations of the members of each group of three. Giving a color to each group of three digits produces a pattern like this. It doesn't matter which color goes with which group.

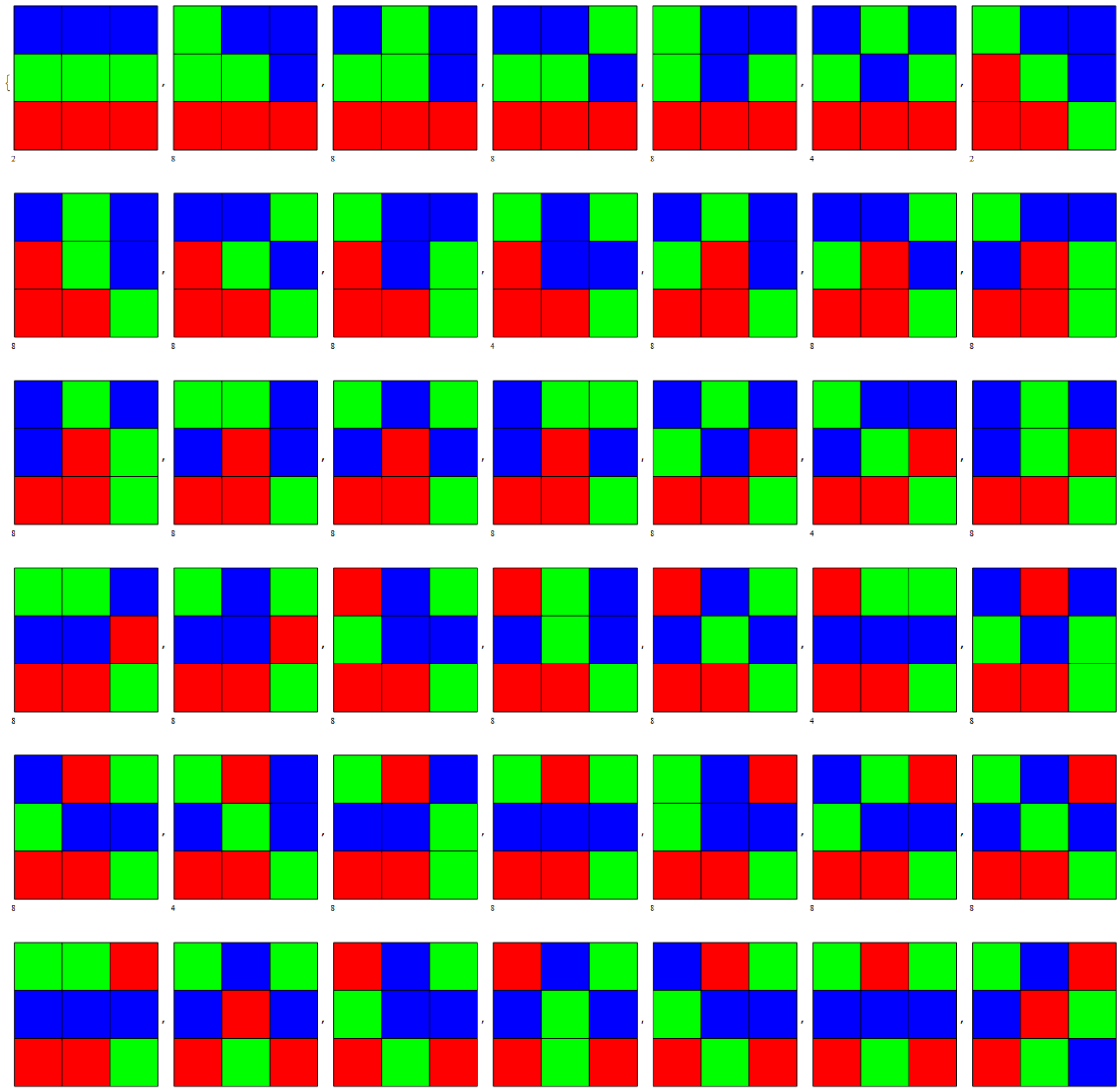


How many different patterns are there? There are $\binom{9}{3\;3\;3} = 1680$ ways to color a block. I consider colorings to be equivalent if they differ only by a combination of rotations, reflections, and color changes. Colors can be permuted 6 ways, and the group (D_4) of reflections and rotations has order 8. The pattern illustrated above represents $6 \times 8 = 48$ of the 1680 colorings because all of its rotations and reflections are different, but some patterns such as the two examples shown below have symmetries, and only 4 or 2 different rotations/reflections, so they represent only 24 or 12 of the 1680 colorings. The first example pattern is unchanged by a reflection in the y-axis, while a reflection in the x-axis only swaps colors red and green. The second is unchanged by a reflection in the $x = y$ diagonal.





The net result is that there are only 42 essentially different patterns, 30 with 48 colorings, 8 with 24 colorings, and 4 with 12 colorings. They are pictured on the following page. The number of the different rotations/reflections for each of the 42 is given below its picture. These numbers are divisors of 8, and they sum to $1680/6 = 280$. I found the 42 patterns by computer search. Burnside's Lemma could be used, but it doesn't make the counting any easier. My challenge to the G4G group is to find a more elegant method to show that 42 the correct number.



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