

A New Twist on an Old Number Divination Trick

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Background. In Martin Gardner's books *New Mathematical Diversions* and *Aha! Insight*, he describes two mathematical magic tricks that are based on a simpler, well-known number-guessing trick. Here, we describe the classic number-guessing trick from which Gardner drew his inspiration and provide a new generalization that allows the magician to divine nearly twelve times the number of values as the original.

Classic Illusion.

The Effect. A magician asks a volunteer to think of a special date of the calendar year, perhaps a birthday or anniversary. Calendar days come in month-date pairs, and the magician asks the volunteer to focus on the date number 1 to 31. The performer then presents the volunteer with five cards, each containing some of these numbers and asks them to find every card that contains their number. After the volunteer identifies all cards with the selected number, the magician immediately reveals the volunteer's secret number.

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17	19	21	23
25	27	29	31

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18	19	22	23
26	27	30	31

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Updated Illusion.

The Effect. A magician asks a volunteer to select any day of the calendar year 2019, perhaps focusing on an important day of the year such as an anniversary or birthday. The magician presents the volunteer with six calendars (provided at the end of this packet). Every day of the year on each calendar is colored red, green, or left uncolored, and the magician gives the volunteer red and green chips to identify the color of the chosen day on all calendars by placing the marker on the calendar. Once the volunteer has marked the calendars, the magician divines the volunteer's special day.

How it Works: The Classic Illusion.

When the magician is presented the subset of five cards, he simply adds the numbers in the top, left-hand corners to find the value of the secret number. For example, if the volunteer selects the cards

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17	19	21	23
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16	17	18	19
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the magician finds the sum $1 + 2 + 16$ and reveals the secret number is 19.

Why it Works: The Classic Illusion.

The trick works by exploiting the nature of the binary number system. This system is special because every nonnegative integer can be uniquely represented by a sum of distinct powers of 2. For example, there is exactly one way to write 2026 as a sum of distinct powers of 2, namely

$$2026 = 2 + 8 + 32 + 64 + 128 + 256 + 512 + 1024.$$

Each power of 2 less than 2026 is either included in the sum or excluded from it; 1, 4, and 16 are the powers that are excluded. To create the cards used in the trick, each number n from 1 to 31 was written in binary and if n required a certain power of 2 in its binary representation, the number n was placed on the card containing that power. For example, $20 = 16 + 4$, so 20 is on the cards whose top-left corners are 16 and 4, but no others.

How it Works: The Updated Illusion.

Days on each calendar are colored in groups of 1, 3, 9, 27, 81, or 243, all powers of 3. Suggestions for performing the trick are provided later including a suggestion for how to identify the power of 3 associated with each calendar. The date is identified in a two-step process. First, the magician first uses the volunteer's marked calendars to calculate a number between 0 and 364, inclusively. This will identify how many days into the year the date falls, where January 1 is day 1. If a green marker is placed on a calendar associated with a given power of 3, the magician *includes* (add) this power in the final total. If a red marker is placed on a calendar, the magician *excludes* (subtracts) the associated power of 3 from the running total. If no marker is placed, the magician disregards the card. The final calculated number is the day of the year corresponding to the chosen date.

The second step is to find the date of the year associated with the calculated number. To find it, the magician divides the calculated value n by 30 to give a rough estimate of the month of the secret date. Suppose $n = 30q + r$, where $0 \leq r < 30$. The magician adds 1 to the quotient to estimate the corresponding month (e.g., if $q = 6$, the date is likely in July since month $6 + 1 = 7$ is July). The remainder is used to count the number of "extra" days into the month. We provide two options for identifying the date and describe how to use them in the example below. One method uses a year-at-a-glance calendar (provided) and the other requires memorizing adjustments for each month that are then added to the remainder to identify determine the exact day of the month.

If no color is identified on any of the cards, the date is December 31.

Why it Works: The Updated Illusion.

Just as every positive integer can be written in binary, it can also be written using the *balanced ternary system*. Ternary refers to a base 3 system just as binary refers to base 2. We use the term "balanced" because if a number n is represented in the balanced ternary system, we can imagine an object of mass n resting in the left pan of a balance scale. Weights of mass 1, 3, 9, 27, ... are available to place into *either* pan until the scale is in equilibrium/balanced. Weights in the right pan are counted as positive, those in the left with the object as negatives. For example, we can determine the mass of a 47-gram object by placing it in the left pan, placing 3-gram and 81-gram weights in the right pan and weights of mass 1, 9, and 27 in the left pan. The mass in the left pan is $47 + 1 + 9 + 27 = 84$ and in the right, $3 + 81 = 84$, so the scale balances. This demonstrates that

$$47 = 81 - 27 - 9 + 3 - 1.$$

In general, every positive (and negative!) integer can be written as a sum of combinations of powers of 3 where we add, subtract, or ignore each power of 3. In essence, every integer is a combination of powers of 3 where the multipliers are +1, -1, or 0.

Example: The Updated Illusion.

Suppose the volunteer chooses Martin Gardner's birthday, October 21. This date is green on the 81 and 243 cards and red on the 3 and 27 cards.

Step 1: Calculate the number of days into year. The sum associated with the green values is $81 + 243 = 324$ and the sum associated with the red values is $3 + 27 = 30$, so the magician calculates the number $n = 324 - 30 = 294$. This tells us that secret date is the 294th day of the year. Dividing 294 by 30, the quotient is 9 and the remainder is 24. Adding 1 to 9, we are likely in the tenth month.

Step 2 Version 1 – Year-at-a-glance calendar. Find October (month 10) on the provided year-at-a-glance calendar and count 24 days into the 30-day span of days next to the month. There are fifteen calendar days in each row for ease of finding the secret day and the fifth and eleventh columns are inverted in color to help visually divide each row into thirds. Since the remainder is 24, we are in the second October row and be in column $24 - 15 = 9$ of this row, which is the 21st.

28	29	30	1	2	3	4	5	6	7	8	9	10	11	12	OCT
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	

Step 2 Version 2 – Utilizing a memorized set of remainder adjustments. If the magician prefers to rely on memory over using the year-at-a-glance calendar, they can memorize the table below to help the magician adjust the remainder by the appropriate number of days depending on the month (or near-month) determined by the quotient. Instead of memorizing these adjustments, they could devise another convenient way to retrieve them. These values are added the remainder to find the true date. For example, for Martin Gardner's birthday, since we are in or near October, the table indicates we subtract 3 from our quotient of 24 to determine the day of the month. These adjustments are necessary because of 31-day months and the shorter February. Occasionally, it is necessary to dip back into a prior month. For example, if the calculated day is 302, the quotient upon dividing by 30 is 10, which indicates a November day. However, because the adjustment for November is -4 , we must subtract 4 from the remainder of 2, yielding -2 . This means the secret day is 2 days *before* the last *last* of October, thus October 29.

Jan = +0	Feb = -1	Mar = +1	Apr = +0	May = +0	Jun = -1
Jul = -1	Aug = -2	Sep = -3	Oct = -3	Nov = -4	Dec = -4

Mathematical Considerations.

It might be natural to ask why to use the balanced ternary system instead of the traditional base-3 system to perform the trick. First, in the traditional ternary system, the multipliers for the powers of 3 are 0, 1, and 2, so we may be required to double numbers mentally and that would slow the calculation process. Moreover, with the balanced ternary system, we are dealing with numbers that end in 1, 3, 7, or 9, thus simplifying the calculations – we need be add 81 and subtract 1, simplifying to 80, or add 243 and 27 for a simpler 270. The second reason for using the balanced ternary system is that with these six cards, the maximum sum that be created is $1 + 3 + 9 + 27 + 81 + 243 = 364$, one shy of the number of days in a year! Including 0, we get the exact number of days, which is why if no card is marked either green or red, which would indicate a sum of 0, we identify the selection with December 31.

Tips for Performing the Updated Illusion.

In the original binary trick, the numerical reminders, 1, 2, 4, 8, and 16 all appear in the top corner. For the updated illusion, there is no clear number in each corner to identify the card “values.” The number associated with each card can be determined by the size of the groups of like colors. For the card associated with 9, for example, there are nine consecutive days colored green, followed by nine red days, then nine uncolored days. This is slightly longer than a week. The card associated with 27 nearly has entire months colored green or red (April, May, and December are good examples). The card for 81 has only three large groups of colors, two green, one red, and the card for 243 has no red. An ambitious magician could recreate these calendars with an image on the obverse side that could be associated with the number in some way; a triangle for 3, a baseball team for 9 since there are nine players, and so forth. Another way to keep track of the values of the cards is to have them spread out on the table in a predetermined order.

As mentioned earlier, when doing calculations, the powers of 3 end in 1, 3, 7, or 9, so exploit sums or difference that result in a multiple of 10. For example, if the 3 and 27 are both red as in the example above, since their sum is 30, this is a simpler calculation than attempting to add and subtract the powers of 3 from largest to smallest. Or, if a red marker is used for the 81 card and a green for the 1 marker, keep $1 - 81 = -80$ tucked away as a simpler calculation.

An alternative to having some sort of marker or chip to identify the calendars with green or red dates would be to have two folders, one green, one red, and instruct the volunteer to place the calendar in the respective folder.

JANUARY

S	M	T	W	T	F	S
	1	2	3	4	5	
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

FEBRUARY

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					8	9
3	4	5	6	7		
10	11	12	13	14	15	16
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MARCH

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3	4	5	6	7		
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APRIL

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	1	2	3	4	5	6
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MAY

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JUNE

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JULY

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AUGUST

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SEPTEMBER

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OCTOBER

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NOVEMBER

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DECEMBER

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JANUARY

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SEPTEMBER

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NOVEMBER

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DECEMBER

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JANUARY

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DECEMBER

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JULY

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OCTOBER

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MARCH

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APRIL

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MAY

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JUNE

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JULY

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AUGUST

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SEPTEMBER

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29	30					

OCTOBER

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27	28	29	30	31		

NOVEMBER

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DECEMBER

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8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

