Gathering for Gardner 12345678910

G4G10 Exchange Book

Art, Games, & Magic

Atlanta, Georgia

MARCH 28 – APRIL 1, 2012

VOLUME 1

G4G10 Exchange Book
Volume 1

The Gift Exchange is an integral part of the Gathering 4 Gardner biennial conferences. Gathering participants exchange gifts, papers, puzzles, and other interesting artifacts. This book contains gift exchange papers from the conference held in Atlanta, Georgia, from Wednesday, March 28th through Sunday, April 1st, 2012. It combines all of the papers offered as exchange gifts in two volumes.

Acknowledgments

Gathering 4 Gardner would like to offer thanks to the following individuals:

Axel Holm – for laying out the pages of this book;
Katie LaSeur – for project management;
John Miller – for helping manage the papers and following up with authors;
Nancy Blachman – for getting this book started.

There are many things you can buy or make yourself, but there are some things that can only be created by a group. This book is such an item.

All papers appear unedited, exactly as they were submitted.

G4G10 Preface

An aura of magic permeates a Gathering 4 Gardner. It's not just the presence of magicians, eager to display amazing feats of sleight of hand and sleight of mind. It's the pervasive spirit of ferocious creativity and antic playfulness among all the participants, whether magician, mathematician, artist, writer, inventor, engineer, scientist, toymaker, or puzzle master, that makes a Gathering such an enchanting and exhilarating experience.

Numbering in the hundreds, the members of this potent jumble are there to honor and remember Martin Gardner, whose many writings, particularly on recreational mathematics and magic, have had such a profound and lasting influence on their lives.

The contributions published here underscore the diversity of participation and thought at the tenth Gathering 4 Gardner, held in Atlanta in 2012. They encapsulate the spirit of enthusiastic sharing so characteristic of a Gathering. Where else could one find origami instructions for crafting a hyperbolic crane or folding Martin Gardner, outrageously punny cartoons, a recipe for chocolate chip pi, or rules for a game based on the chemical element Seaborgium, along with the inside scoop on a ten-card magic spell, all in one place?

These offerings, however, represent only a part of the experience of attending a Gathering 4 Gardner. Just as important are the chance encounters, enthralling conversations, spontaneous collaborations, and bouts of impromptu puzzle posing and solving that spark new ideas and inventions. Undoubtedly, future volumes of Gathering proceedings will offer glimpses of these newly inspired bursts of creativity.

The experience also goes well beyond merely listening to formal presentations in the packed conference hall: a roomful of giddy participants gamely tossing colored handkerchiefs in an attempt to learn the rudiments of juggling; a famed mathematician deftly performing card tricks in an impromptu corridor session; the group construction of an intricate mathematical structure during an afternoon picnic; an artist recounting the passion and process that led to a work on display in the art exhibit.

Martin Gardner died on May 22, 2010, at the age of 95. Though he practically never attended the Gatherings held in his honor, his spirit pervaded the events, and they remain very much a part of his legacy. They speak to his broad interests as an intrepid explorer of ideas and to his quiet generosity and whimsical nature.

Ivars Peterson

http://mathtourist.blogspot.com

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Date: Thursday, 29/Mar/2012

8:30am - 10:00am

ThuAM1: Thursday AM Early
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Presentations

What to do with a rubber band
Hart, George

Update on North America's only Museum of Mathematics
Lawrence, Cindy

Flexible Polyforms
Muniz, Alexandre

The Whimsical Side of Martin Gardner
Sonenshein, Charles

10 flexes on a flexagon
Sherman, Scott

Ponder-This
Margalit, Oded

It's Not Music, It's Theory
Orman, Hilarie

Clouds in My Coffee / Pattern Recognition or Wake up & Photo Your Coffee!
Goldklang, Lew

10 MatheMagics for G4G10
Kauffman, Louis

Limited placement of Polyominoes
Golomb, Solomon W.

Celebration of Mind Party for Chicago
Railing, Max

10:30am - 12:00pm

ThuAM2: Thursday AM Late
Location: **Ritz Carlton Large Meeting Room**

Presentations

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Brecher, Kenneth

The Cover and Column of the Feb 1971 Scientific American
Smith, Alvy Ray

Fruitloopery
Crease, Robert

Virtual Mechanical Puzzles
van Grol, Rik

Organising a successful national math week
Gill, Eoin; Donegan, Sheila

"Go First" Dice
Harshbarger, Eric

Fractal graphs by iterated substitution
Segerman, Henry

Let's play a concentration game with a few cards
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Burstein, Mark

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ThuPM1: Thursday PM Early
Location: **Ritz Carlton Large Meeting Room**

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Hollingsworth, Blane

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Fisher, Gwen Laura

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Harriss, Edmund

Truchet Tile Mosaics
Bosch, Robert

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Glass, Darren

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Esterle, Richard

Computing and the Art of the Letter X
Szczepanski, Amy

Stereographic Photography and Numerology
Torrence, Bruce

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Yackel, Carolyn

What I Learned From Hanging Out With Creationists
Rosenhouse, Jason

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ThuPM2: Thursday PM Late
Location: **Ritz Carlton Large Meeting Room**

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16 is Not Enough: The Minimum-Clue Conjecture for Sudoku
Taalman, Laura

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Blasco, Fernando

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Gosper, R. William; Ziegler Hunts, Julian

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Moulton, David

Teaching Bright Kids Math Using Hackenbush
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Generic Numerical Challenges
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Date: Friday, 30/Mar/2012

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FriAM1: Friday AM Early
Location: **Ritz Carlton Large Meeting Room**

Presentations

Ten Tetrahedra Emerge
Torrence, Eve

Space-Filling Curves with Space-Filling Borders
McKenna, Doug

Have a "GO" at "LIFE"
Elran, Yossi

Imaginary Cube Puzzles.
Tsuiki, Hideki

Ten Piece Icosahedron Puzzles
Bell, George

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Atkinson, Adam

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McLean, Thomas Bruce

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Miller, John

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Schindler, Jay; Hurd, Lyman

Progressively More Difficult Polyomino Trays
Waite, William

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FriAM2: Friday AM Late
Location: **Ritz Carlton Large Meeting Room**

Presentations

Powers of Ten
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Sandfield, Robert

Sleights of Mind
Macknik, Stephen

What the Neuroscience of Magic Reveals About our Brains
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Barnhart, Anthony

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FriPM1: Friday PM Early
Location: **Ritz Carlton Large Meeting Room**

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Location: **Ritz Carlton Large Meeting Room**

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Jespersen, Bjarne

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Manderscheid, Roger

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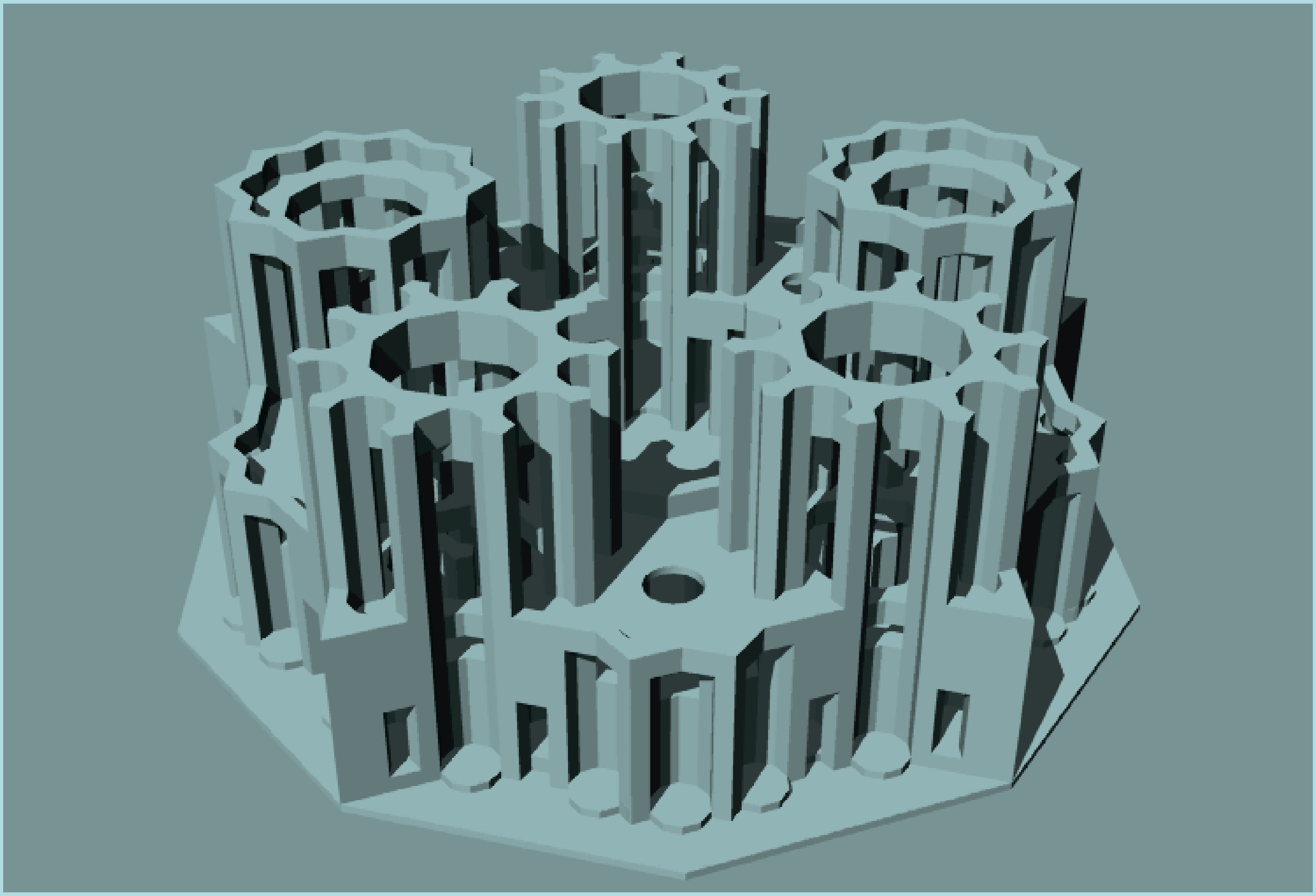
ART



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Constructing the Dodecahedral Trail

by
Neil Calkin
Clemson University
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On November 19, 2010, I attended a talk by Lee Ann Brown, opening a workshop celebrating the 50th anniversary of Oulipo, held at the University of North Carolina at Asheville. In this talk she mentioned that she is fascinated by dodecahedra, as well as by constrained poetry.

Shortly thereafter, I composed this piece describing how one might walk around the vertices of a dodecahedron, selecting words written on the edges of the dodecahedron to use in a constrained poem.

Walking The Dodecahedral Trail

Words on an edge,
Walking the dodecahedral trail.
Encounter vertices, nodes.
Make a choice,
To turn left or right,
Writing down discoveries,
Surprise conjunctions,
And finding once only
Read poems.

There are thirty distinct words in this piece, corresponding to the thirty edges of a dodecahedron.
I then constructed a PHiZZ units dodecahedron (see Tom Hull's lovely book Project Origami), and wrote one word on each unit. I used three units of each of yellow, green and orange, and randomly constructed the dodecahedron from the units, subject to the constraint that any two of the colours form a hamilton cycle through all the vertices (this is the solution to the original problem posed by Hamilton in 1859).

Next, I randomly selected an edge, the word "right", and a direction along which to traverse it: and then I wrote down a sequence of choices to make, RLRL LRLR RLRL. I followed this from the edge "right", turning right or left as the sequence indicated, obtaining the following sequence of words:

Right
Surprise
Turn
Read
Discoveries
On
Poems
Walking
To
Dodecahedral
Choice
Once
Left

I decided to use these as the first words of each line of a poem, and composed the following.

Right Surprised

Right surprised we are, we are,
Surprised as hell, surprised as well.
Turn the eye to see afar,
Read the words the ball will tell.
Discoveries astound us still:
on finding ideas in the air,
Poems emerge from hidden will.
Walking, turning, here to there,
To wander, maybe to explore
Dodecahedral skeletal ball.
Choice is made, and made once more:
Once made, it cannot change at all
Left or right until the end.

As a final observation, note the rhyming scheme is such that the final phoneme of each line follows the pattern
RLRL LRLR RLRL
so that the poem itself encodes the directions in which I turned to obtain the words.

Folding Martin Gardner

Erik D. Demaine* Martin L. Demaine*

The 22″ × 28″ print shown on the next page ("Martin Gardner", Rives paper, 2012) is part of the *!AHA! Art exhibition with a magical mathematical twist*, presented by the Gathering 4 Gardner in partnership with the Atlanta–Fulton Public Library System, March 19–31, 2012.

The crease pattern on top of the print folds into two forms of Martin Gardner, as shown on the bottom of the print (not to scale). The rectangular paper sheet folds into the 3D structure of the words MARTIN GARDNER, as shown in Figure 1, while the grayscale inking in the sheet (top) forms the photograph of Martin Gardner in the background (bottom).

Martin Gardner taught us to look at everyday things from different perspectives, in particular through mathematics. We decided to look at Martin Gardner in different ways, using mathematics as our toolset.

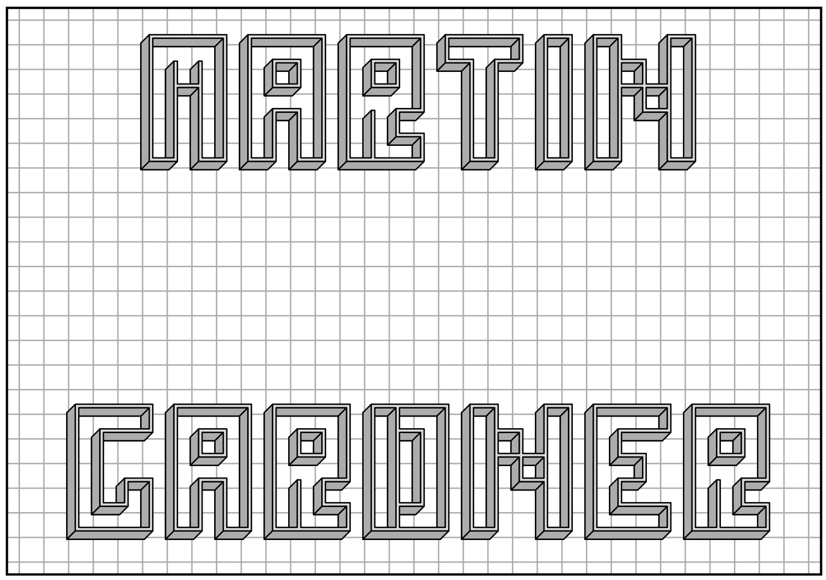


Figure 1: The 3D shape of the folding.

The crease pattern was designed using an algorithm by Demaine, Demaine, and Ku [DDK10a, DDK10b], which describes how to efficiently fold any orthogonal "maze" (including word outlines like MARTIN GARDNER) from a rectangle of paper. Red lines fold one way and blue lines fold the other way. To experiment with other designs, try our Maze Folder or read our papers on the web: http://erikdemaine.org/maze/

Given the complexity of the crease pattern, we expect it never to be folded. If you want to try your hand at it, though, you can download and print the crease pattern from the web: http://erikdemaine.org/prints/MartinGardner/

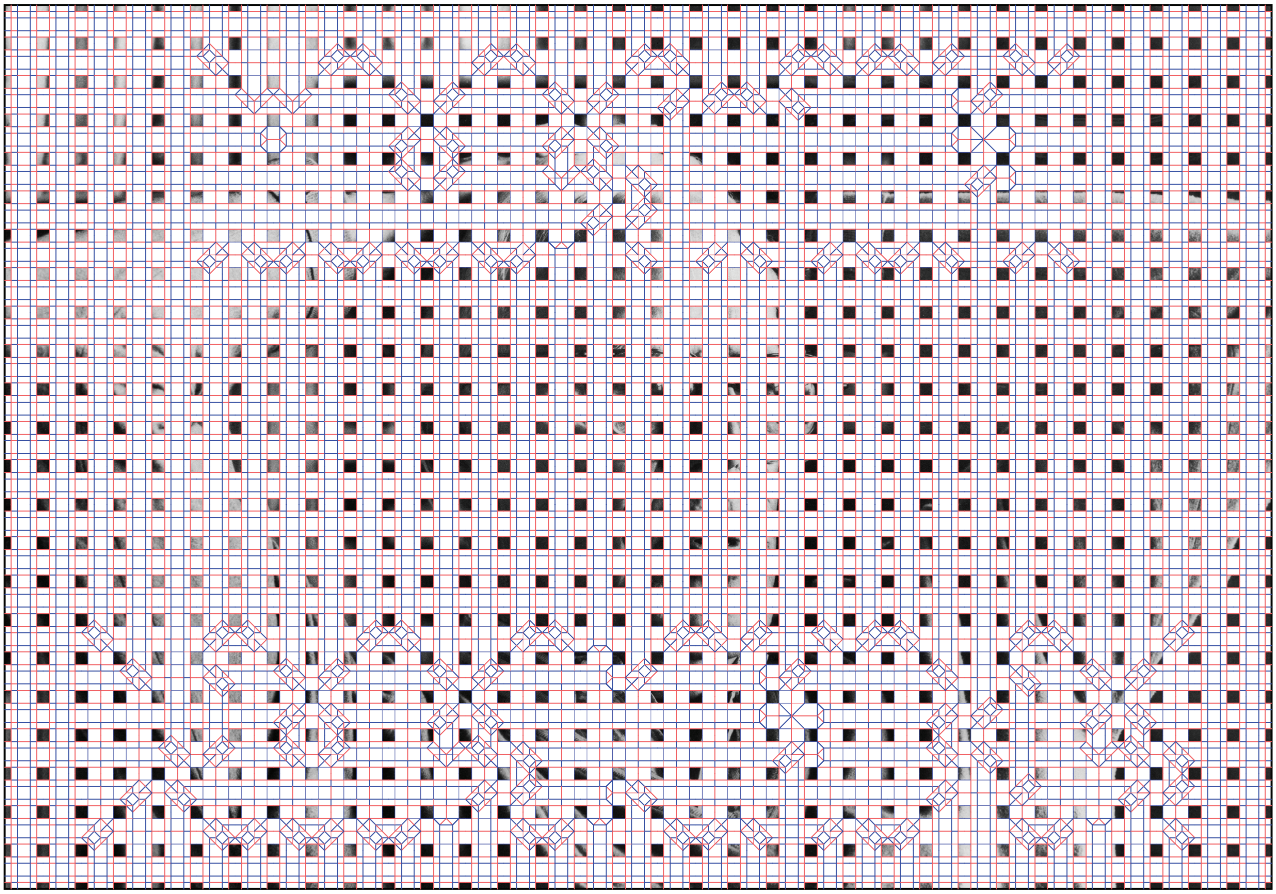
The photograph of Martin Gardner is from the Archives of the Mathematisches Forschungsinstitut Oberwolfach, and used with permission.

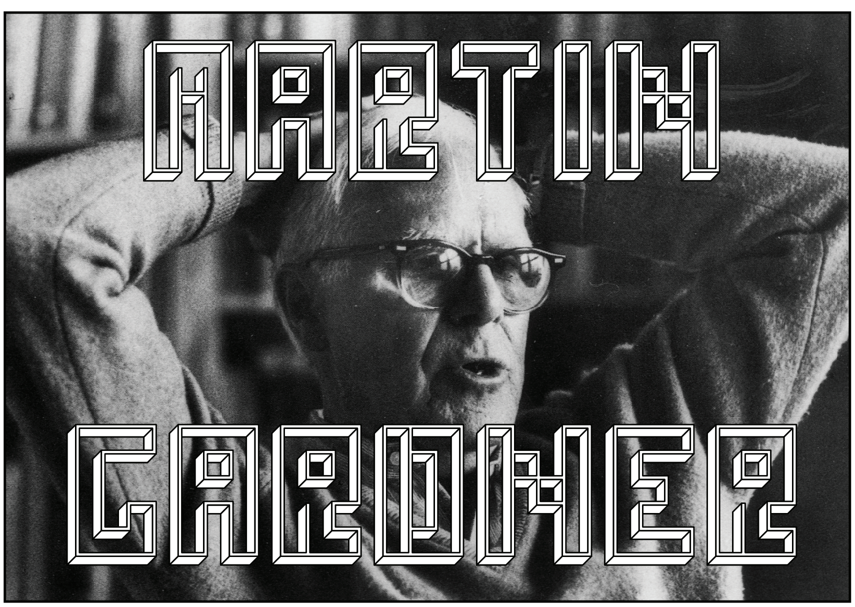
References

[DDK10a] Erik D. Demaine, Martin L. Demaine, and Jason Ku. Folding any orthogonal maze. In *Origami*^{5}*: Proceedings of the 5th International Conference on Origami in Science, Mathematics and Education (OSME 2010)*, pages 449–454. A K Peters, Singapore, July 2010.

[DDK10b] Erik D. Demaine, Martin L. Demaine, and Jason Ku. Origami maze puzzle font. In *Exchange Book of the 9th Gathering for Gardner*, Atlanta, Georgia, March 2010. To appear.

*MIT Computer Science and Artificial Intelligence Laboratory, 32 Vassar St., Cambridge, MA 02139, USA, {edemaine,mdemaine}@mit.edu





Folding the crease pattern on top (red lines mountain, blue lines valley) brings together the photograph, while simulataneously forming the 3D shape of the letters MARTIN GARDNER, shown below.
photograph used with permission from the Archives of the Mathematisches Forschungsinstitut Oberwolfach print designed by Erik Demaine and Martin Demaine, 2012

Photographic Fractal Trees

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Abstract

We present several fractal trees created by iterating building blocks constructed from photographs of real trees. The use of photographs of different types of trees, along with variation of the parameters available for construction of the trees, allows a wide variety of forms to be realized. The trees shown have self-similarity fractal dimension varying from 1.45 to 2.17, and infinite series have been used to characterize the number of branches and the area of the trees. Randomization of the construction process is demonstrated to yield less regular and more naturalistic tree forms.

1. Introduction

Fractals are objects that exhibit self similarity on different scales. In other words, repeatedly zooming in on a fractal reveals similar structure over and over again. In a mathematical fractal, the level of detail is infinite. In a fractal object in nature, this zooming in can typically only be carried out a few times, and the similarity is less regular than in mathematical fractals. Examples of fractals in nature include mountains, clouds, coastlines, and arteries in the human body. The last is an example of a branching fractal, in which a feature branches into smaller features repeatedly. Pythagorean trees are simple mathematical branching fractals constructed of alternating triangles and squares [1,2]. Trees in nature are also branching fractals.

Mathematical fractals are created by iteration, in which a step or series of steps are carried out repeatedly. In this paper, we describe a technique for creating fractal trees by iteratively arranging copies of photographic building blocks. The resulting constructions are more fractal than natural trees and can vary in appearance from naturalistic to fantastical. The work described in this paper had its roots in earlier tree-like constructs created by iteratively arranging spirals [3].

All of the photographs were taken using a relatively low-cost digital camera. The drawing program FreeHand was used to create preliminary designs of many of the trees, and Photoshop was used to create the final tree from the photographs.

2. Method for Creating Photographic Fractal Trees

The process of creating the fractal trees can be broken down into four basic steps.

1. Create a preliminary design.
2. Identify the tree or bush that will be used and photograph it.
3. Digitally alter the photograph(s) to fit the template designed in Step 1.
4. Iteratively construct the tree.

For some of the trees, the design was driven by a desired fractal form, in which case the first two steps were in the order shown above. In other cases, it was driven by a photographed tree form, in which case the first two steps were reversed in order.

Designing a fractal tree involves designing a building block and then iteratively constructing a tree from it. Design choices include how many branchings will occur in each segment and exactly where and how the second generation of segments will mate to the first segment. Each segment will be scaled down by some factor, rotated by some amount, and possibly reflected.

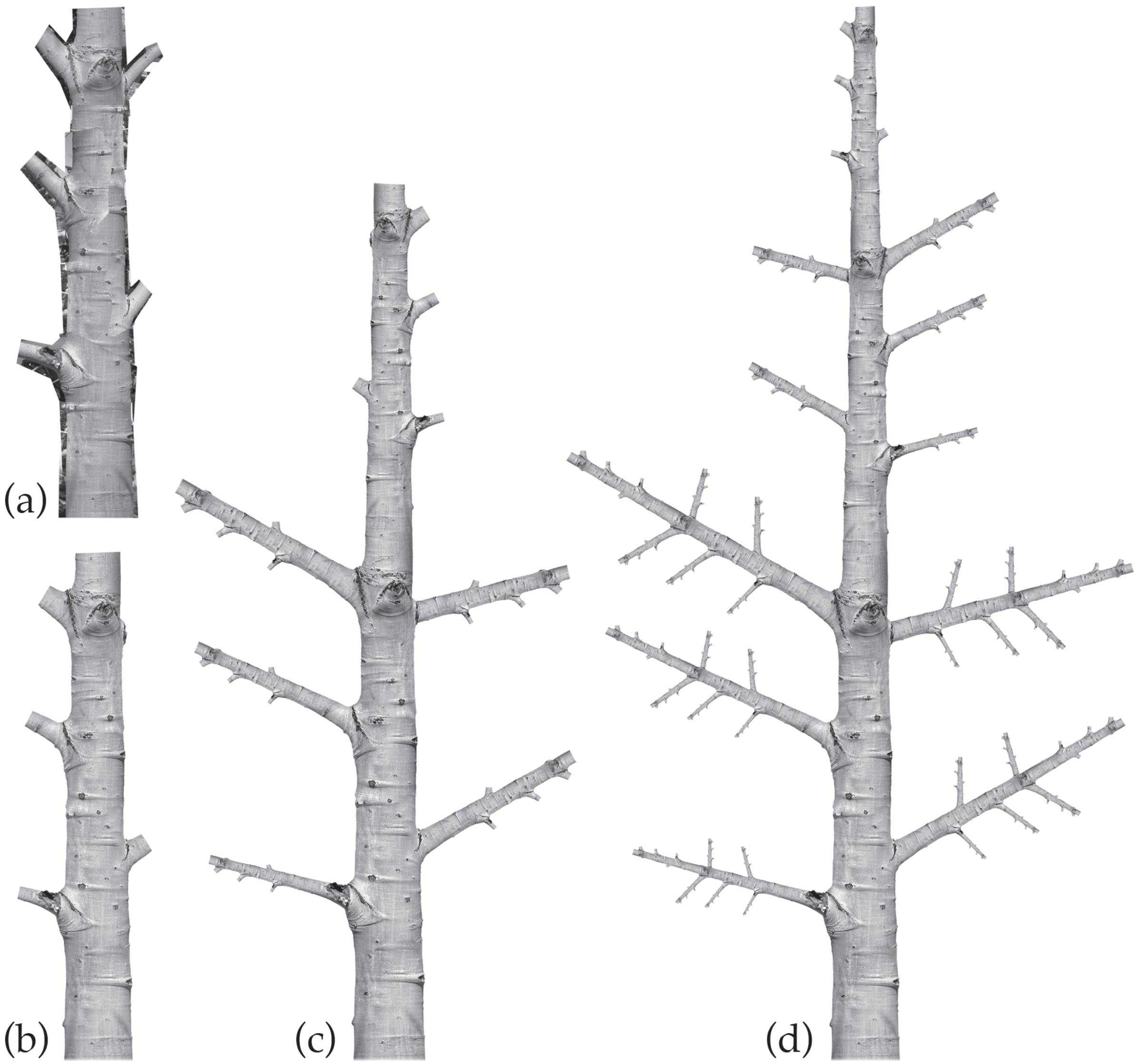

Figure 1: *Example of the construction process for a photographic fractal tree. (a) One or more photographs are combined to create a roughed-out photographic building block. (b) Adjustments are made to allow the different photographs to join seamlessly, and for the scaled-down building blocks to join seamlessly to the larger building block. In addition, the background is carefully trimmed away. (c) Scaled-down copies of the building block are arranged around the original (first generation) building block to form the second-generation tree. (d) Scaled-down copies of the second-generation tree are arranged around the original building block to form the third generation tree.*

An example is shown in Figure 1. In this case, the preliminary design was done before photographs were taken, with the intent of using an aspen tree for the photographic building block. Since it would be nearly impossible to find a real tree that had the desired combination of branch sizes and locations, several photographs were taken of more than one tree. Pieces were then cut out of a few different photographs and pasted together, with some preliminary adjusting of shading and scaling to get the initial version of the building block shown in Figure 1a. Further distortions in shape and adjustments in shading, followed by trimming around the edges resulted in the final building block shown in Figure 1b. This building block is the first generation of the tree, which includes the lowest portion of the trunk.

Figure 2: *The final fractal tree that results after 15 iterations of the sort shown in Figure 1. The inset shows a detail of the tree, against a black background for better contrast.*

The next step in constructing the tree was to make six copies of it, to scale, rotate, and reflect (as desired) each one, and then position them at six different locations along the first generation tree. These seven objects were then merged to create the second-generation tree shown in Figure 1c. Six copies of the second-generation tree were then made, and they were transformed and positioned relative to the first generation tree using the same set of transformations to form the third generation tree shown in Figure 1d. This process was continued until the additions to the previous generation were so small as to be insignificant to the eye at the full scale of the finished print, as shown in Figure 2. In this case, 15 iterations were performed in order that the tree looks line an infinitely-detailed fractal even at dimensions of 28″ x 40″. At that size, the height of the building block is reduced to approximately one pixel. In the final print, a black background was used, as in the detail in Figure 2.

3. Further Examples

The different types of trees and bushes that can be used as photographic source material, along with the different choices available in the design of the structures, as described above, allow a wide variety of forms to be obtained using this construction method. In this section, we present a few additional examples. More examples not shown here can be found in other work by the author [4,5].

In Figure 3, a fractal tree full of spiraling segments has been constructed from a photograph of a cholla cactus skeleton. In general, structures that curl in one direction will form when reflections are not employed. If there is sufficient turning in each generation, spirals will result. Note that only two smaller copies of the photographic building block are added with each iteration.

The tree shown in Figure 4 has straight, thin branches that overlap heavily, creating a complex collection of nearly straight-line segments. This tree is not very naturalistic, but emphasizes its fractal character through the boundary of the branching regions. The photographic basis for this tree was a group of twigs from a palo verde tree. The straightness of the twigs and the nearly right angles between the twigs give this tree a distinctive appearance.

The tree shown in Figure 5 is much more naturalistic than that of Figure 4, but its fractal character is still quite evident. The branches roughly form a series of triangles that reduce in size moving from lower left to upper right. Photographs of a royal poinciana tree in Hawaii were used to create the photographic building block for this tree. In contrast to the trees shown in Figures 2–4, for which 15–20 iterations were performed, the construction of this tree was terminated after eight iterations. More iterations were found to muddy the appearance of the tree due to the large amount of overlap of the branches. This has the effect of making the smallest features more naturalistic, as they are similar in size to the smallest twigs on a large natural tree. Note that no reflections are employed in this tree. However, in contrast to the tree of Figure 3, there is not enough turning to one side to allow spirals to form.

4. Mathematical Properties of the Trees

Relatively simple mathematical analysis can be used to characterize the properties of these trees. Issues that can be addressed include the number of branches, the area of the trees, and the complexity of the trees.

If b_i is the number of smaller branches added per larger branch in the ith iteration, then the total number of branches added in the ith iteration, N_i, is given by $N_i = (1)\ b_1\ b_2\ b_3\ \ldots\ b_i$. ($b_0 = 1$, the trunk). For all of the trees shown in this paper, b is the same for each iteration, in which case $N_i = b^i$. For example, if $b = 3$, as in Figure 4, three branches are added in the first iteration, nine in the second, etc. This tree was iterated 15 times, so the number of branches added in the final iteration was 3^{15}, over 14 million.

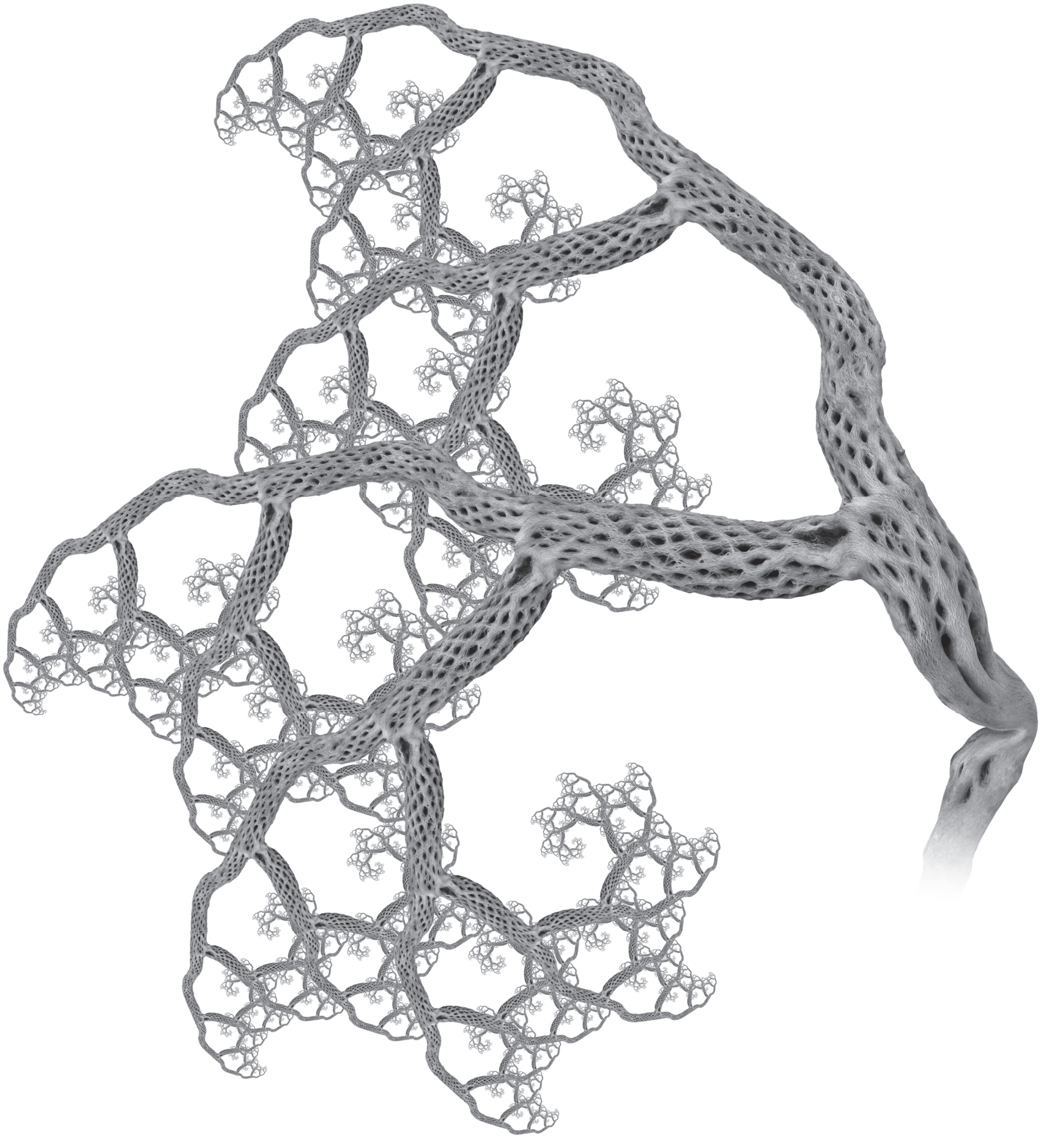

Figure 3: *A fractal tree formed from photographs of a portion of a cholla cactus skeleton.*

The total number of branch segments after i iterations is given by $N_T = N_1 + N_2 + N_3 + \ldots + N_i$. If b is the same for each iteration, $N_T = b + b^2 + b^3 + \ldots + b^i = b(1 - b^i)/(1 - b)$.

The area of one of these two-dimensional trees can also be examined. If the area of the first generation is arbitrarily set to 1, then the area of the branches added in the second generation is the sum of the area of each of the b branches: $A_2 = s_1^2 + s_2^2 + \ldots + s_b^2$. (The area of the first added branch is the area of the first generation, 1, times the square of the scaling factor s_1 for that branch, etc.) For example, the tree in Figure 4 adds three branches with scaling factors of 0.55, 0.60, and 0.66, giving an area of $0.55^2 + 0.6^2 + 0.66^2 \approx 1.10$ for the added branches. Each of these second generation branches will have a similarly larger area added to it in the third generation, so the area of the added branches in each generation increases by the same factor relative to that of the preceding generation. I.e., the total area A_T is given by a geometric series, so $A_T = 1/(1 - A_2)$. For the four trees shown in Figures 2-5, A_2 is approximately 0.83, exactly 0.85, approximately 1.10, and exactly 1.0. As a result, the total area diverges (become infinite) in the limit of an infinite number of iterations for the trees of Figures 4 and 5. By inspection, the infinitely iterated trees clearly fit in a finite area on the page. The infinite area is possible because of the overlap of branches.

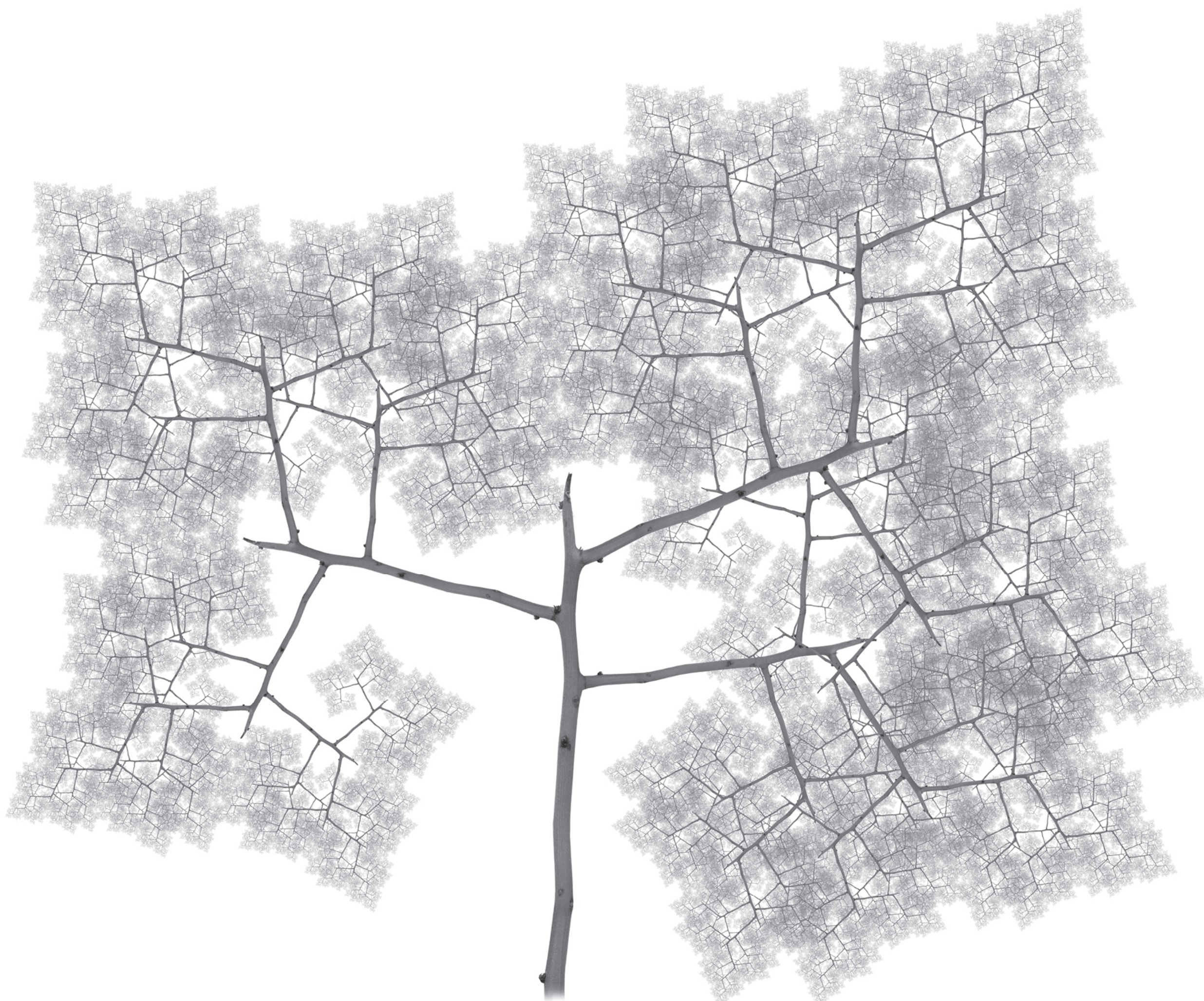

Figure 4: *A fractal tree formed from photographs of palo verde twigs.*

A measure of the complexity of a fractal is provided by the fractal dimension, which evaluates how fast a parameter like length increases as scale decreases. There are several different notions of fractal dimension [1]. For regular structures like those shown here, the self-similarity dimension provides a ready measure. This is given by $D = (\log b) / (\log 1/s)$, where b is the number of pieces into which the structure can be divided, and s is the scaling factor. In this case, b is the number of branches added per larger branch. The scaling factor s is different for each branch, so an average value for the branches added was used for each tree. The approximate values of D calculated for the four trees shown in Figures 2-5 are 1.45, 1.61, 2.17, and 2.00 respectively. Qualitatively, this trend agrees with the amount of overlap observed in the branches of these four trees. Notice that a fractal dimension greater than 2 for a planar structure is only possible with overlapping features.

Figure 5: *A fractal tree formed from photographs of a royal poinciana tree. There are no reflections of the branches in the construction of this tree.*

5. Randomized Fractal Trees

For the trees shown in Figures 2-5, the series of transformations carried out was identical for each iteration. However, one or more of the parameters can be varied in order to achieve additional tree forms. Varying parameters in a random manner would be expected to generate less mathematically regular structures, which therefore have the potential to appear more naturalistic.

An example is shown in Figure 6, where the same photographic building block was used as for the tree of Figure 5. For these trees, however, the choice of whether or not to reflect each branch at each step was made randomly. This was accomplished by rolling a 20-sided die at each iteration. The numbers 1-16 were used to determine which branches reflected. (The die was rolled again if 17-20 came up.) The choices can be set by assigning “0” to unreflected and “1” to reflected in the binary representation of the number. For example, the number 5 is 0101 in binary, which can be read from left to right as determining the first and third branches from the left to be unreflected, and the second and fourth to be reflected. With eight iterations, there are 16^8 (over 4 billion) distinct trees that can be formed, three of which are shown in

Figures 5 and 6. The two trees shown in Figure 6 are the result of two different sets of eight rolls of the die. Additional examples of randomized fractal trees can be seen in References 4 and 5.

Figure 6: *Two randomized fractal trees created using the same photographic building block used for Figure 5. In this case, the choice of whether or not to reflect each of the four branches at each iteration was made randomly.*

6. Conclusion

We have presented a variety of fractal trees created by iterating photographic building blocks. The use of photographs of different types of trees, along with variation of the parameters available for construction of the trees, allows a wide variety of forms to be realized. The trees shown have self-similarity fractal dimension varying from 1.45 to 2.17, and infinite series have been used to characterize the number of branches and the area of the trees. Randomization of the construction process has been demonstrated to yield less regular and more naturalistic tree forms. Even more naturalistic forms could be realized, for example by varying the number of branches added with each iteration.

References

[1] Heinz-Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe, *Fractals for the Classroom – Part One, Introduction to Fractals and Chaos,* Springer-Verlag, New York, 1993.

[2] An interactive Pythagorean tree construction is available on the Wolfram Demonstrations Project website: http://demonstrations.wolfram.com/PythagorasTree/.

[3] Robert Fathauer, *Fractal Patterns and Pseudo-tilings Based on Spirals,* in Proceedings of the Bridges 2004 Conference, edited by Reza Sarhangi and Carlo Séquin, pp. 203-210, 2004.

[4] Robert Fathauer, *Fractal Trees,* Tessellations Publishing, Phoenix, 2009.

[5] Website featuring Robert Fathauer's art: http://members.cox.net/fathauerart/FractalTreesArt.html.

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Infinity Bead

a hollow see-through beaded bead

an original technique by
Gwen Fisher & Florence Turnour
www.beadinfinitum.com

Download the complete 23 page, full color version of this pattern at
http://www.beadinfinitum.com/infinityG4G10.pdf

Materials
A large (30mm) Infinity Dodecahedron uses
Size 6/0 seed beads colors A [30]
Size 8/0 seed beads colors A [120], B [60], C* [60]
10mm to 14mm rattle bead (optional)
6.5 feet Nymo nylon beading thread, size D
Beading needle, size 10

A large Infinity Cube uses
Size 6/0 seed beads colors A [30]
Size 8/0 seed beads colors A [120], B [60], C* [60]
4.5 feet Nymo nylon beading thread, size D
Beading needle, size 10

***The Puzzle Kit** has the C beads divided into five different colors. It has11 enough beads to make a large dodecahedron and cube as in these photos.

Dodecahedron: Five Coloring
You can color the 20 vertices of a dodecahedron with 5 colors so that every hole (face) shows all 5 colors. This coloring gives 12/5 = 4 vertices of each color. These 4 vertices are equally spaced around the dodecahedron, and if you imagine lines connecting them, you would have a regular tetrahedron in each color.

Cube: Four Coloring
You can color the 8 vertices of a cube with 4 colors so that every hole (face) shows all 4 colors. This coloring gives 8/4 = 2 vertices of each color. These 2 vertices will be at opposite ends of the cube.

The Infinity Dodecahedron
Start with two sizes of seed beads (large and small in color A).

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1. Thread your needle. * Pick up
   - 1 large bead (Bead 1),
   - 2 small beads (color A).

Repeat from * 4 more times (5 times total) to add a total of 15 beads.

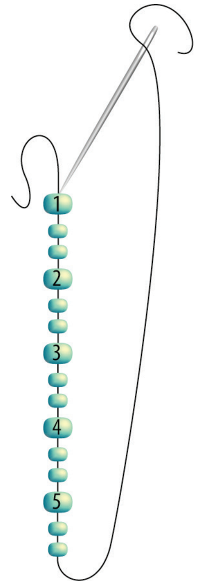

Figure 1:
String beads

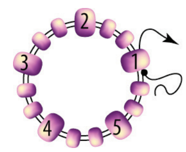

Figure 2

Photo 2

2. Sew through all 15 beads again, in the same direction, to make a ring, and slide the ring down leaving a 8 inch tail that you will later weave into the bead work. Tie a square knot with the working end and the loose end of the thread, but as you tighten the knot, be careful to leave a little bit of slack in the ring to keep it flexible. Sew through Bead 1.

For the rest of the project, keep the tension tight enough to pick up slack, but loose enough to keep the bead work flexible.

3. To make the second ring, * pick up
   - 2 small beads,
   - 1 large bead (Bead 6).

Repeat from * 3 more times. Then pick up
   - 2 small beads.

Sew though Bead 1 from the side with the tail to complete Ring 2.

Figure 3

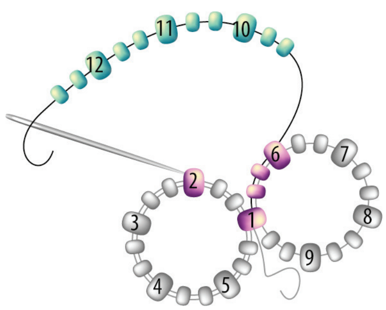

Figure 4

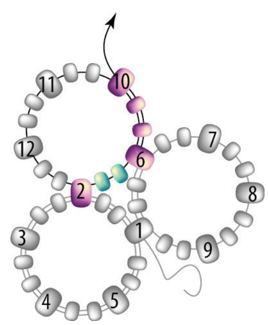

Figure 5

Photo 5

4. Continue sewing around Ring 2 until you exit the first large bead (Bead 6). Ring 3 will take two stitches to complete. *Pick up
   - 2 small beads,
   - 1 large bead (Bead 10).

Repeat from * 2 more times (3 times total). Then pick up
   - 2 small beads.

You should have 11 beads as in Figure 4. Sew through Bead 2 towards Bead 6. Pull any slack in the thread.

5. To complete Ring 3, pick up
   - 2 small beads.

Sew through Bead 6 to make a ring, Weave through two small seed beads and Bead 10. Pull any slack in the thread. See Figure 5 and Photo 5. Notice how the three rings connect only at the larger beads. All 12 rings on the Infinity Bead will have 5 large beads and 10 small beads and two adjacent rings will connect only at the larger beads.

6. Flip the bead work over, and repeat Steps 4 and 5 until you have 8 rings.

7. Position your needle by sewing around Ring 8 through 2 small beads, Bead 26, 2 small beads, and Bead 27. Ring 9 will take four stitches to complete. Pick up

- 2 small beads,

Sew through bead 21. Pick up

- 2 small beads.

Sew through Bead 15. Pick up

- 2 small beads.

Sew through Bead 8.

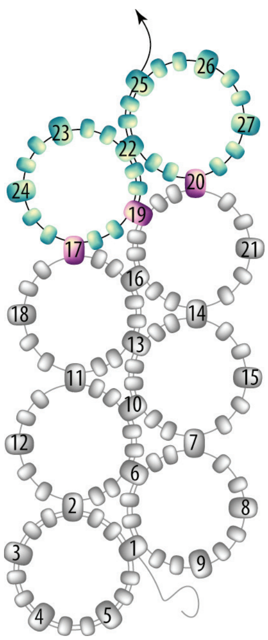

Figure 6

Photo 6

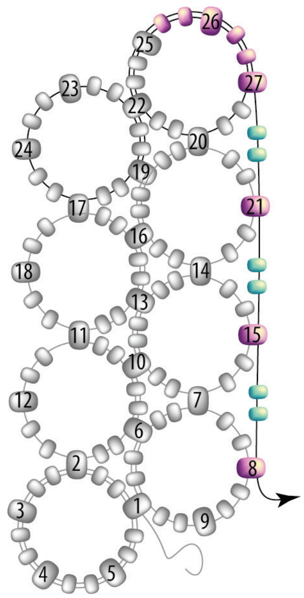

Figure 7

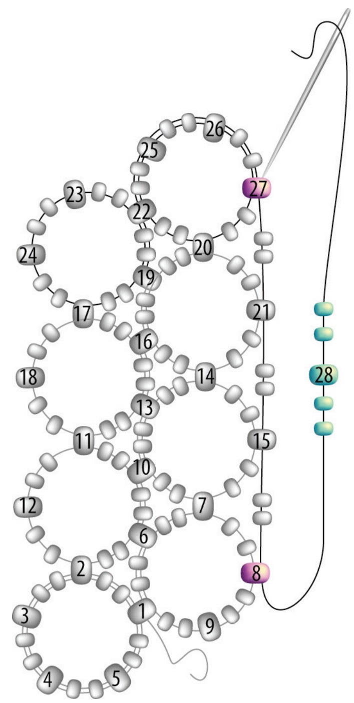

Figure 8

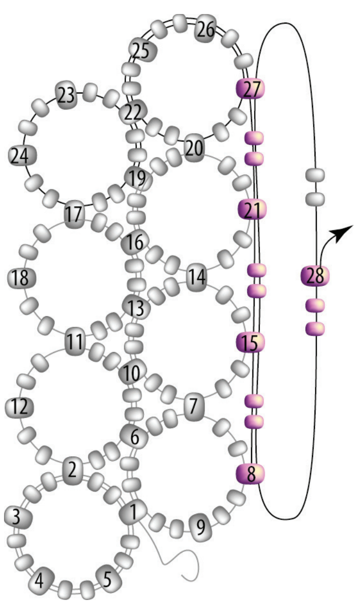

Figure 9

Photo 9

Photo 10

8. Pick up

- 2 small beads,
- 1 large bead (Bead 28),
- 2 small beads.

Sew through Bead 27. Do not pull the thread tight.

9. Position your needle by sewing though the beads you added in Steps 9 and 10 (and the large beads), stopping just after you sew through Bead 28. See Figure 11 and Photo 11.

10. As you pull the thread tight, the bead work will begin to curl into a cup.

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11. Position the bead work so that it is a cup (rather than a mountain) and the single ring with Bead 4 is pointing towards you like in the photos and figure. Ring 10 will take four stitches to complete. Pick up

- 2 small beads.

Sew away from you through Bead 26 in the adjacent ring. Pick up

- 2 small beads,
- 1 large bead (Bead 29),
- 2 small beads.

Sew away from you through Bead 5. Do not pull the thread tight.

12. To complete Ring 10, pick up

- 2 small beads.

Sew through bead 9. Pick up

- 2 small beads.

Sew through Bead 28. To position your needle, sew through 2 small beads, Bead 26, 2 small beads, and Bead 25.

13. Pull the thread tight and see how the bead work curls into a cup. Orient your cup of beads so that the rim faces towards you and the bead you are exiting (Bead 25) is on the top right. All of the beads in Figures 13 are on the cup rim.

14. Ring 11 will take four stitches to complete. Pick up

- 2 small beads.

Sew counter clockwise through Bead 23. Pick up

- 2 small beads,
- 1 large bead (Bead 30),
- 2 small beads.

Sew counter clockwise, across the cup, through Bead 4. Pick up

- 2 small beads.

Sew counter clockwise through Bead 29. Pick up

- 2 small beads.

Sew counter clockwise through Bead 25. Pull the thread tight. The beadwork now resembles a squishy ball of beads, hollow with one large hole where we weave the final ring, Ring 12.

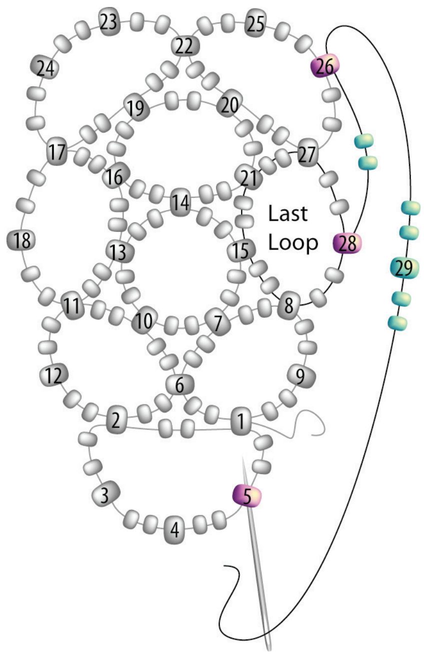

Figure 11

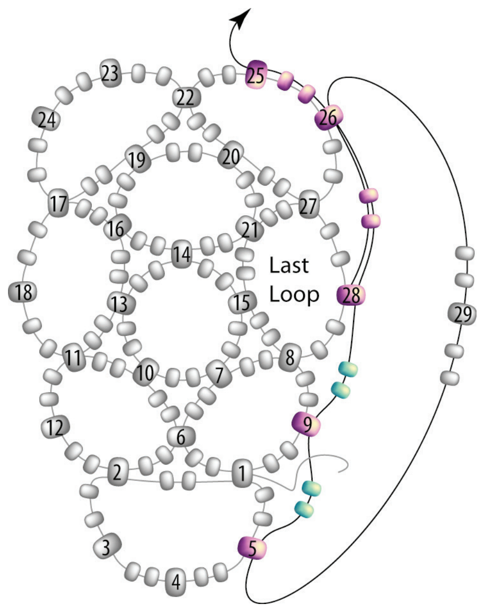

Figure 12

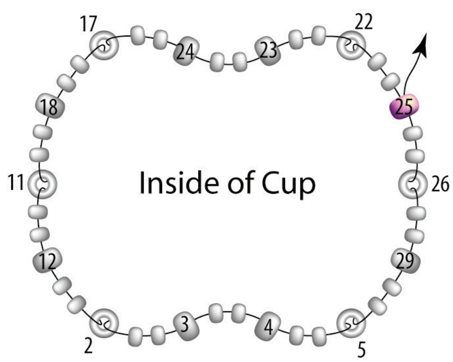

Figure 13

Photo 13

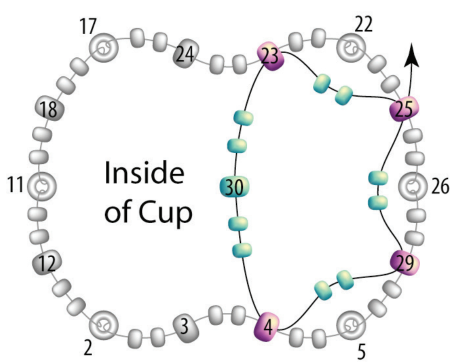

Figure 14

Photo 14

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15. Position your needle by sewing through 2 small beads in Ring 11, Bead 23, 2 small beads, and Bead 24.

Optional: Drop a rattle bead or other precious object into the cup. The holes will get a little smaller as you add more beads. As long as the rattle bead is at least close to not falling out, it should stay trapped later.

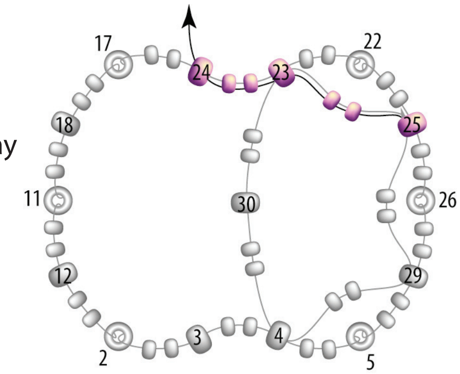

Figure 15

Photo 15

16. The final ring, Ring 12, will take five stitches to complete. Pick up
- 2 small beads.

Sew through Bead 18. Pick up
- 2 small beads.

Sew through Bead 12. Pick up
- 2 small beads.

Sew through Bead 3. Pick up
- 2 small beads.

Sew through Bead 30. Pick up
- 2 small beads.

Sew through Bead 24. Pull the thread tight. Continue sewing counter clockwise half way around Ring 12 to tighten it.

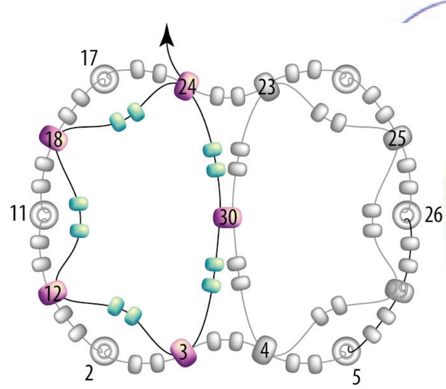

Figure 16

Photo 16

17. Check your work. Around your beadwork, you should see 5-pointed stars. Notice how each 5-pointed star is composed of 1 pentagon and 5 triangles. You should have 12 pentagons and 20 triangles total.

The Outside Layer

18. So far, you have added the large (Lg) beads and the small color A beads. In the next steps, you will be adding small beads in colors B and C by sewing first through the Lg beads, and later through the color B beads until all 20 triangles look like Figure 18.

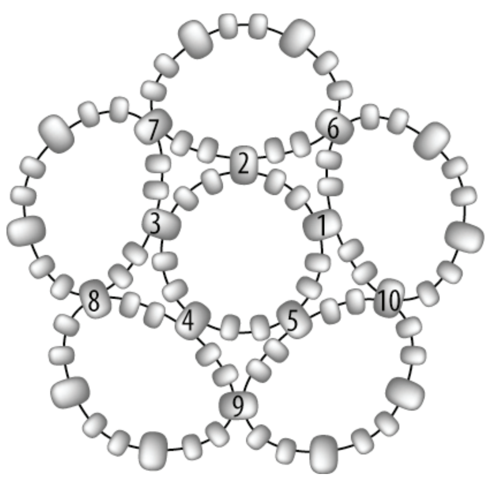

Figure 17

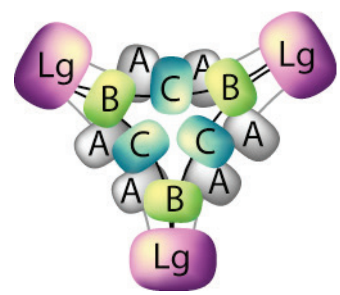

Figure 18

19. Exit any large bead (call it Bead 1) and orient your beadwork so that the thread is pointing away from you (up) as it exits Bead 1. Pick up

- 1 small color B bead,
- 1 small color C bead,
- 1 small color B bead.

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Sew through one of the large beads in the same triangle as Bead 1. You have 2 choices about which large bead to sew through, either Bead 2 or 2*.

Pick up 3 small beads (BCB) as above, and sew through one large bead in the same triangle as Bead 2. Again, you have two choices, either Bead 3 or 3*. Figure 19 shows the thread passing through Bead 3. Photo 19A shows these first 2 stitches (Bead 1 is at 5:00).

Continue picking up 3 small beads (BCB) and sew through one of the large beads in the same triangle. Meander over the surface of your bead work, through Beads 4, 5, 6, … planning ahead so that you do not sew towards a triangle where you have already added beads (BCB). For example, Figure 19 show that when exiting Bead 7, you could sew through Bead 8 or Bead 8*, but Bead 8 is a better choice.

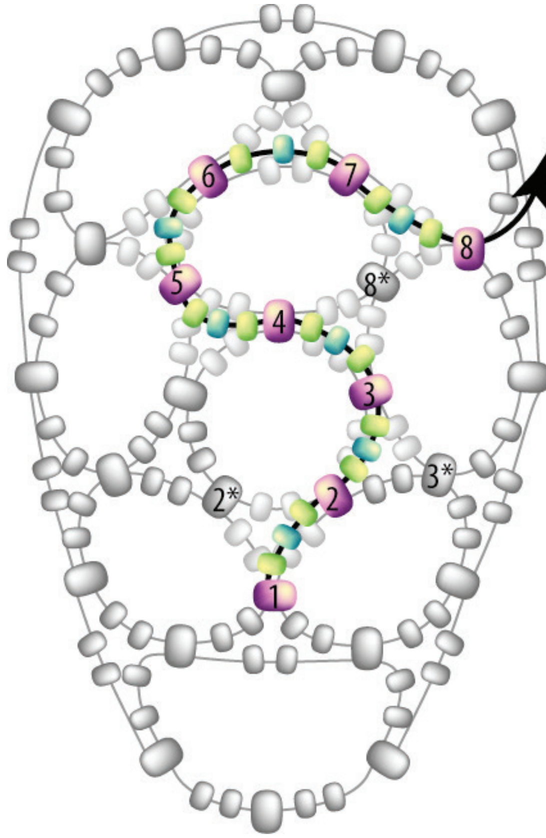

Figure 19

Photo 19A

Photo 19B

Photo 19C

When you are forced to enter a triangle that already has outer layer beads (as in Photo 19C), go to Step 20. At this point, your bead work won't hold its shape. From this point on, use fairly tight tension as you weave.

Aside: You can use Infinity Weave to make any convex polyhedron in which every vertex has valence three, such as the truncated rhombic dodecahedron shown in the three photos below.

20. Once you are forced to enter a triangle with outer layer beads (BCB) added, there are 2 possibilities for how this can happen. Both are shown in Figure 20: either entering Bead 1 or Bead 9. What is most likely, is you will sew through a bead like Bead 9, where there is no outer layer bead added directly after Bead 9. In this case, pick up

- 1 small color B bead,
- 1 small color C bead.

You have two choices for which direction to sew, either towards Bead 5 or 6. Figure 20 shows sewing towards Bead 5. In either case, you will need to sew through 3 beads: 1 small color C bead, 1 large bead, 1 small color C bead. See also the upper middle triangle in Photo 20.

If there is a small outer layer bead directly after the large bead, as with Bead 1 in Figure 22, sew through the large bead and the small bead directly after it. Pick up

- 1 small color C bead,
- 1 small color B bead.

Then, you only have 1 choice for which direction to sew: into the large bead that you have not yet passed through, as Bead 2* in Figure 20.

Repeat Step 20 several times. Eventually you will enter the beads in a triangle where only one color C bead needs to be added, as in Photo 20. Go to Step 21.

21. From Bead 2*, sew through the small color B bead right after it, and pick up

- 1 small color C bead.

Sew through the small color B bead and Bead 2. The outer layer of beads should lie neatly over the inner layer. You are finished when all triangles look like Figure 18. Repeat Steps 19 to 21 as many times as needed to complete each triangle, tightening as you sew.

22. If any your Infinity Bead is squishy and needs tightening, weave through the outer layer of beads (Lg, B, C) tightening as you go. Complete the beaded bead by tying two half hitch knots in the same spot. Then weave the thread through a few more beads, and cut all of the thread ends close to the beads. Finish off the second end in this same way.

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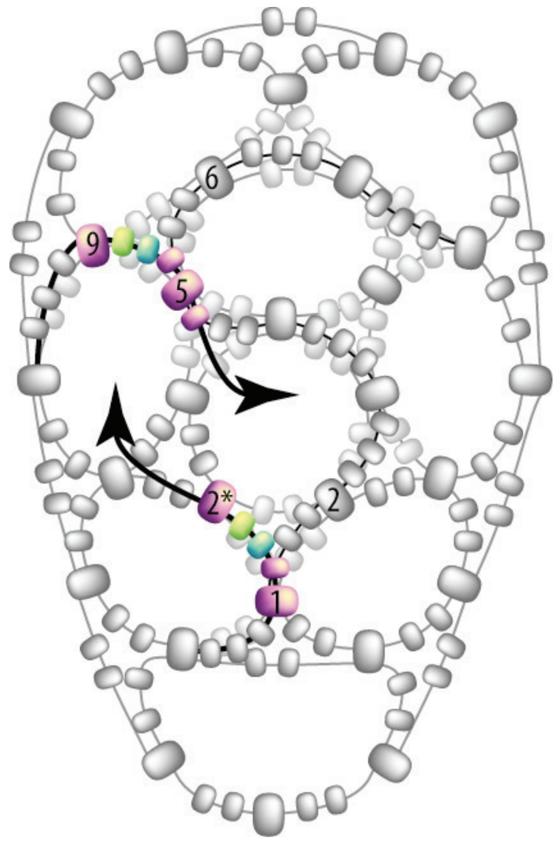


Figure 20

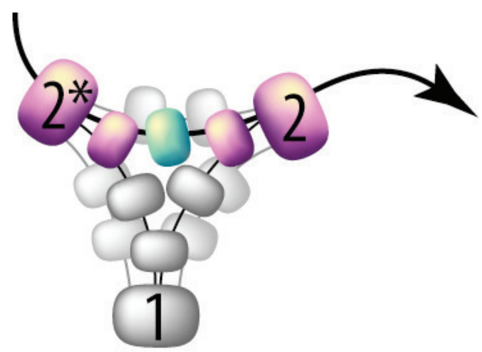


Figure 21

Photo 20

Photo 21

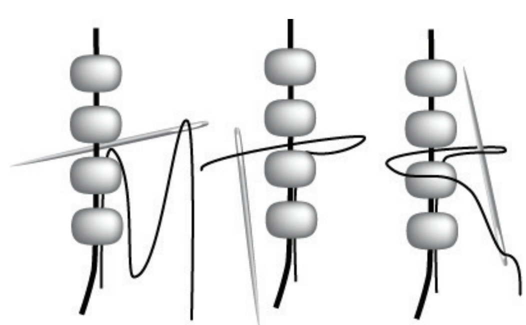


Figure 22: Half hitch knot

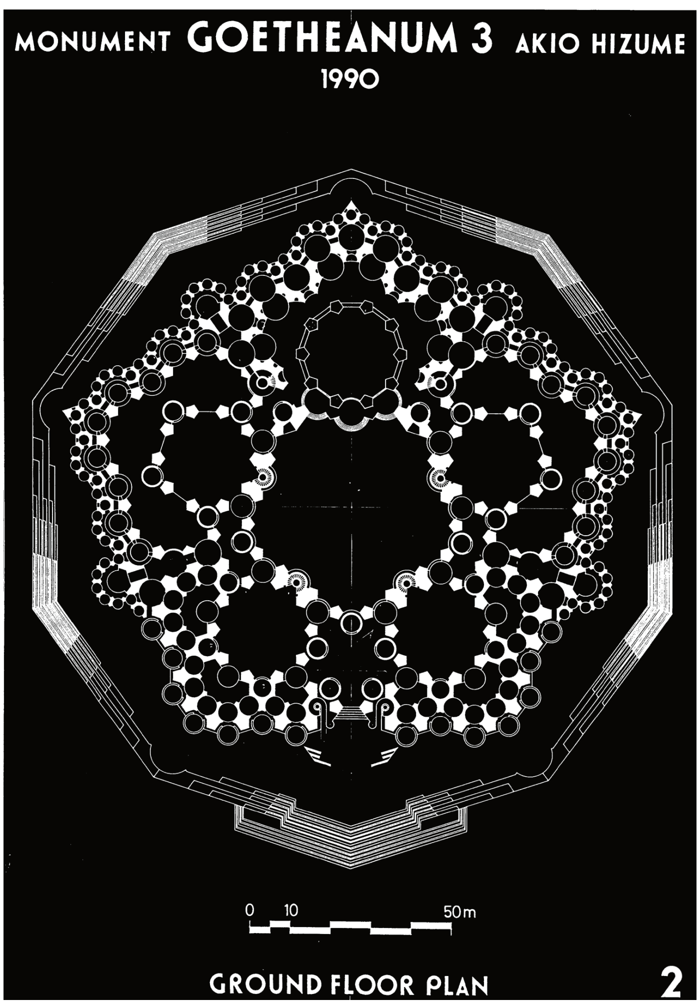
Quasi-Crystal Pavilion

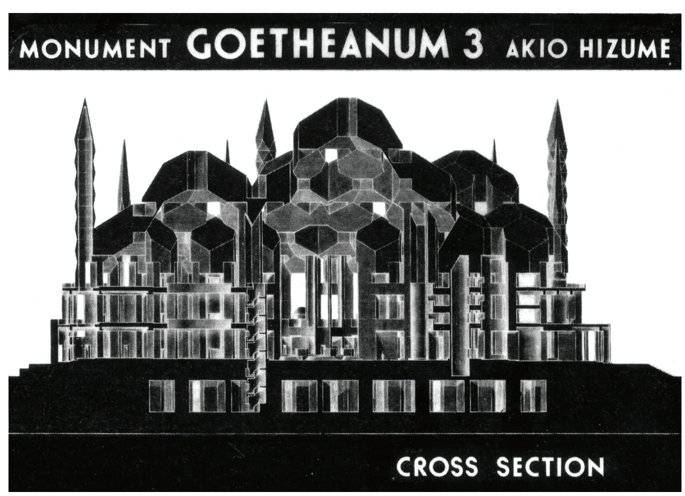
Akio Hizume
Artist
akio@starcage.org
http://www.starcage.org/

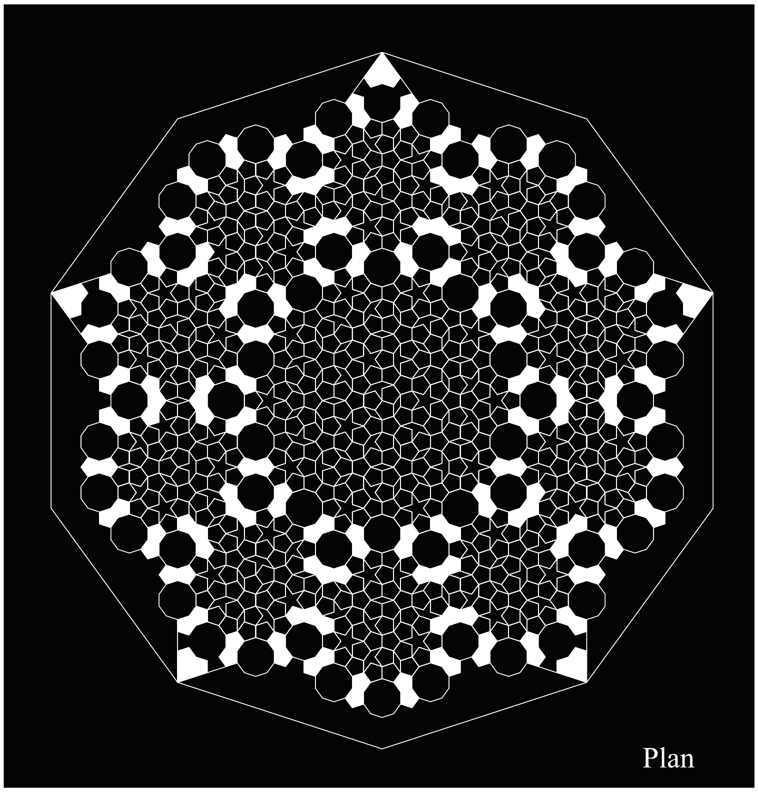
I designed huge Architecture based on the Penrose Tiling in 1990 on the same scale as The Saint Peter's Basilica, the Vatican. It was an imaginary quasi-periodic architecture because of too big scale. But last year, I re-design more compact pavilion under the same mathematical concept. Some of the proposals should be realized somewhere in the world before long.

0. The Goetheanum 3 : MATRIX of the six dimensional architecture (1990)

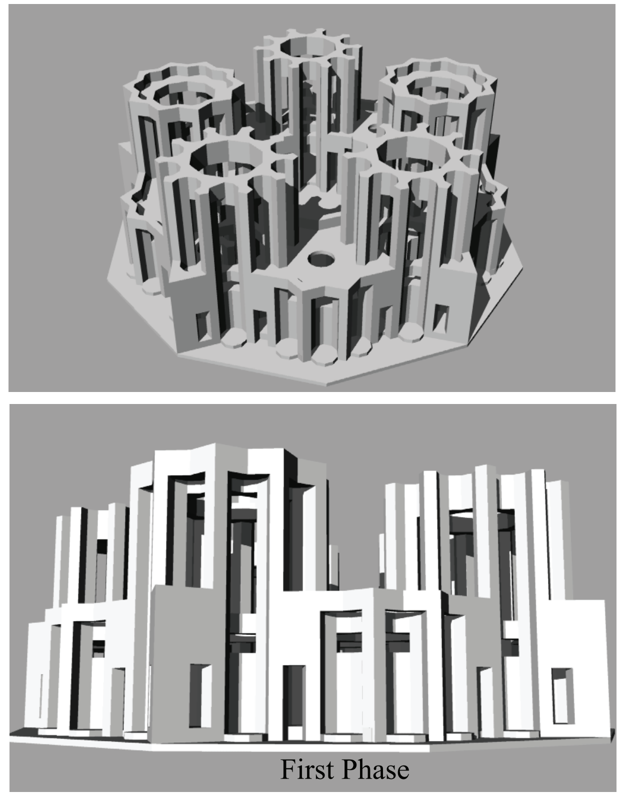
In 1990, I published architectural design called the Goetheanum 3. The figures below are just three of the 17 drawings by hand. It took more than four years to draw. This work follows through the pentagonal Penrose Tiling. I must handle six coordinate axes. There are a lot of novel forms of architecture in detail. Of course it is not realized yet. It is too huge scale to build today. Anyway I could success to prove that the Penrose Tiling must be applied for architecture.
Recently a family of origami artist offered me to design a private museum in Japan. I accepted the request, and started to design. I extracted some elements of design from the Goetheanum 3, and reconstructed them compactly. Then I got four proposals like pavilion. In general, successful pavilion must be prototype for the futural architecture. The Goetheanum 3 is its pregnant matrix. There are still a lot of possibilities.



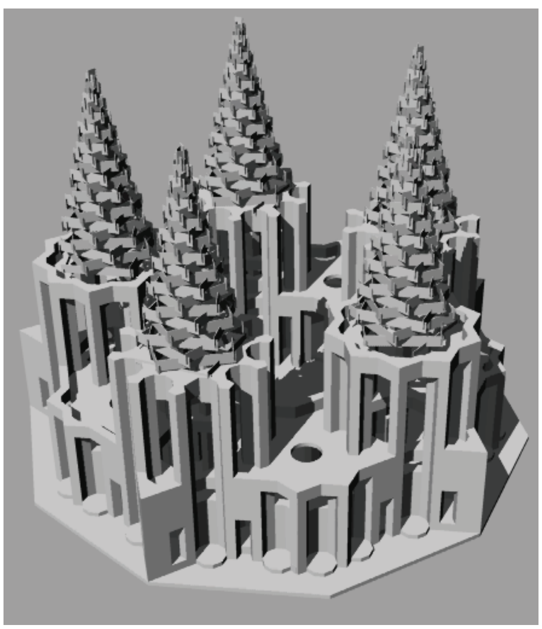




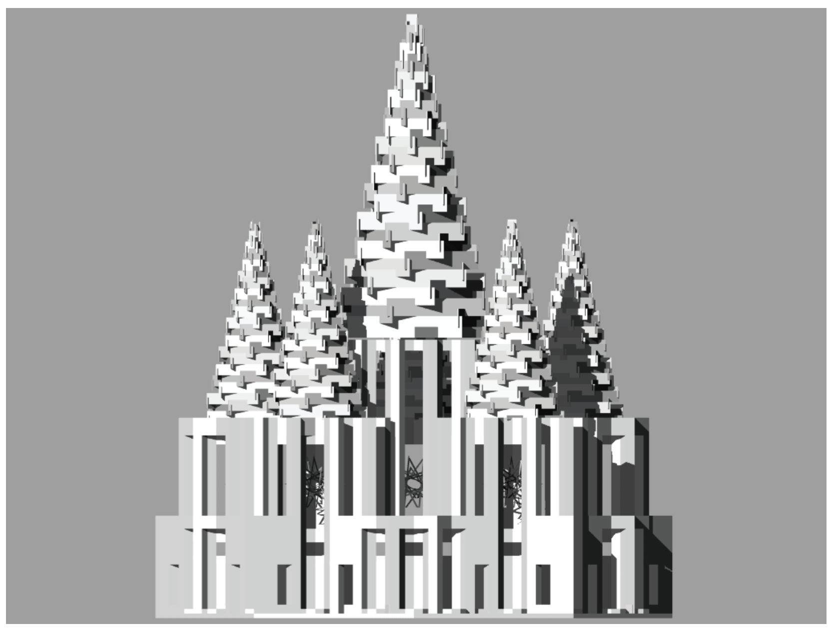
Plan



First Phase



Second Phase



Third Phase

1. Gothic Type (2011)

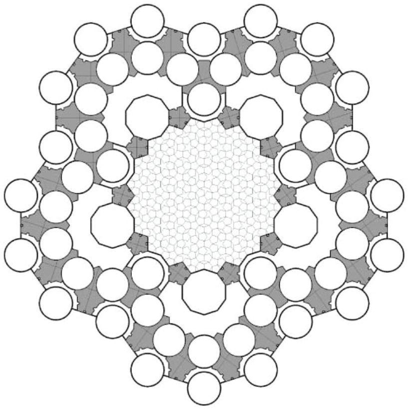
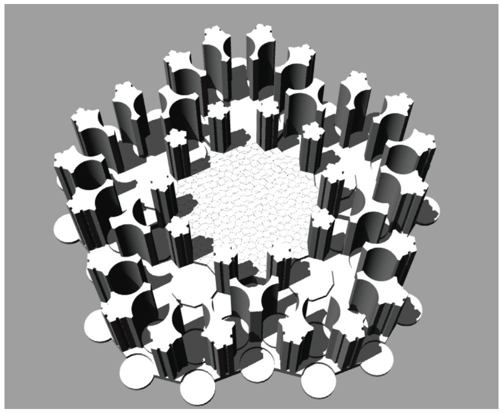
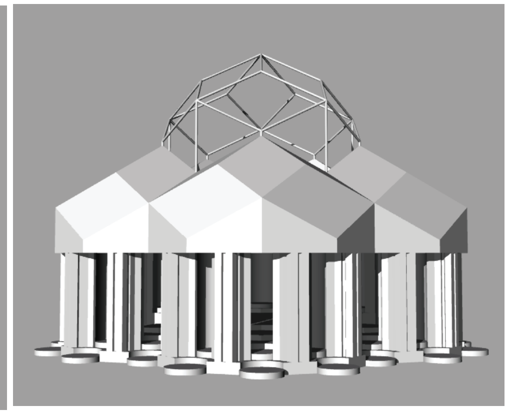
First proposal called the Gothic Type. It should need several phase of construction. The first phase, there are only wall and flat slab. This structure was designed as flying buttress for the Goetheanum 3.

Second phase, five towers cover the roof. The tower is also my invention called Fibonacci Tower which is based on the Phylotaxis. The Star Cage structures called Pleiades are hung from the top of each tower. The hanging Star Cage is not only symbol but also illumination and pendulum. It makes the tower stable against earthquake.

Third phase, the biggest Fibonacci Tower covers the central court.

The structure can follows the method of Gothic Cathedral in medieval Europe but there is no square and right angle at all. If the Freemason still alive, they should construct this pavilion with pleasure I believe. The sound and light effect should be unique and beautiful.

I like this proposal best. In this occasion, it was not realized but it should be realized somewhere in the world because it is the mathematical destiny.

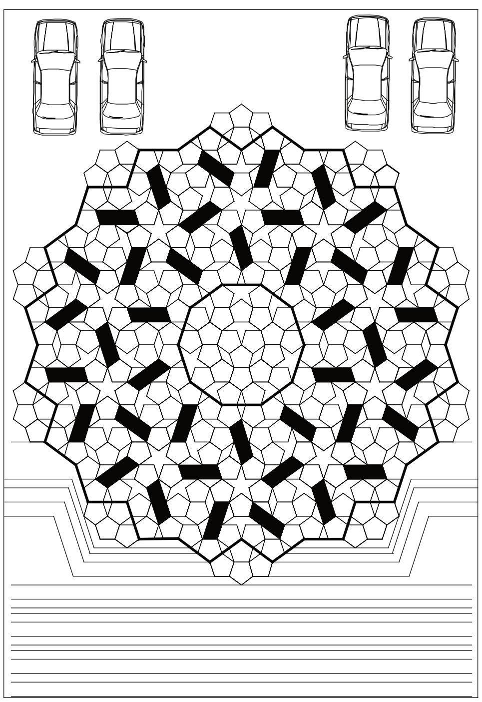
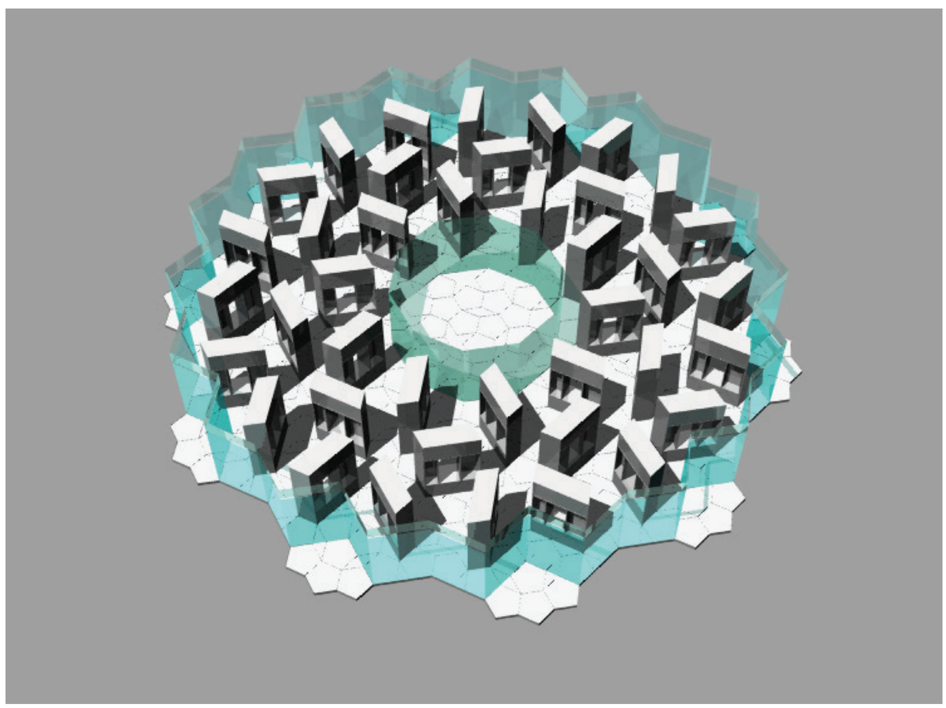
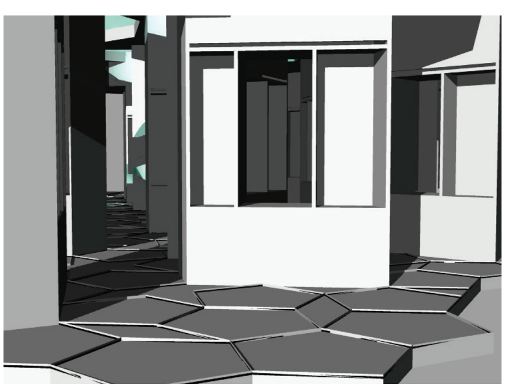
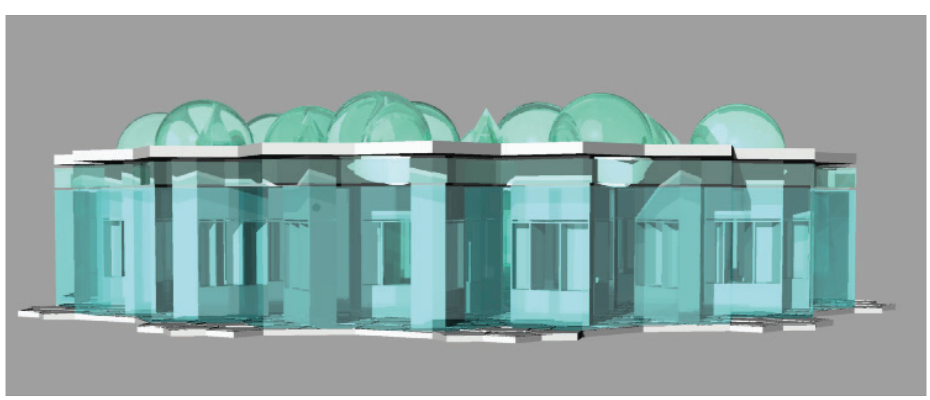
2. Rotonda Type (2011)

If the Renaissance Architect Andrea Palladio had known about the Penrose Tile, he should build such Pavilion I believe. That is why I call the proposal as the "Rotonda".

The Rhombic Triacontahedral roof can cover on the pentagonal Penrose Tiling compatibly. I found it in 1986 then I started to design the Goetheanum 3. It was not always easy to find, but after the discovery, everybody may feel that it is very simple and elegant solution.

3. Bookshelf Type (2011)

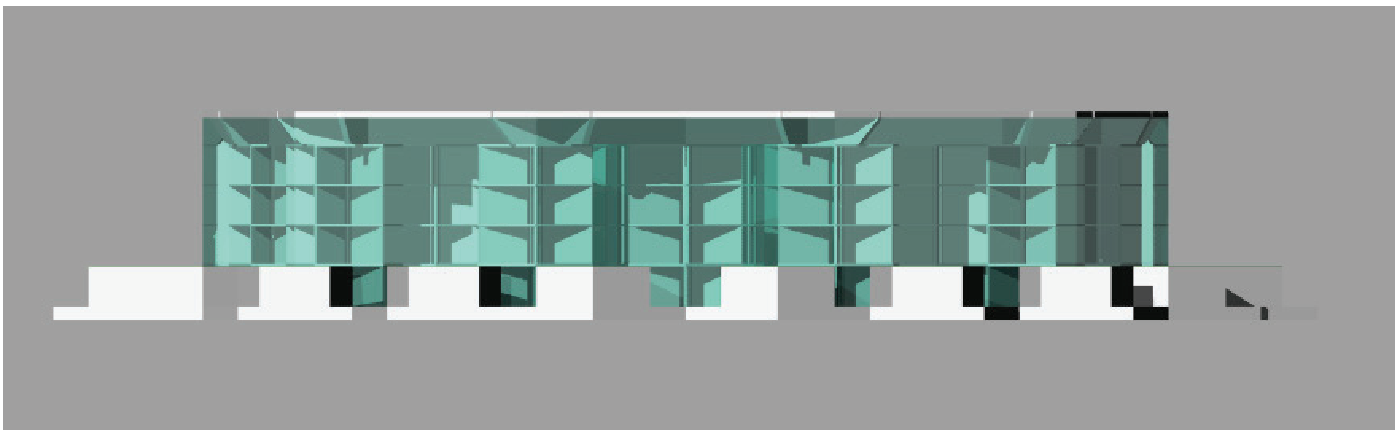
The third proposal was only the bookshelves supports the flat roof as post. There is neither other post nor bearing wall.

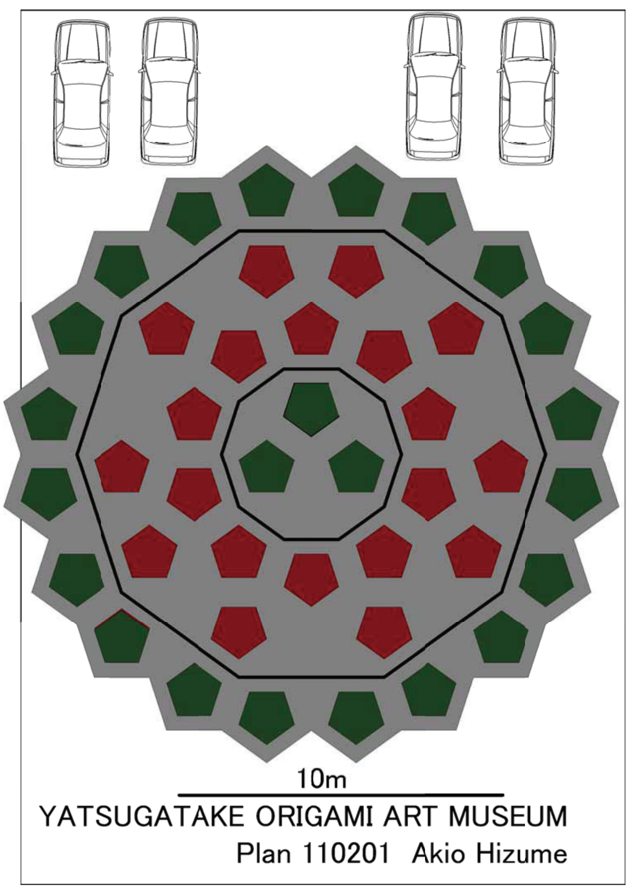





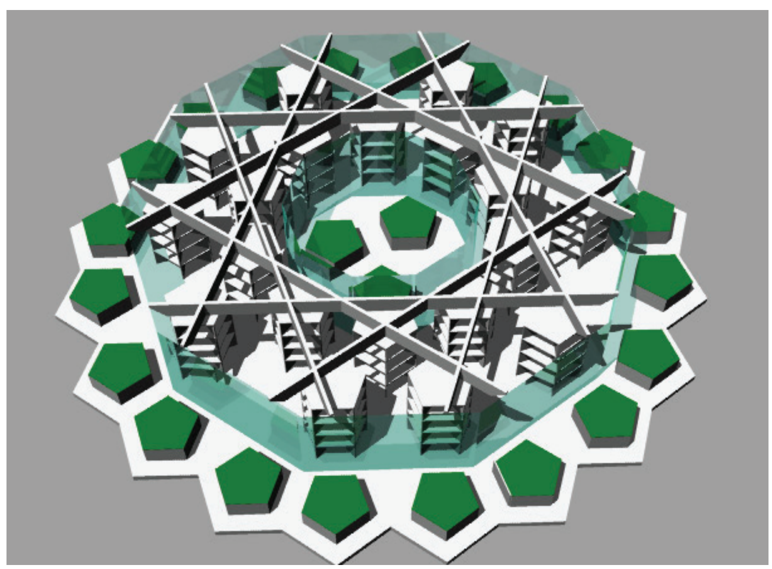
If Ludwig Mies van der Rohe had known about the Penrose Tiling, he might build such glass house.
The labyrinth of the bookshelves should be suitable for library or museum because many different fields of culture should encounter each other in such library, for example mathematician and artist. Compared to this, existing library seems to obstruct such interdisciplinary events I always feel. By the same token, the existing city design keeps people from interacting with each other.
If Jorge Luis Borges had known this architecture, he might wish to rewrite "The Library of Babel".

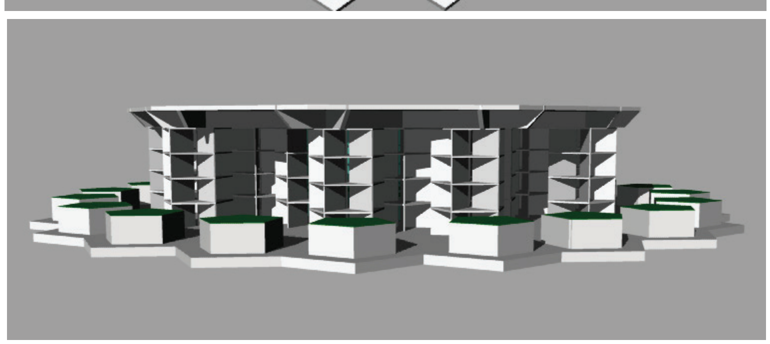
4. Penta-Booth Type (2011)

Every Penta-Booth is not only bookshelf and display table, but also posts supporting the flat roof. The top of each Penta-Booths are connected by pentagonal basket structure that is also my invention called “GOMAGARI” in 1986. The Penta-Booths make exciting labyrinth based on the Penrose Tiling. I have built many such installation as an experimental city planning. The audience seemed to really enjoy living in such city temporary.
The clients like this proposal best. We decided the direction. But they prefer pitched roof to flat roof. So I must change design dynamically.









5. The Latest Proposal (2012)

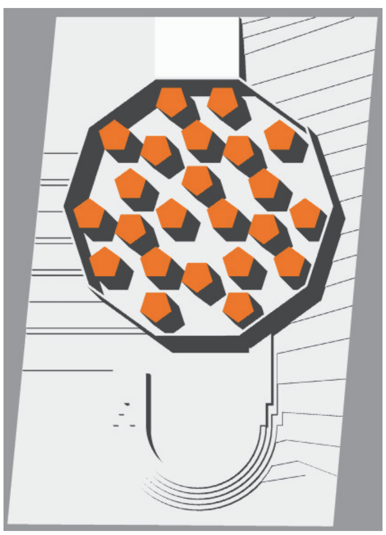
After surveying ground, I redesigned new proposal. The Penta-Booth labyrinth supports the decagonal platform. It is a kind of very stable artificial ground. I will make permanent exhibition there with sound and light controlled by computer with the quasi-periodic rhythm and metallic tone generated by the Golden Ratio. The labyrinth is not only permanent exhibit space but also residence room for gests.

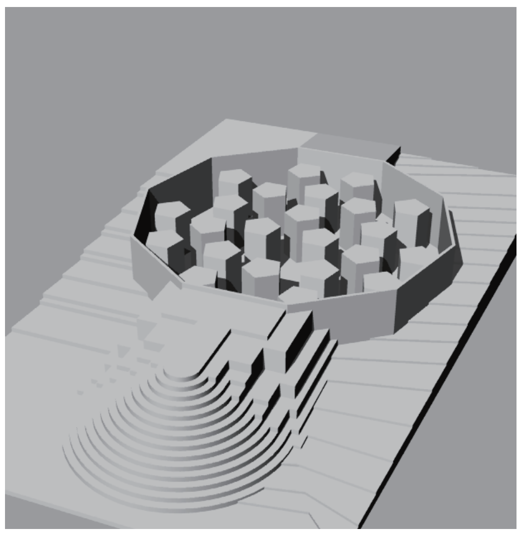
The triple spiral Fibonacci Tower is placed on the platform. It is an open multi-purpose space. One lamp is hung from the top of tower as a pendulum, which makes the tower stable against earthquake. Just one lamp should be enough for the space because the roof structure perfectly diffusely reflects the light. There are some ways to cover roof. We might use some ORIGAMI technique.

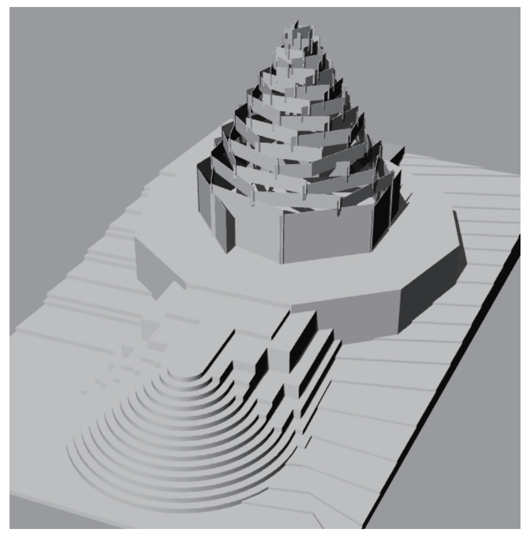
I added new element on the latest proposal, that is, outdoor staircase based on the “Fibonacci Cascade” I called. Children and monkeys can't help shinning up, sitting and playing there. You can find the same staircase on the platform of the Goetheanum 3. Underneath the staircase there is utility room.

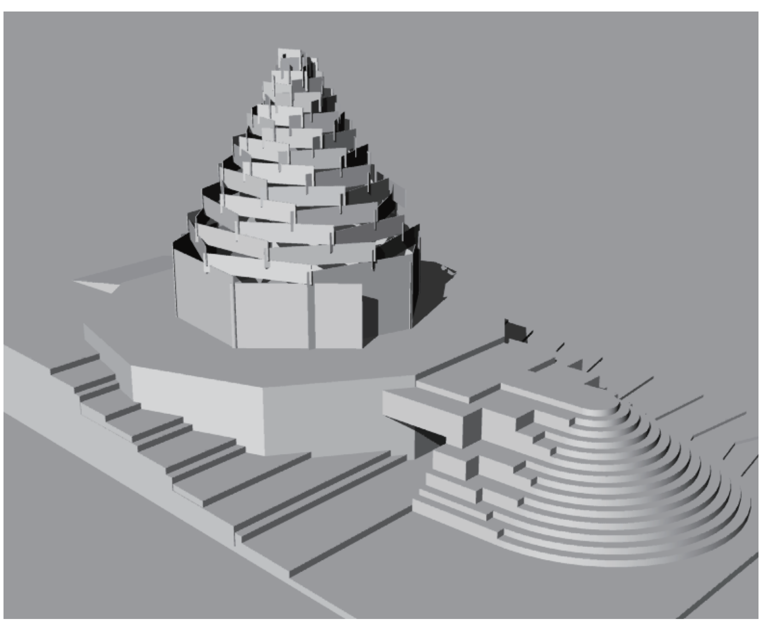
On the gentle slope of natural ground, I will build the Democracy Steps based on the Fibonacci Lattice, that is, I built the same one in Tom Rodgers’ and Sarah Garvin’s garden in G4G8. Even the Democracy Steps was also contained in the Goetheanum 3 already. That is why the Goetheanum 3 is MATRIX.

The Quasi-Crystal Pavilion is literally a quasi-crystal of the Golden Ratio. There is nothing except the Golden Ratio.









My Fractal Tool

Masayoshi Iwai
Academy of Recreational Mathematics, Japan (ARM, Japan)

Although “Mandelbrot set” and “IFS code” are conventional methods for generating fractal patterns, they are too artificial.

I wanted to draw realistic and funny fractal patterns easily, and succeeding in designing a convenient tool for generating fractal patterns from graphic files on a PC.

The outline of the program will be explained with several examples.

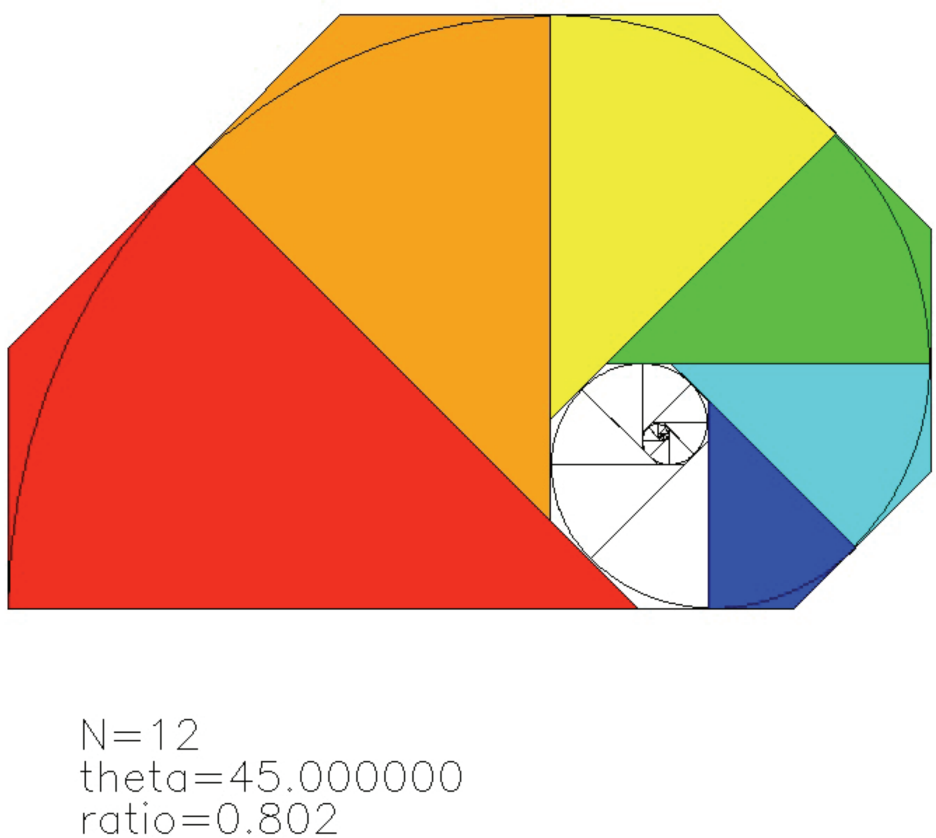
My G4G10 Exchange Item

Masayoshi Iwai
http://ch.cri-mw.co.jp/iwai/
http://www.facebook.com/masaka.iwai

This zip includes the drawing tool and its codes.

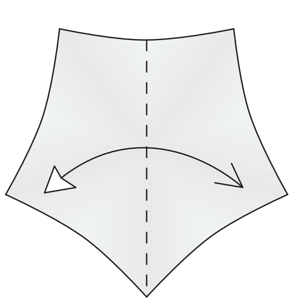
↓ ↓ ↓ ↓ ↓ ↓ ↓
http://masaka.up.seesaa.net/image/g4g10iwaihtm.htm
↑ ↑ ↑ ↑ ↑ ↑ ↑

The drawing tool is used for my G4G10 Talk “Kite Spiral”.

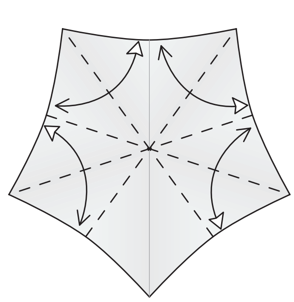


Hyperbolic Crane

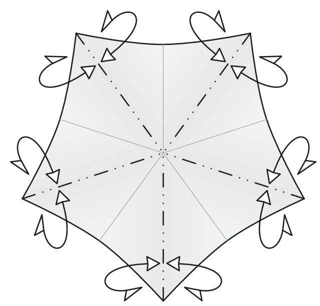
by Robert J. Lang, www.langorigami.com



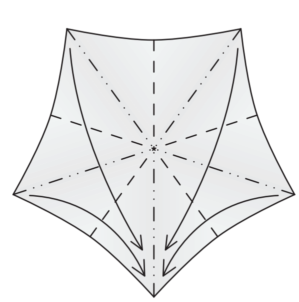
1. Begin with a hyperbolic pentagon. Fold in half through one corner and unfold.



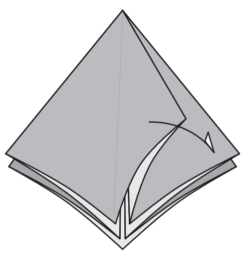
2. Repeat with each of the other 4 corners.



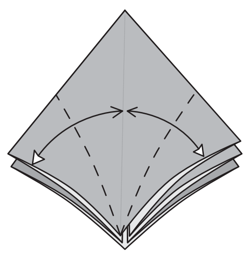
3. Change each of the creases that run from corner to center to mountain folds, i.e., crease each fold in the other direction.



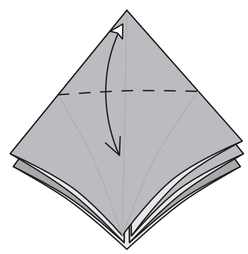
4. Using the existing creases, gather all 5 corners together at the bottom.



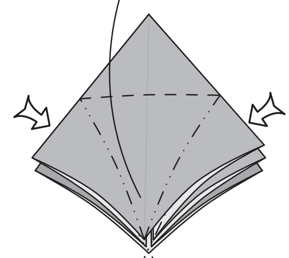
5. There is one extra flap in the middle. Fold it over to the right.



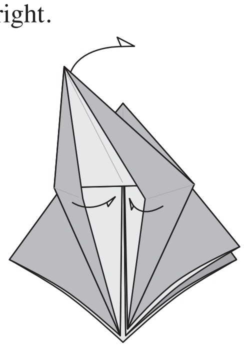
6. Fold one flap (two layers) so that the raw edge aligns with the center line; press firmly and unfold. Repeat on the right.



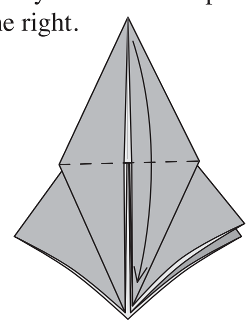
7. Fold the top flap down along a line connecting the points where the creases you just made hit the edges and unfold.



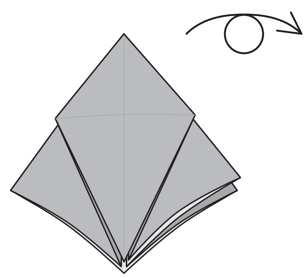
8. Lift up the bottom corner and push the sides in so that they meet along the center line.



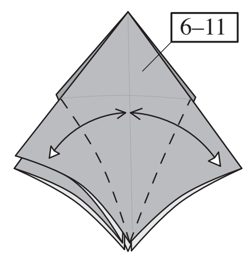
9. In progress. Flatten so that all layers lie together.



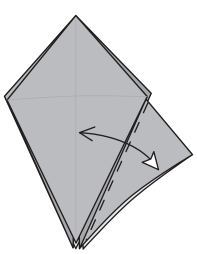
10. Fold the flap down.



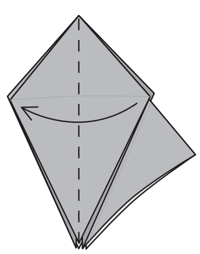
11. Turn the paper over from side to side.



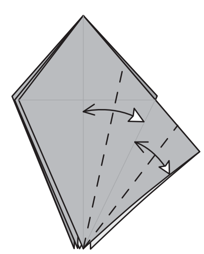
12. Repeat steps 6–11 on this side.



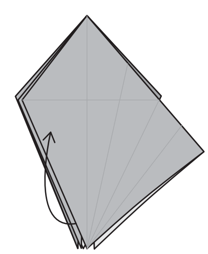
13. Fold and unfold along a crease aligned with existing edges.



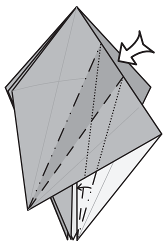
14. Fold one flap to the left.



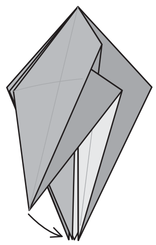
15. Bisect two angles on the wide flap on the right.



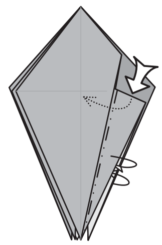
16. Lift up the near flap a bit in preparation for the next step.



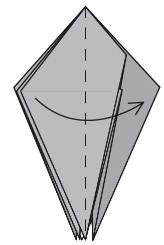
17. Using the existing creases, invert the shaded region so that the paper zigs in and out. Look at the next figure to see the result.



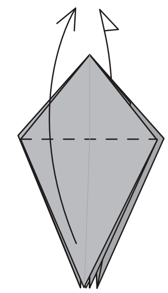
18. Flatten.



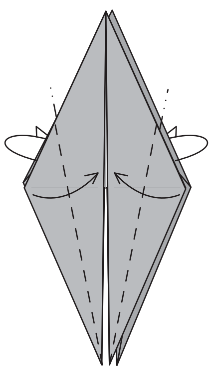
19. Invert the corner and tuck the edges inside.



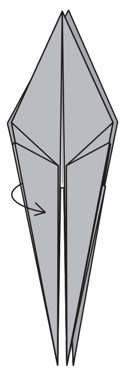
20. Fold one flap to the right.



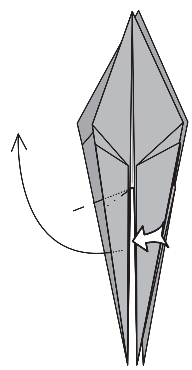
21. Fold one layer up in front and one layer up behind.



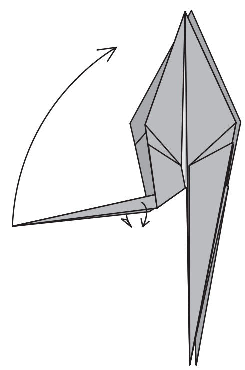
22. Fold two edges to the center line in front and two to the center line behind.



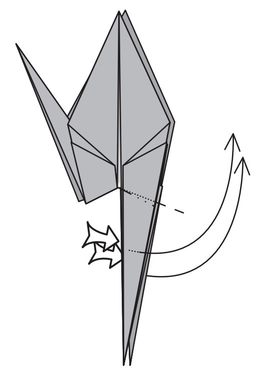
23. Open out the single point on the left side slightly.



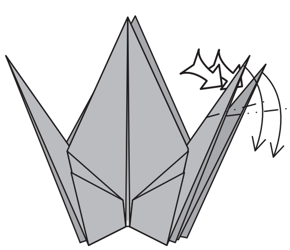
24. Push on the central edge of the flap so that it turns inside out between its upper edges.



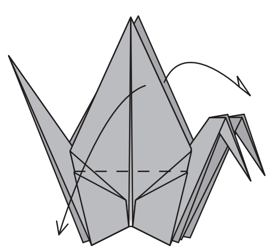
25. Swing the point up almost all the way to the top and fold its edges together. Flatten into the new position.



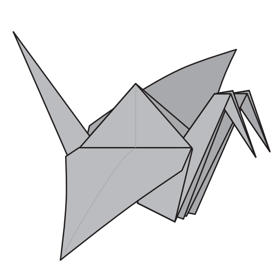
26. Repeat steps 24–25 on both of the points on the right.



27. Reverse the tips of both points so that they point downward, similarly to what you did in steps 24–25.



28. Fold one wing down in front and the other down behind.



29. The finished Hyperbolic Crane.

It's Not Music, It's Theory

Hilarie K. Orman

Abstract

Can the geometry of a 3D object be represented in music? These experiments in assigning musical notes to the faces of polyhedra and interpreting the result as scales and chords show that the problem has only partial solutions. The study suggests a more general problem in graph theory, similar to coloring.

1 Introduction

The structure of music has some mathematical properties, but do mathematical structures have musical properties? Can geometric symmetry in 3 dimensions be translated to music? These are the questions that motivated this work.

Linear mathematical structures like arithmetic sequences have been investigated by some composers, and the recent popularity of "π day" is perhaps responsible for several interpretations of the digits as musical notes or chords. On the other hand, two or three dimensions seem more difficult to represent.

This work focused on representing two particular Platonic solids with musical tones. The goal was to create something with a faithful interpretation of the geometry and to convey the visual symmetry through some kind of tonal symmetry. The secondary goal was to make a short, playable piece of music that sounded, if not exactly nice, at least not too awful. A tertiary goal was to discover something interesting about music structure.

Of the 5 Platonic solids, the octahedron and the dodecahedron are interesting targets for tonal interpretation because the octahedron has 8 faces and there are 8 tones in the usual musical scale (7 tones plus one an octave higher than the base) and the dodecahedron's 12 faces suggest the 12-tone scale on which most Western music is based.

2 Note = Face, Scale = Adjacent Faces, Chord = Vertex

The strategy we adopted was to assign a note to each face and to interpret the faces musically as the ordered set of tones on adjacent faces. Vertices were interpreted as chords determined by the notes assigned to the faces meeting at the vertex.

The first question we asked was whether or not it was possible to find an assignment of notes that would let each face have a scale with the same intervals. A computer program was set to work searching all assignments of notes to faces, eliminating those that had "bad" chords or "dissimilar" scales. For the octahedron, the scale was C major, for the dodecahedron the scale was the 12-tone.

Scales are similar if they have the same inter-note intervals. The pentatonic scale "C D E G A" has intervals (deltas) "2 2 3 2". There are pentatonic "modes" based on other intervals, particularly the pentatonic minor: "C - Eb - F - G - Bb" (3 3 2 3). Our search criteria did not dictate any particular interval set, but it did look for assignments that results in the same interval set (allowing permutations) for each face. We excluded assignments that had more than one semitone (interval 1), because these sequences were likely to sound dissonant and would not be recognizable as scale sequences. A further criteria was that the vertices (chords) should not have semitone or tritone (5) intervals, because these sound very dissonant.

Face Assignments

a	d	f
e	c	g
c	e	b
c	b	g
d	a	c+
e	g	b
a	f	c+
f	d	c+

Vertex Chords

d	e	c+	b
f	g	c+	b
a	g	e	c+
c	a	f	g
c	d	f	b
c	a	d	e

Figure 1: Octahedron Tones

A (nearly) complete search on the assignment space yielded no results for either the octahedron or the dodecahedron. The octahedron failed on the chord criteria (there are 4 notes in an octahedron chord, but only three in a face scale), and the dodecahedron failed on the scale criteria (the pentatonic scales have 5 notes). We absorbed this disappointment and loosened the criteria to allow more dissonnance.

By allowing one semitone into chords, we were able to get solutions for the octahedron that included some similar scales, illustrated below. The 3-note scales are reasonably musical, but the chords are jarring. The results can be heard via [6] and [5] .

For the dodecahedron, by defining scales as similar if the absolute values of the adjacent tones yielded some identical subsequences, we were able to find an assignment for the dodecahedron that contained some scale similarities. This is illustrated in the table below (we simplified the dodecahedron search by assigning C# to the face opposite the C face). The interval sequence "6 4 6 4 4" occurs twice (faces 1 and 12), the subsequence "3 4 3" occurs 5 times, "7 6 7" twice, and "7 6 3" twice.

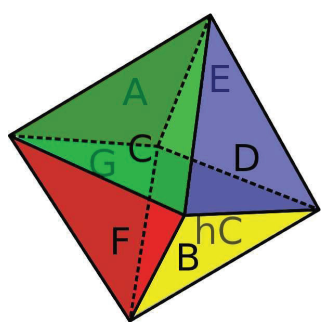


Figure 2: Octahedron

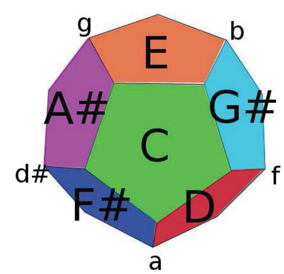


Figure 3: Dodecahedron

Dodecahedron Tones

Face Scales

a#	e	g#	d	f#
c	f#	d#	g	e
c	a#	g	b	g#
c	e	b	f	d
c	g#	f	a	f#
c	d	a	d#	a#
c#	g	a#	f#	a
c#	b	e	a#	d#
c#	f	g#	e	g
c#	a	d	g#	b
c#	d#	f#	d	f
a	f	b	g	d#

Vertex Chords

c#	f	a	e	g	b
c#	f	b	e	g#	a#
c#	g	b	d#	g#	a#
c#	d#	a	d#	f#	a#
c#	d#	g	c	f#	a#
d#	f#	a	c	d	f#
d	f#	a	c	d	g#
d	f	g#	c	e	g#
f	g#	b	c	e	a#
e	g#	b			

3 Prior Work

Mappings of number sequences to tones or chords are a popular feature of mathematical shows, and at least one is online [2]. There are a few serious musical pieces based on dodecahedrons, and one, by Joceyln Ho, is available on YouTube [4]; there is also a paper about the composition [3]. An interesting assignment of notes to the faces of a rhombic dodecahedron and pentatonic scale relationship is the subject of part of a book [1].

4 Mathematical Generalization

We might ask if there are other polyhedra that are more amenable to musical assignments, noting that solutions build on the well-known graph coloring problem. We could define a "numbering" for a polyhedron G from a set S by assigning elements of S to faces of G with the conditions that (1) every element of S is assigned to at least one face of G (2) no two adjacent sides have the same member of S, and (3) no vertex has two or more faces with the same member of S. A face scale for face F is the ordered sequence of members of S on faces adjacent to F.

For the musical problem, we wanted all face scales to be similar and all vertex sets to be "musical". The musical constraints limited the set of possible face scales. Which constraints can be satisfied with for a given G and S?

References

[1] David A. Becker. *Incarnations of the Blaring Bluesblinger: A Multimedia Mathmantra on Manifestations of Mutuality in Music, Molecules, and Morphogenesis.* University Readers, 2010.

[2] Michael Blake. What π sounds like. `http://www.youtube.com/watch?v=YOQb_mtkEEE`. YouTube.

[3] Jocelyn Ho. From 3D to 2D: Using geometry and group theory to model motivic structure in musical composition. In Carlos Agon, Emmanuel Amiot, Moreno Andreatta, Gérard Assayag, Jean Bresson, and John Manderau, editors, *Mathematics and Computation in Music: Third International Conference, MCM*, June 2011.

[4] Jocelyn Ho and Avinash Krishnan. 12 variations on a dodecahedron. YouTube. A piano solo where the structure of the music is mapped on the 12 faces of a dodecahedron and can be visualized as two pianists moving across different faces.

[5] itzaloulou. Chromatic dodecahedron, 2012. YouTube.

[6] itzaloulou. Musical octahedron, 2012. YouTube.

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ANTI – G

I begin by noting that although Scot Morris has said nothing this year about the letter "G", had he said anything, it would have been a bunch of crap! Or as I would prefer – "a bunch of garbage."

At G4G7 I spoke out against not just the number "7", but all numbers and then said I would have nothing more to say anti-number. Much to everyone's surprise, including my own, I have kept my word. I have remained silent, which is more than Scot has been able to do! But I can no longer do so. There is a scourge in the land and it all relates to the alphabet. Wherever there is a problem, the problem is spelled with letters. Problem begins with P and that rhymes with T and that stands for Trouble [or Talk, meaning this one.] Clearly, if there were no alphabet, we could not spell anything and there would be no problems. I could have chosen any letter to speak against, but since we are here for G4G10, I have decided to speak out against the letter "G."

"G" is the seventh letter of the alphabet, so I'd like to incorporate herein by reference, every nasty thing I said about "7" in my prior "Anti-7" talk. Please see the G4G7 handouts for details[1].

Interestingly, or at least I hope so, "G" is one of the many letters of the alphabet that is also a word. But what a dumb word – "Gee," as in Gee wilikers, Gee whiz, or Golly gee. Try saying one of those while seeming to maintain any IQ of over 80. For those of you willing to admit to any IQ of less than 80, I recommend incorporating some of those expressions. They will make you seem smarter.

I know "G"[2] is a somewhat controversial choice for an "anti" lecture, for are we not here to celebrate Martin Gardner, in the state of Georgia? Not only has G-d chosen "G" for an initial, but it is clear how his son's name, Jesus, should have been spelled! But, if you're going to go for it, start with the hard one!

Normally, I include at least one fancy, but meaningless, mathematical formula in these talks, but that would seem pointless with G. However I do happen to have one equation left over from prior talks, so I'm going to toss it in here just to use up a power point slide.

G = [˥*Ђ ^ □] / [□*□ ^ ґ]* □

Of some interest I think, is that all these symbols represent "G" in some language or script.

Obviously I would have almost endless potential examples of what a terrible letter "G" is, if I were to use words or names that contain "G", or even those that start with "G."[3] So to be sporting I will be primarily using only examples with "G" as a significant initial.

"G" has many uses in everyday language.

G represents Gravity – talk about something that will bring you down.

The expression G-men[4] referred to FBI agents. FBI has three perfectly good initials, why G would be used for FBI is beyond me.

4G – Life was much better when I could just make up facts, state them with authority and be believed. Now someone always has a 4G phone to Google the subject and find the real answer. It's just not worth making up stuff anymore.

Gmail – If it were not for Gmail those kind Nigerian princes would not be waiting for me with the many millions of dollars they have decided to give me for somewhat unclear reasons. Gmail has also often brought me the good news that I have won a lottery I did not enter. By next year I will be rich. Rich! I tell you[5].

G–String – A terrible misogynist bit of clothing used only to tempt men from the straight and narrow path. It is detestable and a cause for disgust. Here is a photograph from the Internet. Let's take just a few moments to be repulsed. [Pause] [PowerPoint slide]

G–Line subway in NYC – Goes from Court St. to Church St. Who cares?

G Sharp – Do we really need it? No, seriously, do we really need it? I can't tell.

G – in Morse code – " - - ." No comment, just thought it might come in handy someday.

G – in Braille – "::" See above comment.

G spot – I'm a little uncomfortable with this one, for I don't even believe it exists, but if any woman feels differently I am not going to debate it. I will only say that if it does exist, it just isn't worth the effort to find it.

F*****G – Not really a "G" word, but I'm just proving I don't spell very well.

"G–Force" – It may not be the worst movie ever made, but it is surely the worst movie ever made featuring Guinea Pigs

Kenny G – Please insert your own jokes.

G'Day – This is Australian[6] for "I'm trying to sound like an Australian." We could do without it.

g = gram – You'd think this is just another harmless measurement of weight, but I've never known anyone to use it for anything other than selling illegal drugs.

G20 economic group – you can see how well that's working.

Warren G. Harding – Try and find him on a list of our great presidents.

G. Gordon Liddy – A man so strange that he once tried to pick up a woman by burning his palm over a candle, then he bragged about it.

Susan G. Komen – Honestly, I would never have thought of including her, but I have strong feelings vis-a-vis Planned Parenthood, so in she goes.

GOP – I'm not going to push this one, but as I've already said, I have some strong feelings.

G. I. Joe – Amazingly, he is NOT a real American hero. Like my talk, they just made him up.

Burton G. Malkiel – A very famous economist, which means no one has heard of him, but he once taught a class I was in so he gets included.

G – The movie rating. A guarantee that not even your children will enjoy the film.

Jihad – G is such a terrible letter that even Jihadists refuse to spell their name correctly.

Anti-G Program - It is not enough to just explain the evils of "G," but I have a seven step [A-G] program to greatly reduce the use of "G" in our society.

[A] Make sure you never go to Gabon, Gambia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea Bissau, or Guyana.

[B] Learn to spell without the forbidden letter. "J" will usually work as a substitute. Besides, Wildebeest sounds much better.

[C] Gonorrhea – Try to avoid it. If you must have an STD, there are many other choices.

[D] Avoid politicians with G names. I mean Gingrich? Really, Gingrich? [That line was much funnier when I wrote it in December.]

[E] Never Gerrymander, even if you have to Google it just to figure out what it is, so you can avoid it.

[F] Teach your children to spit whenever they hear the letter "G." Please also teach them to clean up afterwards, just in case I'm nearby.

[G] This is not just the last step, but it is by far the most important! We must . . . Oh, I'm sorry; I see my time is up. Thank you.

---

[1] G4G7 Exchange Book – Vol. 2, page 294
[2] The Really Big Book of "G."
[3] The Oxford English Dictonary – Special edition without the letter G.
[4] The Big Book of Sounding Tough.
[5] I mean, why would a Nigerian lie to me?
[6] See "Australian for the rest of us."

Perspicuity: Cartoons by Craig Swanson

The following are nine math-themed cartoons that I have drawn over the past 20 years.

The first six come from my book "Perspicuity: The Early Quirks of Craig Swanson" and are printed with accompanying essays.

The last three were drawn after I put the book together and are, for now, essay free. Just a couple of notes on them:

- *Sine Die* has been called my most obscure cartoon (mostly by my wife Cori).
- *What are you trying to prove?* does not feature any particular mathematician, though it probably should.
- *Refer Madness* has a puzzle embedded in it – namely the order of the dashed lines.

Enjoy!

Craig Swanson

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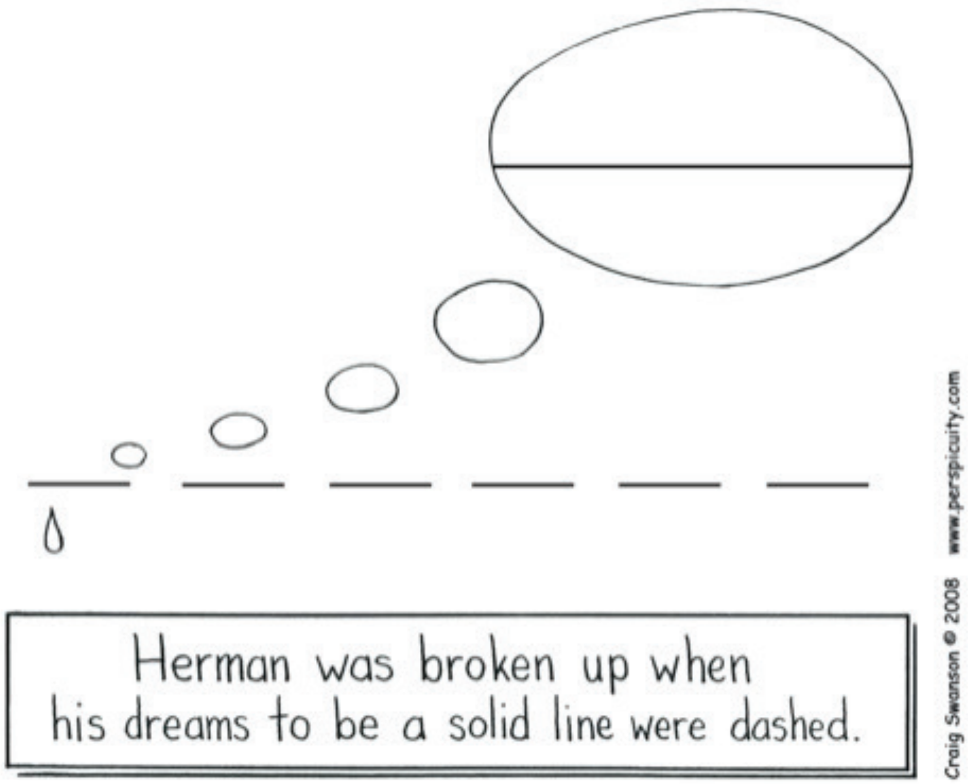
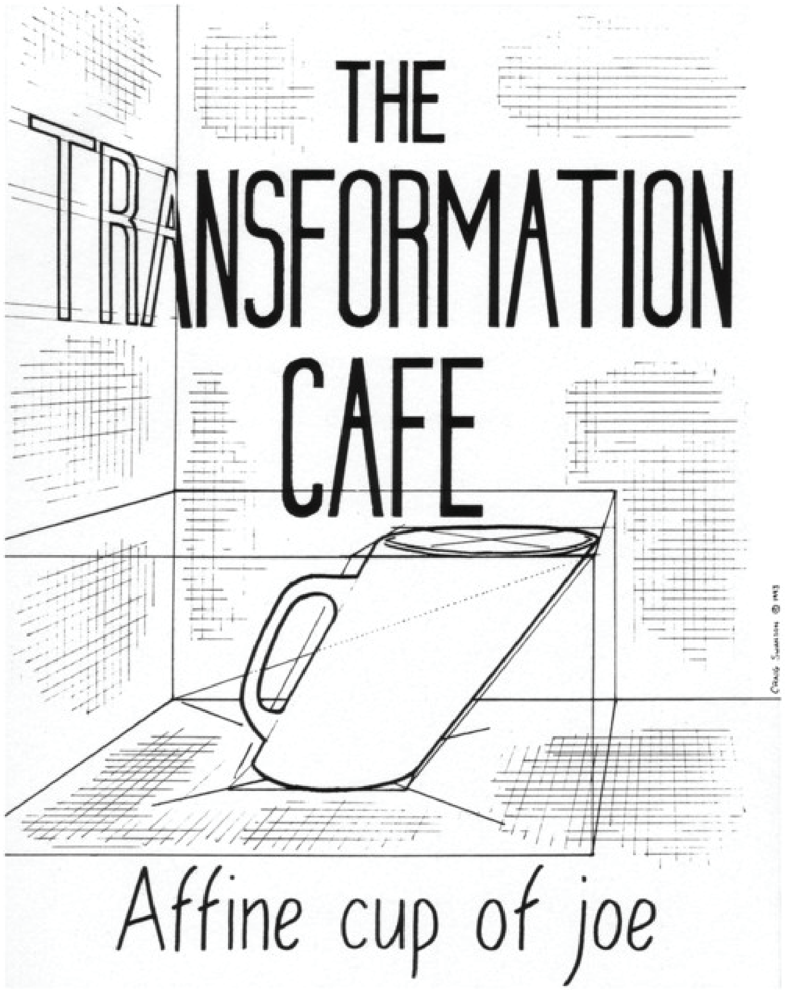
www.perspicuity.com



Herman the Line

And if this wasn't bad enough, Herman had to take a weekend job dividing lanes on the highway.



The Transformation Café

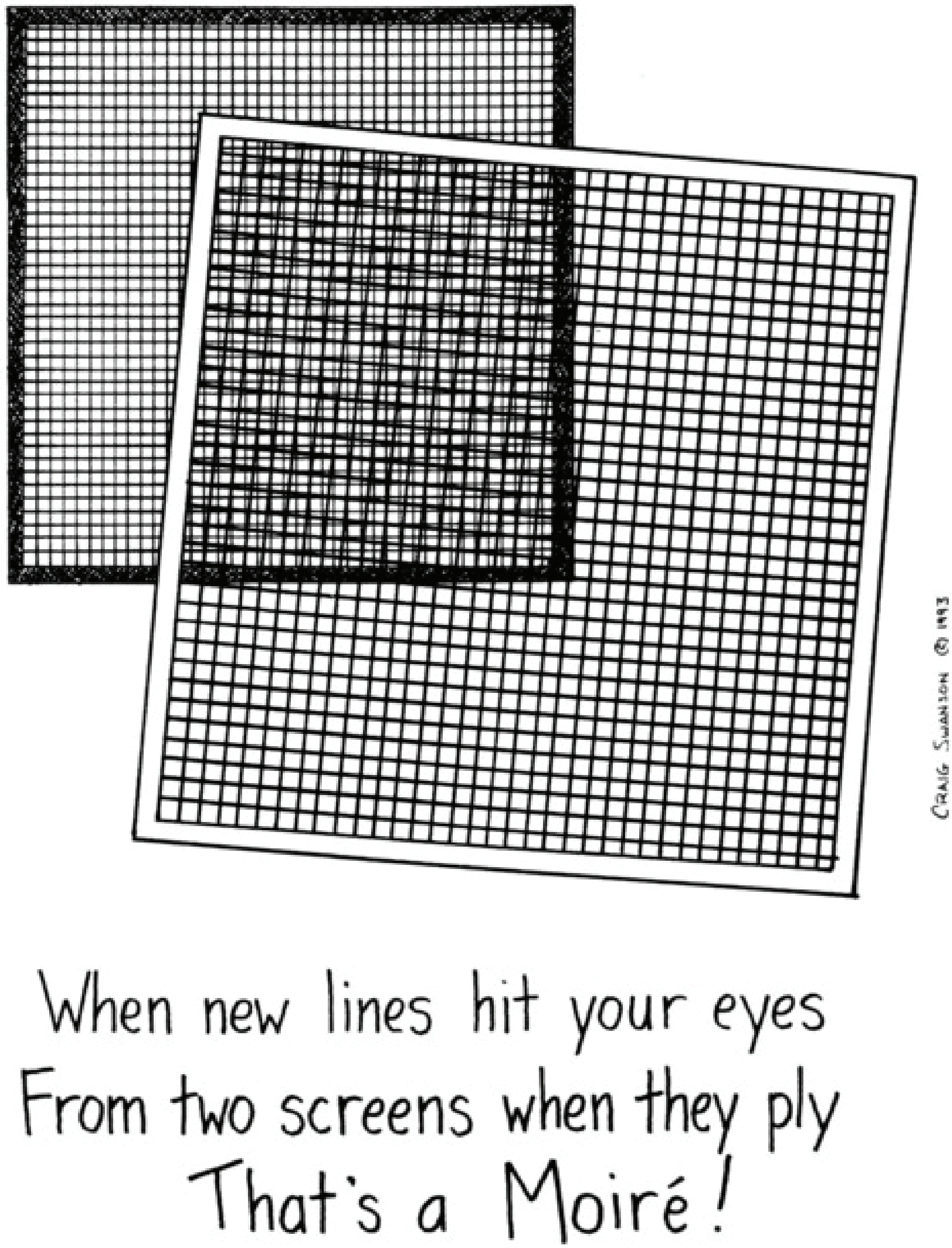
The Someday Café had a sign behind the counter that says "A fine cup of joe." After viewing it hundreds and hundreds of times, my mind saw it as "affine," which is a mathematical process of transforming images by scaling, translating, and rotating them.

Etymologically, "affine" comes from the Latin for marriage. The word "affined" is no longer in use, but once meant to be related by marriage. We still use "affinity" to mean an attraction. So graphical images that are produced via affine transformations are considered to be related to each other, differing only in size, location, or orientation.

Then came the cartoon. The coffee cup image is skewed, which results from a combination of affine transformations, perhaps a rotation, a scaling, followed by a rotation back to its original position.

Got it? Don't worry, few people do. And besides, nobody I have to explain it to thinks it's funny.

Too bad. I like it.



That's a Moiré

I think my little ditty (to the tune of "That's Amoré") describes a moiré pattern quite well. As far as I know, this is the only joke that explains itself.

Here is an example of finding someone else having done the same thing. A friend showed me a book, by humorist Spider Robinson, which contains tons of other variations on "That's Amoré;" all puns on amoré homonyms. At least one of them was one I had planned to do in the future. I guess I've been freed up from that one.

Sneak Preview

Last week, as a number of you may well remember, amidst the whirlwind of many current projects, I took Thursday and Friday off, and flew out to Rootsbury, Vermont in order to attend the annual Mathematical Awards for Metaphysically Motivated Individuals. Each year excitement and enthusiasm is radiated by everyone involved with the MAMMIs and this year proved no exception.

Many of the same faces seen last year and the years before were visible throughout the evening. But this should also be known as the year that some new figures appeared; at least ones that had not been seen for quite some time. One such individual was the Square, who placed third in the Prominent Geometric Figure category. Not since the ;50s, when he was blacklisted as a social deviant, had the Square shown his surface around these award ceremonies. As he accepted the prize, the Square kissed the trophette and sang, "MAMMI, How I Love Ya!" It was the Sphere, however, who won Most Prominent Geometric Figure. Apparently, the Circle and the Sphere shared many of the same qualifying characteristics; both shapely, well-rounded, and highly respected figures among the geometric circuits. But it was the Sphere who rolled away with the award, for as one judge put it, "She just has more dimension to her."

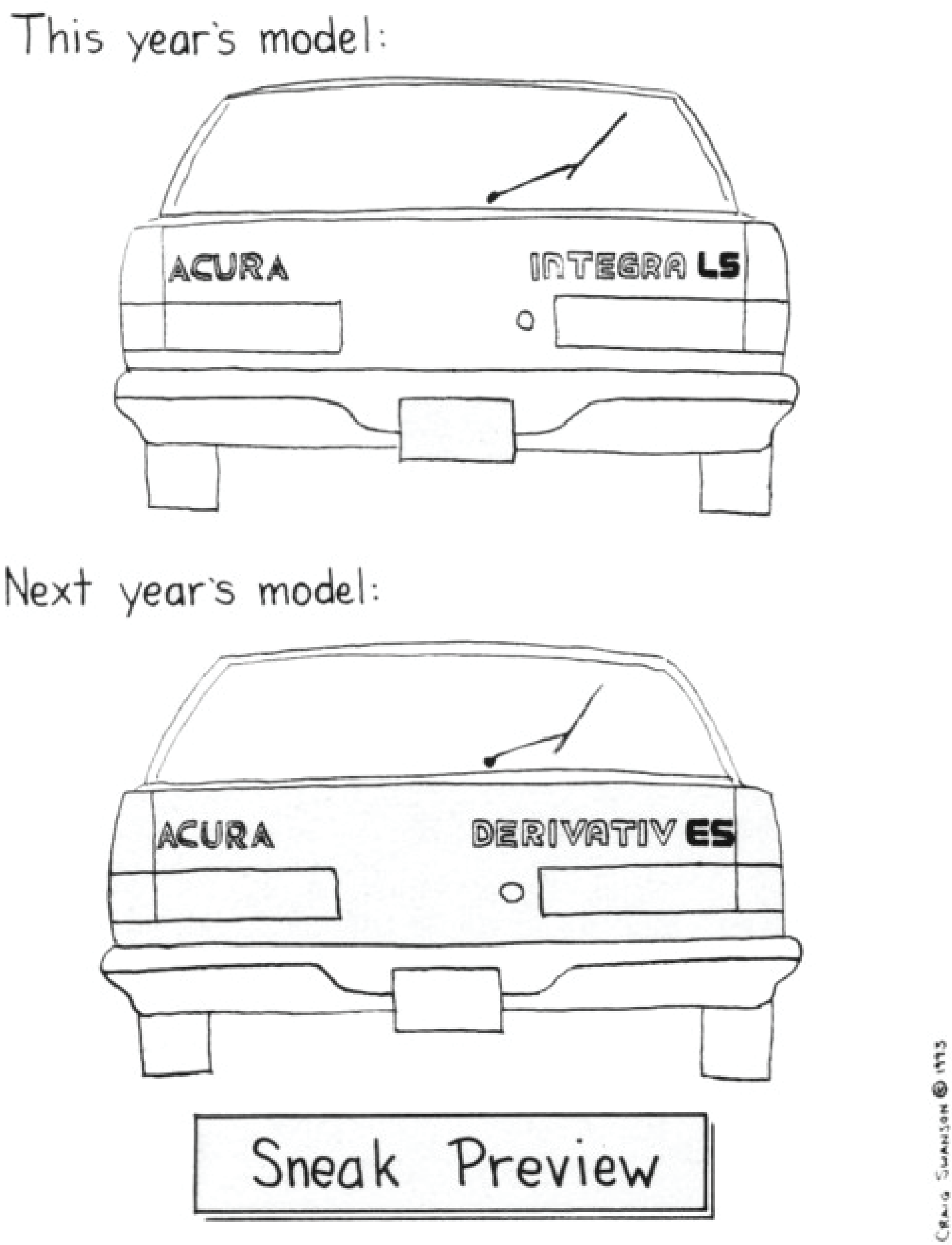
An integral part of the MAMMIs is the Calculus Award. This year it was presented to two nominees: $f(x) = e^{ix}$ and $f(x) = \cos(x) + i \sin(x)$, because the judges could not differentiate the two.

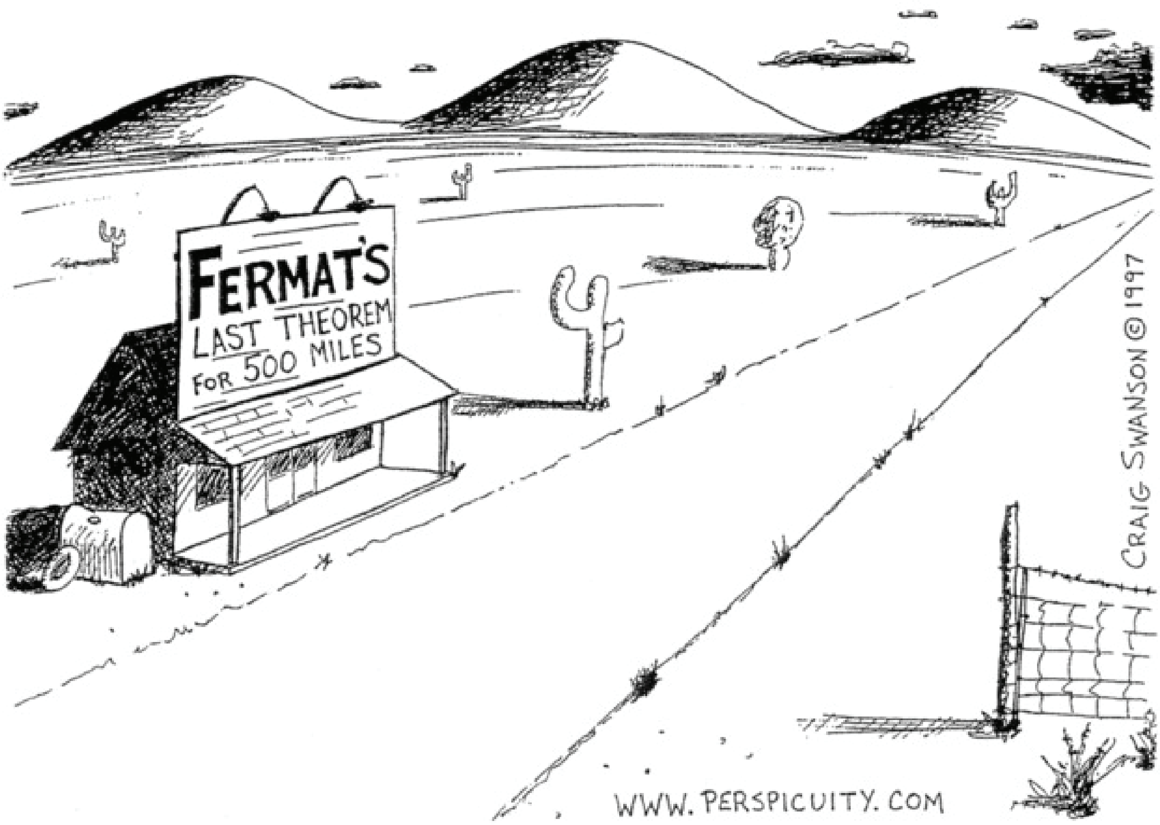
Throughout the year, everyone was certain that the Mathematical Dance Award was going to the Rec-Tango. That was before the Double Helix Twist arrived late in November with such popularity that it shot asymptotically to an international hit. DHT, as it is affectionately referred to by its devoted groupies, also placed first in the dance division of the biological and chemical award ceremonies, making it the first dance to ever sweep the MAMMIs, BAMMIs, and ChAMMIs.

The Mathematical Adage Award was presented to "The sum of the limits is equal to the limit of the sums," while the Math Song Award went to the old country favorite, "You Can't Add Apples to Oranges."

Although, in general, the MAMMIs went fairly smoothly, it was not devoid of tensor moments. Early in the evening a young quadradical caused quite a commotion as he stood on his chair and chanted, "Power to the Polynomials!" Despite efforts by security guards, no one was able to apprehend the delinquent equation, and he was able to escape by integrating with the crowd. In addition, the zero suffered an identity crisis and had to be removed from the group.

In summation, with all unexpected incidents aside, the MAMMIs were once again a success. It would be difficult to walk away from the Mathematical Awards for Metaphysically Motivated Individuals with anything less than positive feelings.


This year's model:
ACURA
INTEGRA LS
Next year's model:
ACURA
DERIVATIV ES
Sneak Preview
CRAIG SWANSON © 1993

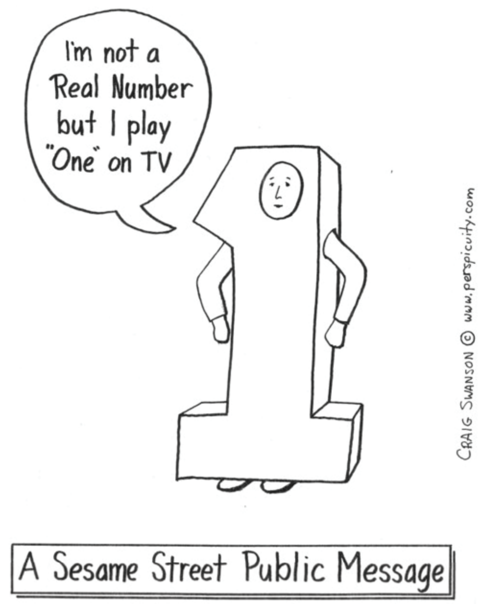


Fermat's Last Theorem...

Homage to Pierre de Fermat's old problem, which stated that $x^n + y^n = z^n$ has no solutions for positive integers x, y, and z, if n is greater than 2. This became known as **Fermat's Last Theorem**.

There are many solutions for n = 2, such as when x = 3, y = 4, and z = 5. Fermat said he had a proof that n could not be larger than 2, but neglected to write it down.

For 350 years no one could prove whether it was true or not. Finally in 1993, Andrew Wiles (with help from Richard Taylor) solved the problem. Fermat was right.



A Sesame Street Public Message

We (as humans) probably first used numbers in order to keep track of our sheep or count our coins. All we needed was 1, 2, 3, and so on. We now call these **counting** or **natural numbers**.

You can add and multiply counting numbers and the results will still be counting numbers. However, once you start doing subtraction, you find taking 7 away from 7 gives you **zero**, which is not a counting number. And if you take 5 away from 3 you get -2, which introduces the concept of **negative** numbers.

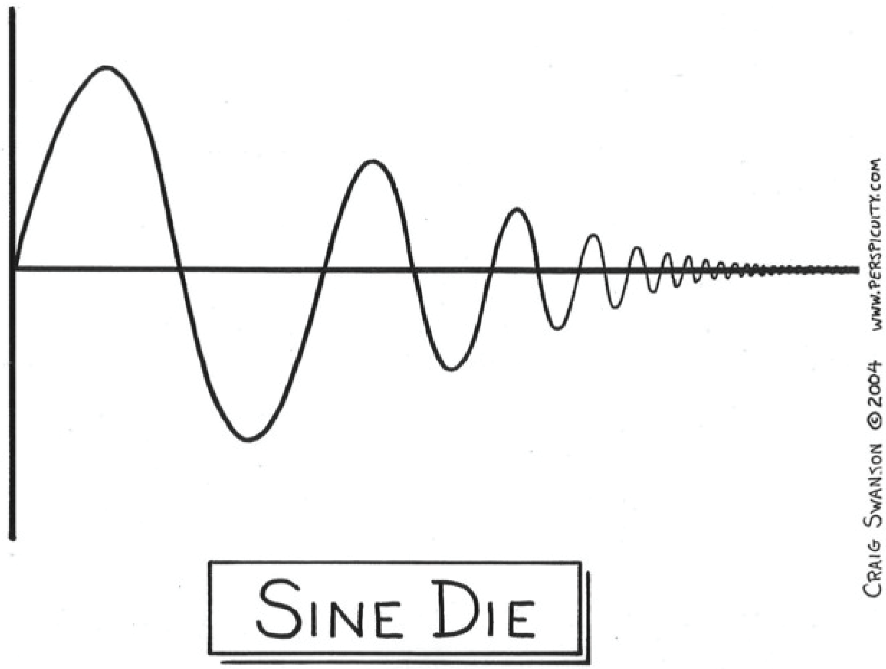
Integers are the set of numbers that include the counting numbers, zero and negative numbers. Integers are quite useful until you include division, as when we divide 7 into 10 equal parts. Here you get what are called **fractions**, in this case 7/10.

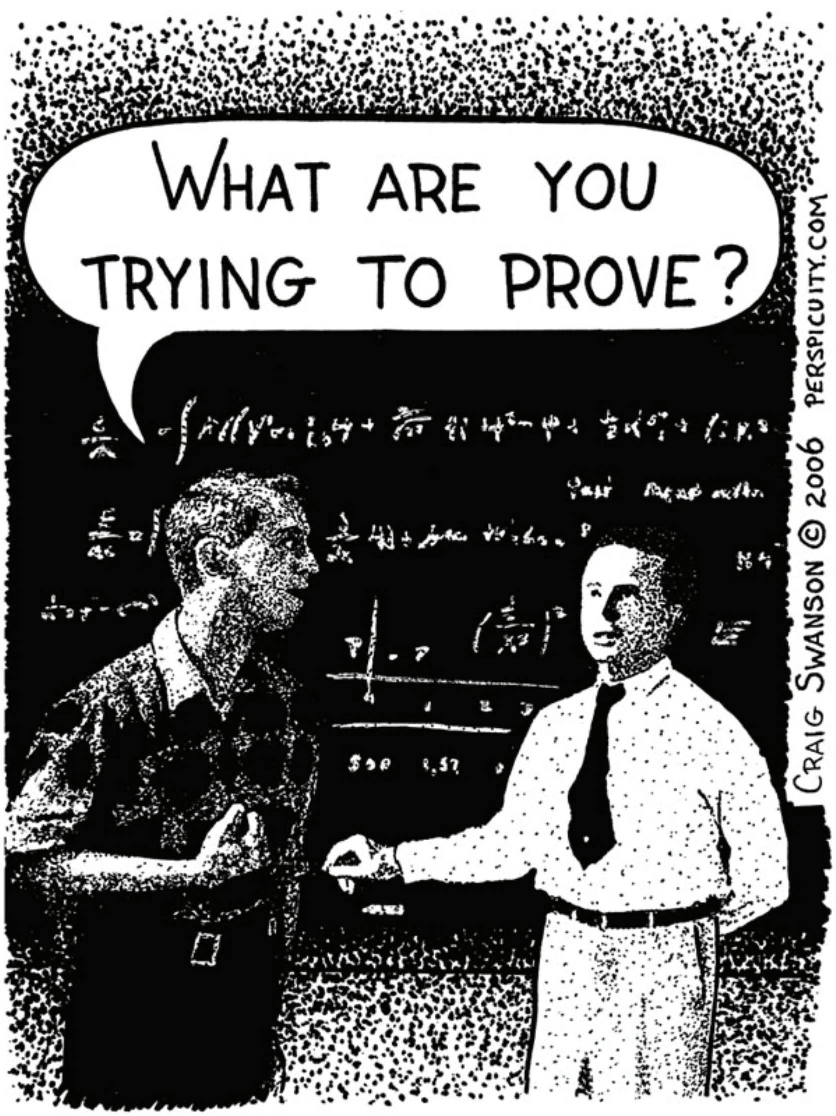
But, of course, these are just a few of the types of numbers you might come across. There are circumstances where numbers cannot be represented by integers or fractions. Two such examples are the square root of 2 and pi, which is the ratio between a circle's diameter and its circumference). These are called **irrational numbers**.

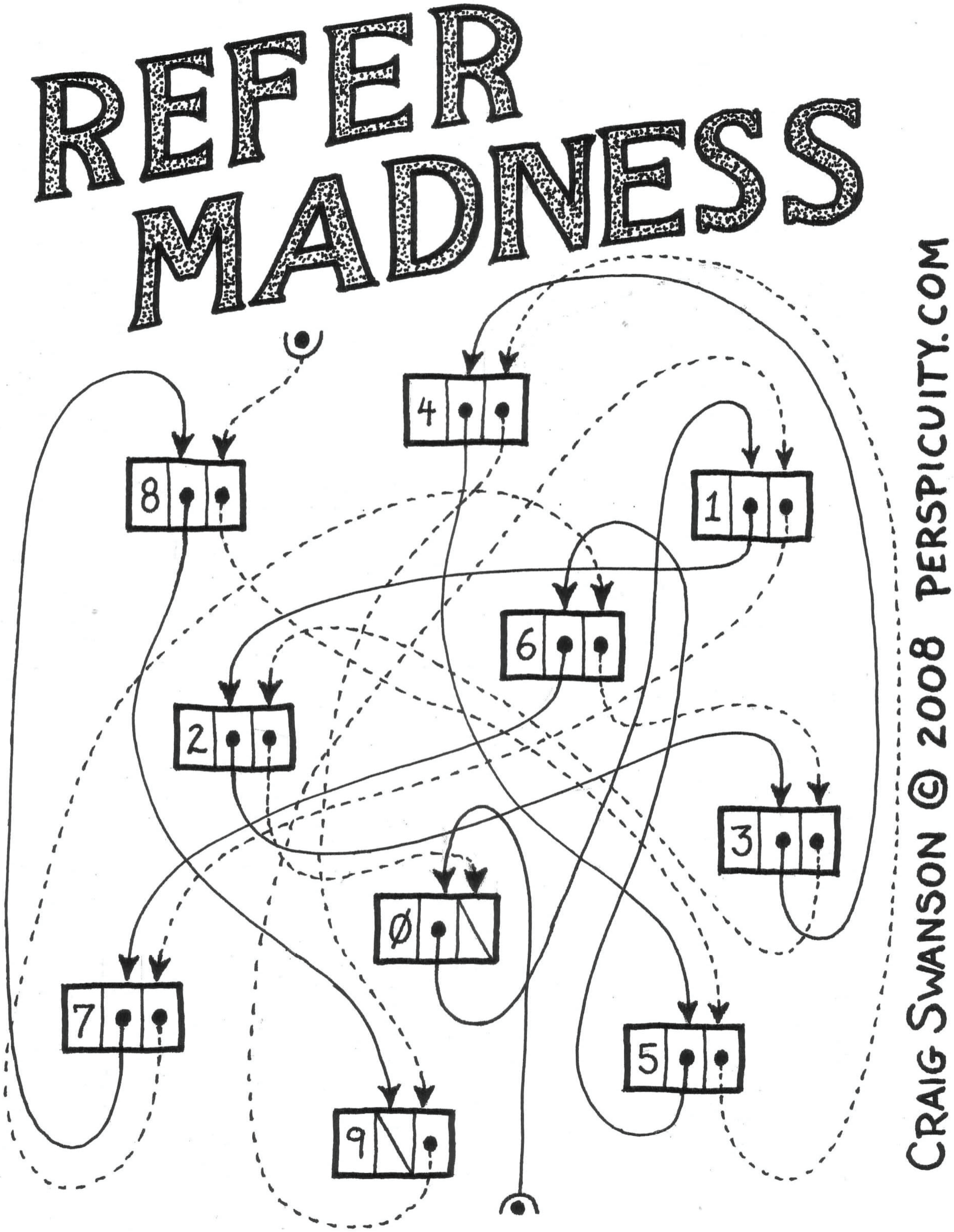
When you take the set of all integers, fractions, and irrational numbers you have **real numbers**.

There are also **imaginary numbers**, which is what you get when you multiply a real number by the seemingly impossible (represented by the symbol i). When you add a real number to an imaginary number you get a **complex number**.

I learned all this on Sesame Street. Thanks, Ern!


SINE DIE
CRAIG SWANSON © 2004 WWW.PERSPICUITY.COM


WHAT ARE YOU TRYING TO PROVE?
CRAIG SWANSON © 2006 PERSPICUITY.COM


REFER MADNESS
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Twas the Night Before Gardner
T. Arthur Terlep

Twas the night before Gardner, when all through the Ritz
Every a Gatherer was stirring, for registration blitz.
The name tags were hung o'er every neck with care,
In hopes that the bar bets soon would be there.

The guests then nestled all snug in their beds,
While visions of polyhedra danced in their heads.
With Setteducati in his suit, and Bruce in his cap,
The planners began for a great week's hap.

When up on the stage, there arose such a clatter,
I woke from my seat to see the Mad Hatter!
Counting off days, he went like a flash,
And informed us acutely of a Doomsday bash.

The moon in the sky, so often in tow
I couldn't count time in the basement below.
When, what to the back of my eyes straying nerves,
But a two headed hyper fowl swimming space-filling curves.

Then a bundle of cards, he arrived on the scene,
I knew in a moment, it must be Lennart Green.
More quickly than quarks, his colleagues engaged.
Fooled though I was of the illusions they staged!

"Now Magic! now, Puzzles! now, Science and Math Games!
On, Music! On, Knitting! On, on Sculptures by big Names!
To the top of the stand! In front of us all!
Present away! Dash away! Go eat lunch at the mall!"

To Tom's house we came, very grateful at this,
Wishing him well, this great man we miss.
But up near the house, my Go game near through,
I watched from afar as the geometric structures grew.

When then, for a moment, I heard a Hart sing
While we some sipped sake through bamboo cut ring.
As we tuned to the taps, my meter turned 'round,
And was sad to find out, the last bus was due bound.

The exchange room was filled, from the door to the wall,
While Thane kept quite calm, despite din in the hall.
A bundle of Toys I had flung on my back,
And I refilled my suitcase, just so I could pack!

Then stones-how they twinkled! My Goban so merry!
His groups were like boas, his shapes didn't tarry!
I recall that my loss was drawn up on this Go,
And Berlekamp's count confirmed what I know.

Later Dennis plied pipes and bent pans in his grip,
And phonebooks likes leaves flew out from a rip.
We thought he was fake, before bowling ball belly,
And twisting horseshoes, like Dunkin' Donuts filled with jelly!

Strick said "Let's dine!" At the Sundial we all ate
But I like a rabbit cried "I'm going to be late!"
These friends newly found and this gathering I was at
Inspired me to write this "Twinkle, Twinkle, Little Bat."

As I set down these words, my gift exchange work,
My timeline's askew; my meter's a-jerk.
But illusionists said that your memory's a farce,
So in judging this, well, I hope you're as sparse!

And now, time to go, I ran for my train,
As days' past events brewed coffee bubbled brain.
But I heard Martin say, as I rode out of sight,
"Happy Snark hunting to all, and to all a good flight!"

Gathering for

Gardner

12345678910

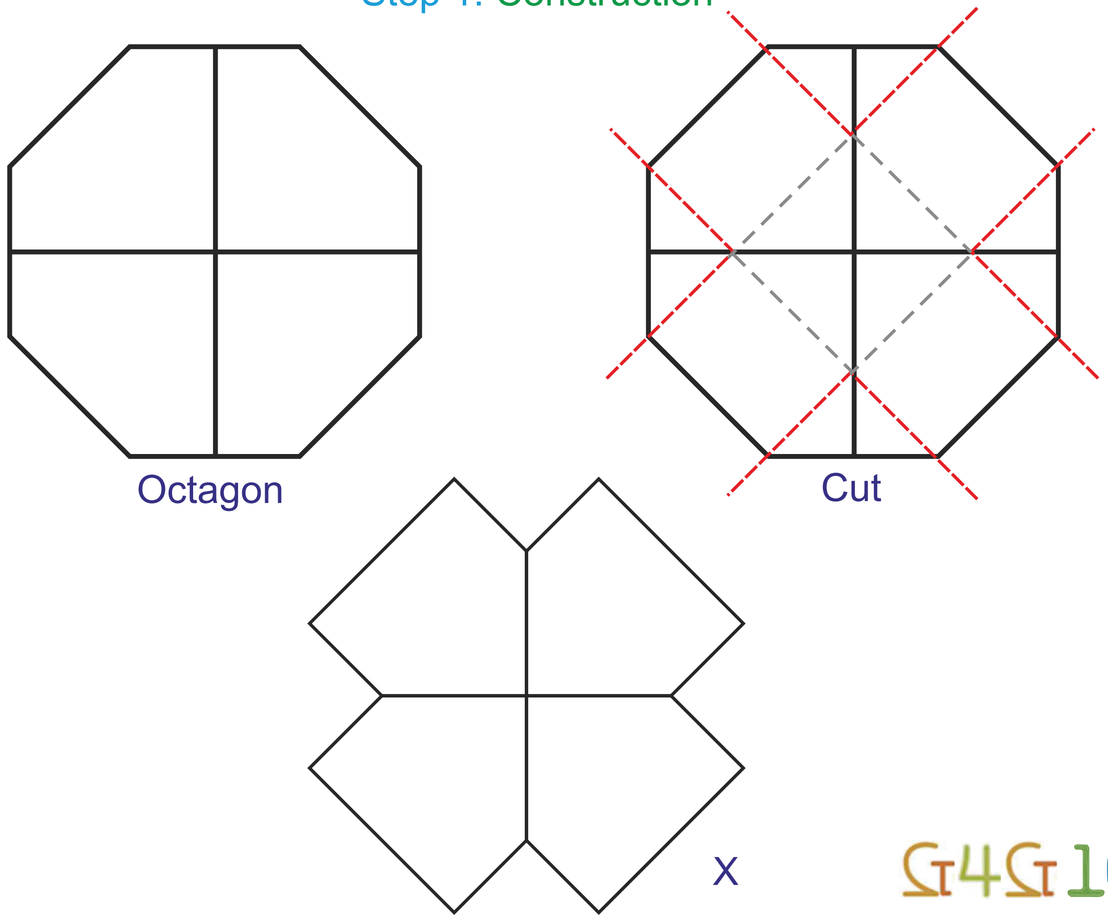
Atlanta

2012

Zdravko Zivkovic

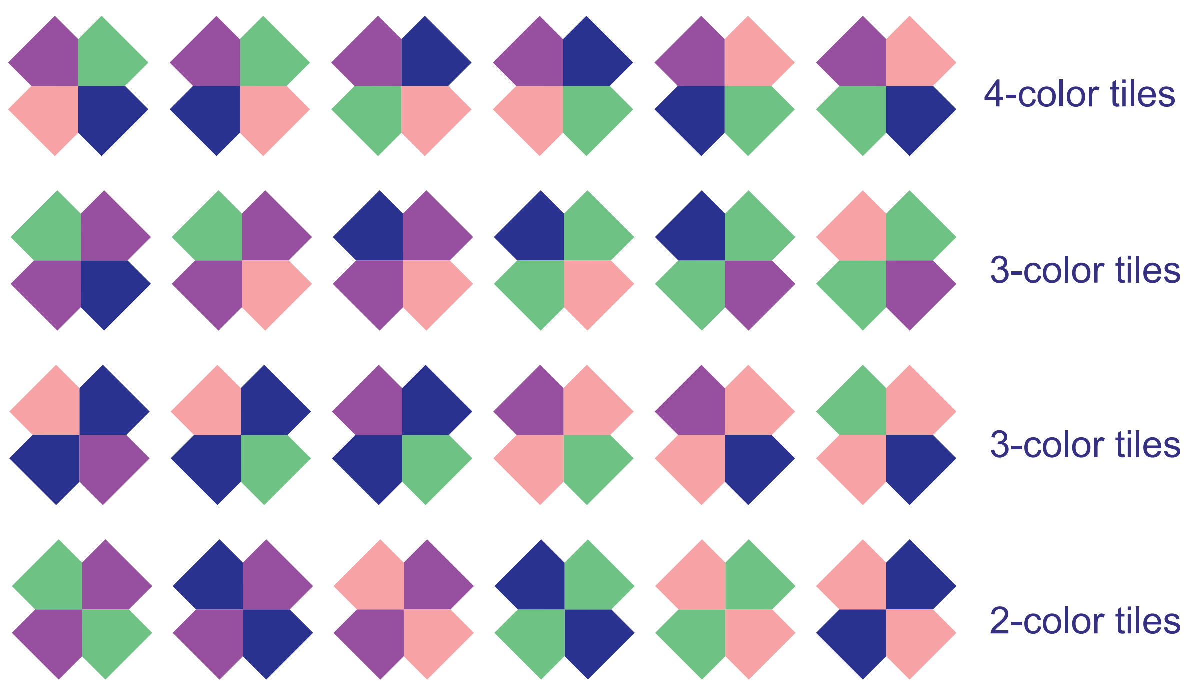
Serbia

Step 1: Construction



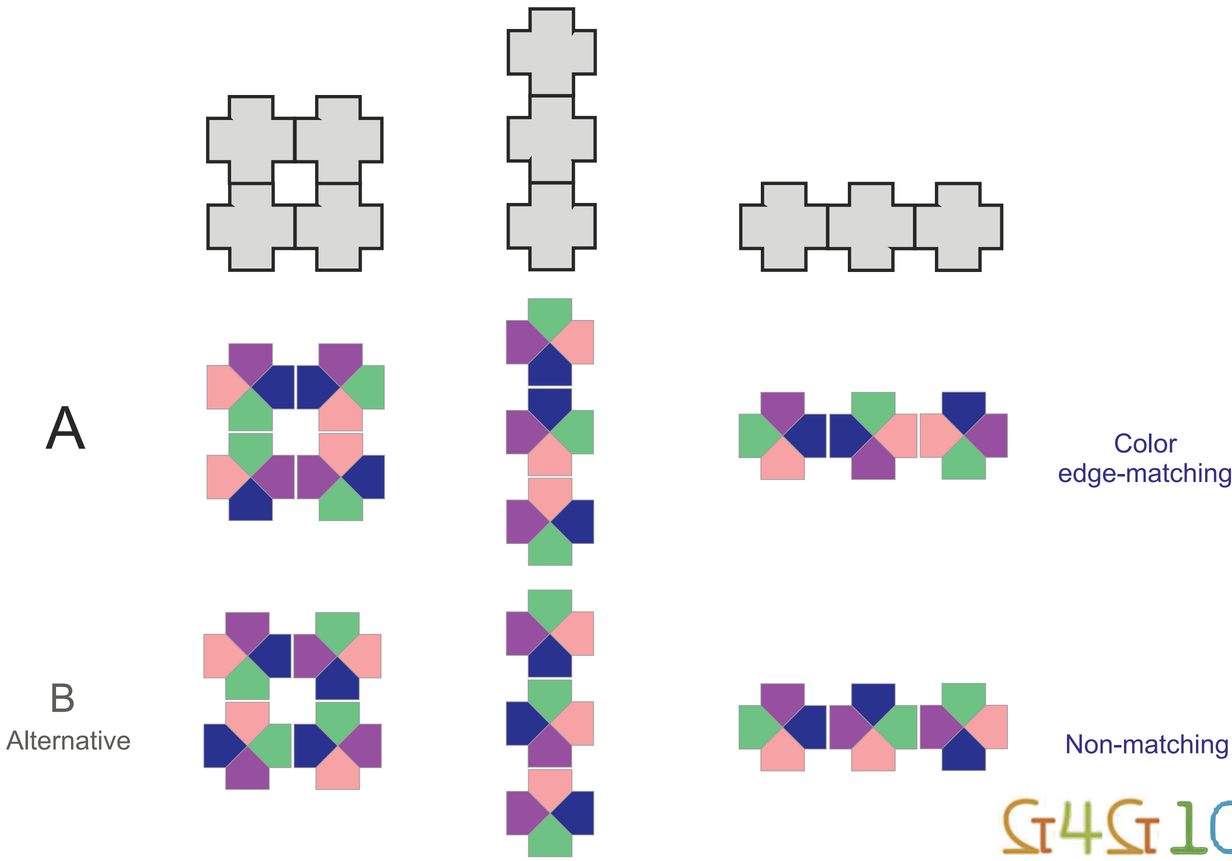
G4G10

Step 2: 24 four-color tiles set

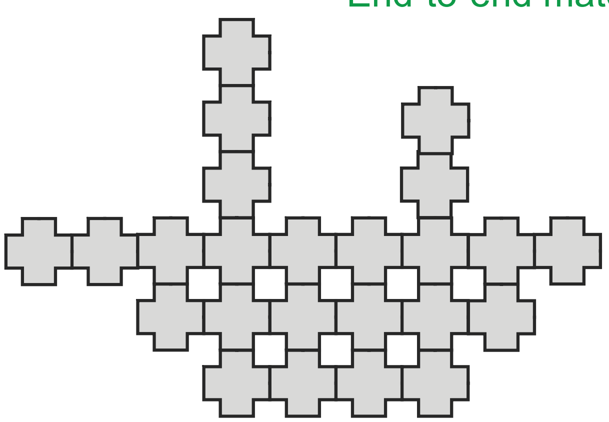


G4G10

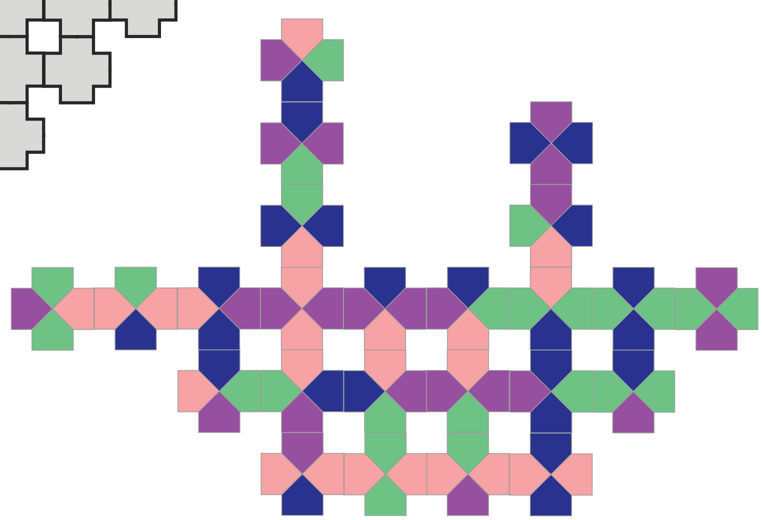
Step 3: End-to-end matching



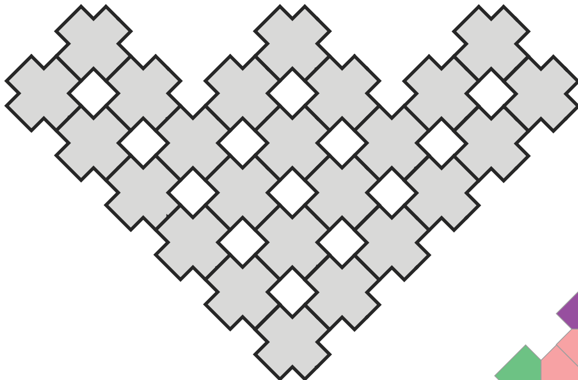
End-to-end matching: examples



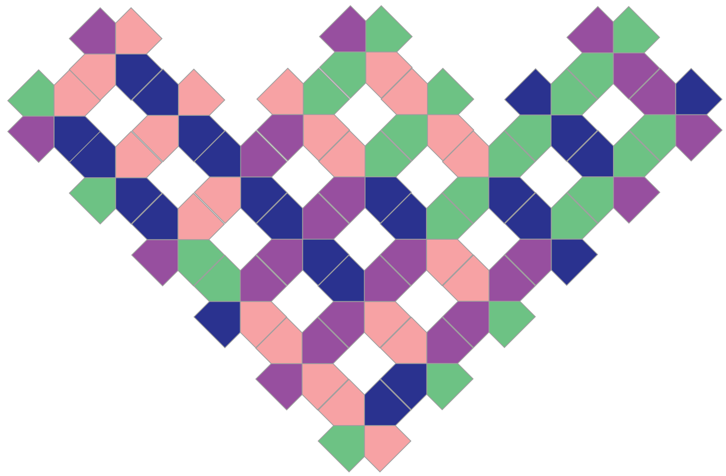
Example 1
(Ship)



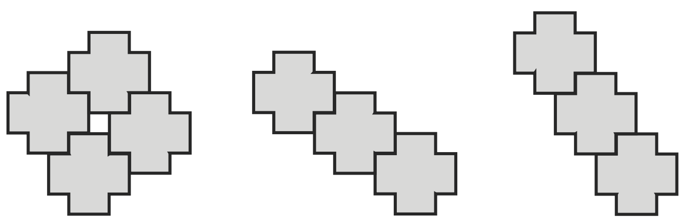
End-to-end matching: examples



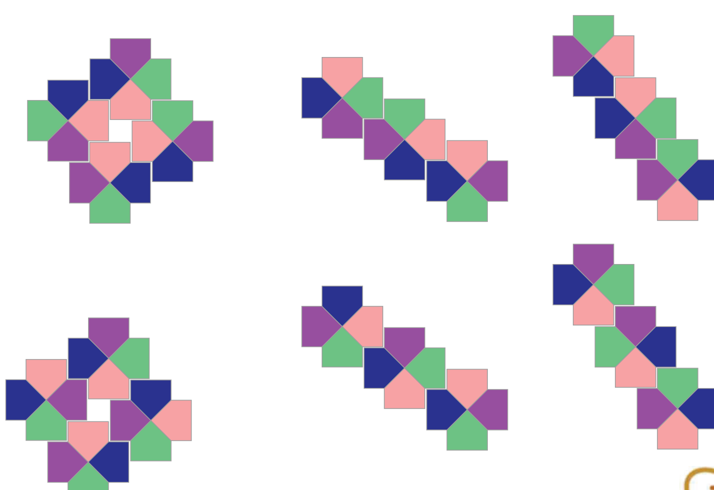
Example 2
(Heart)



Step 4: Corner-to-corner matching



A

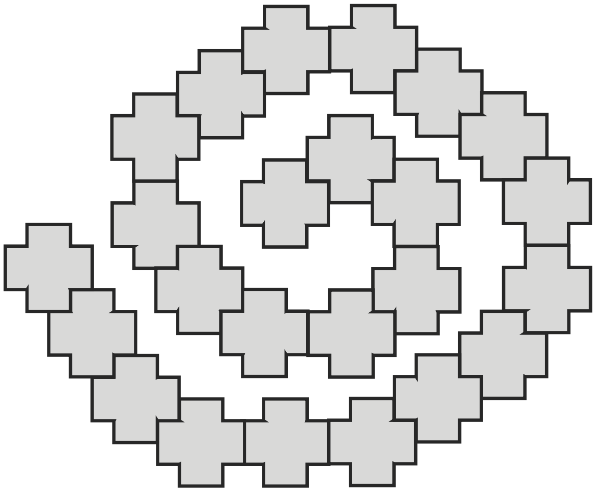


Color
edge-matching

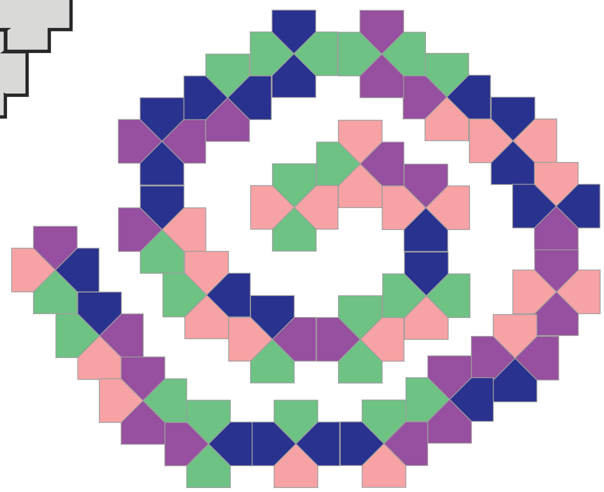
B
Alternative

Non-matching

Corner-to-corner matching: examples

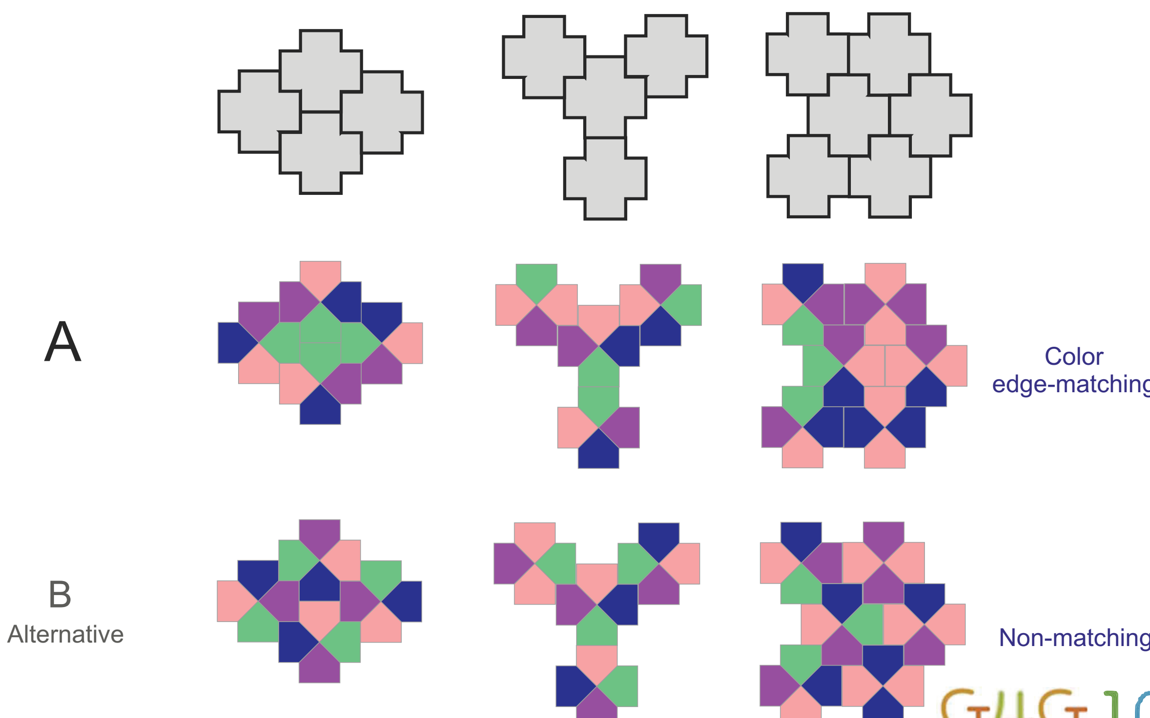


Example 3
(Spiral)

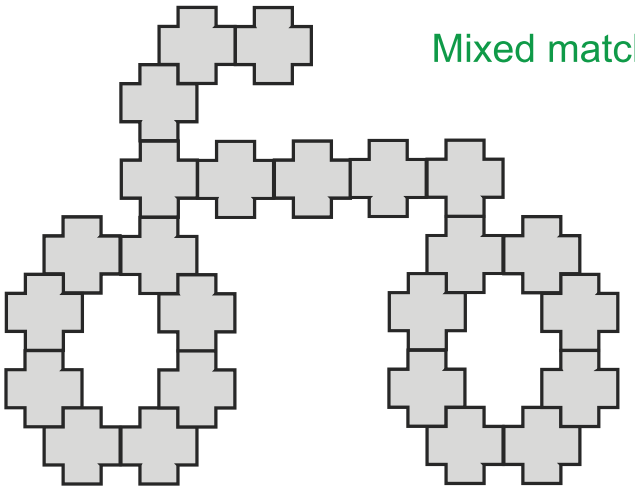


Step 5: Mixed matching

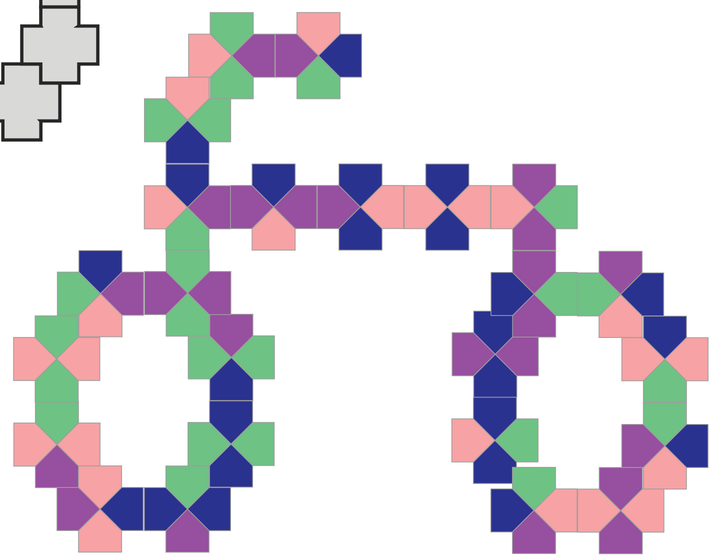
(end-to-end & corner-to-corner)



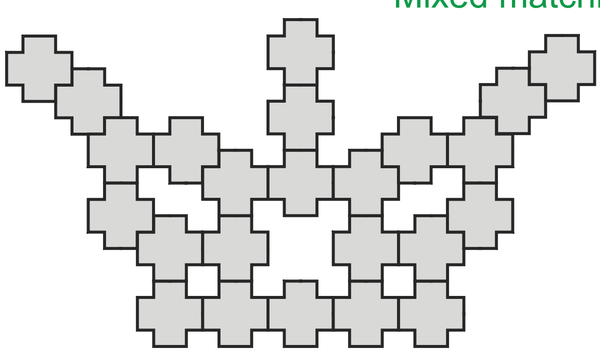
Mixed matching: examples



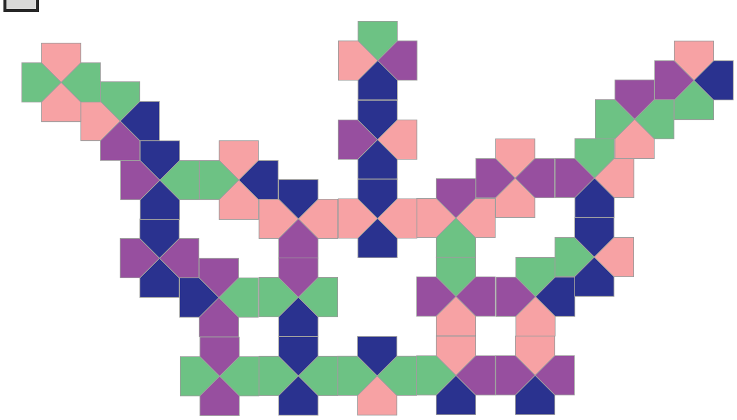
Example 4
(Easy - Bicycle)



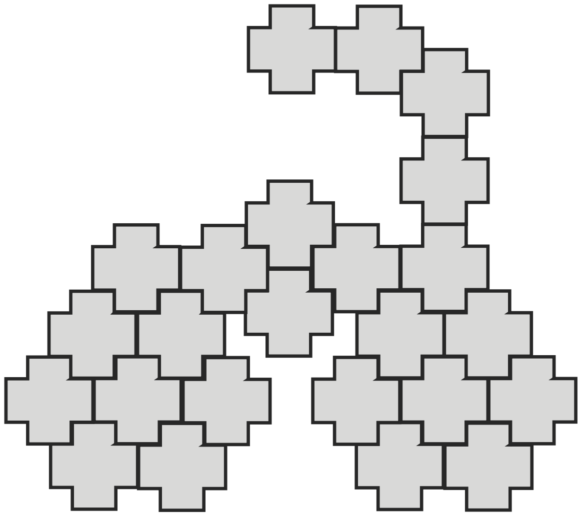
Mixed matching: examples



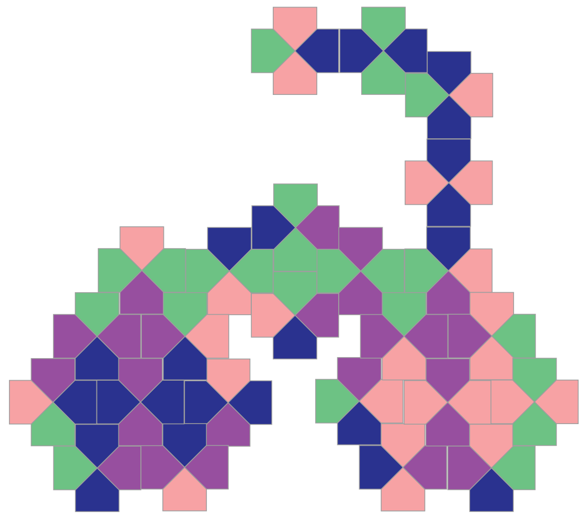
Example 5
(Easy - Crown)



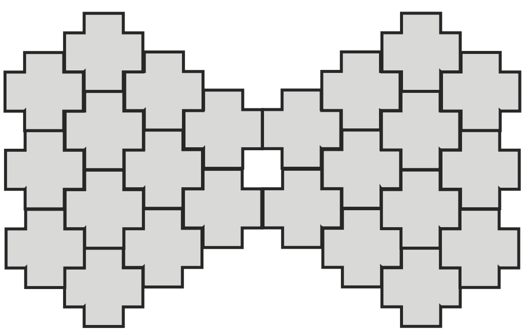
Mixed matching: examples



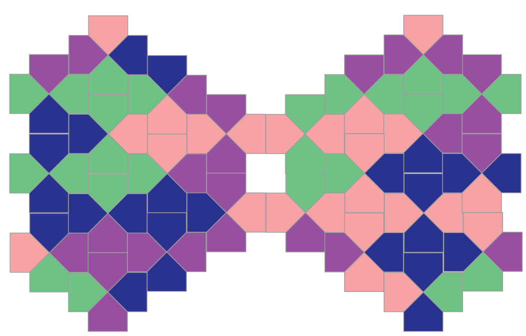
Example 6
(Medium - Bicycle)



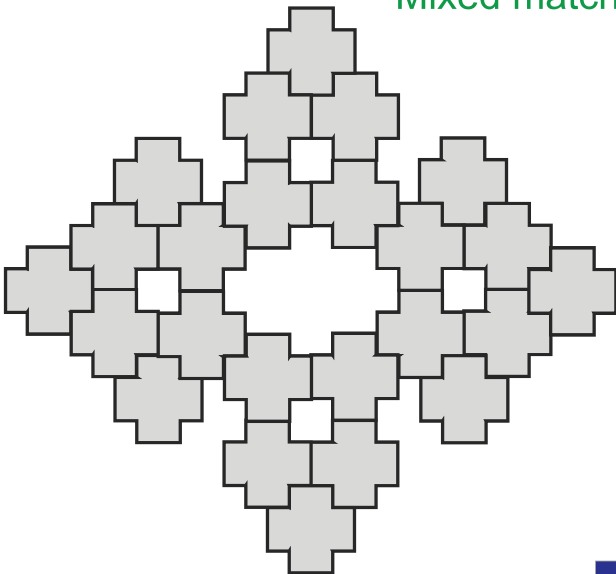
Mixed matching: examples

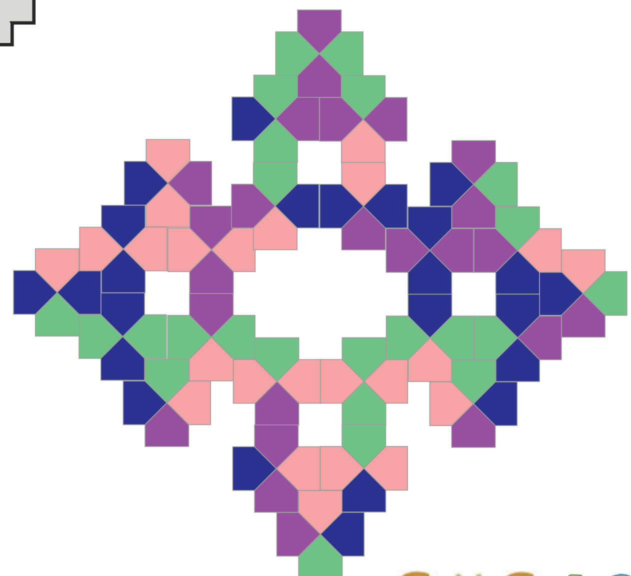


Example 7
(Medium - Bow-tie)



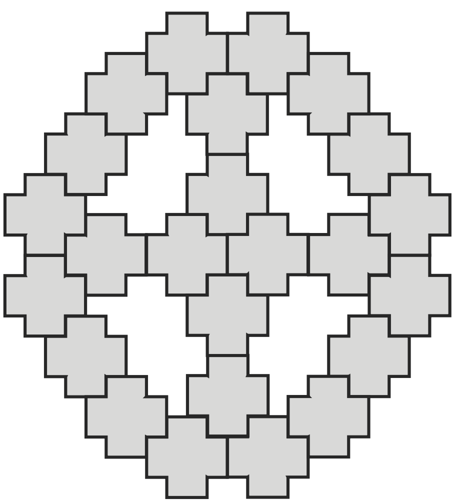
Mixed matching: examples

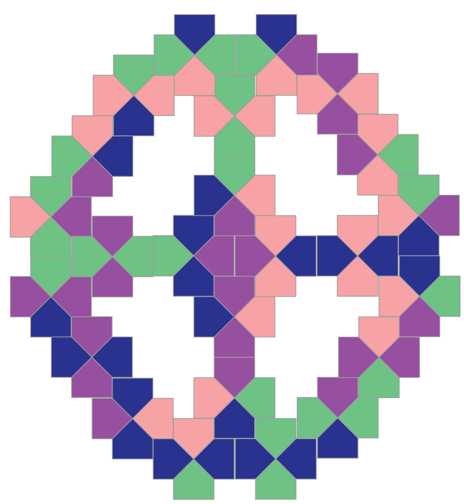




Example 8
(Medium - Symmetry 1)

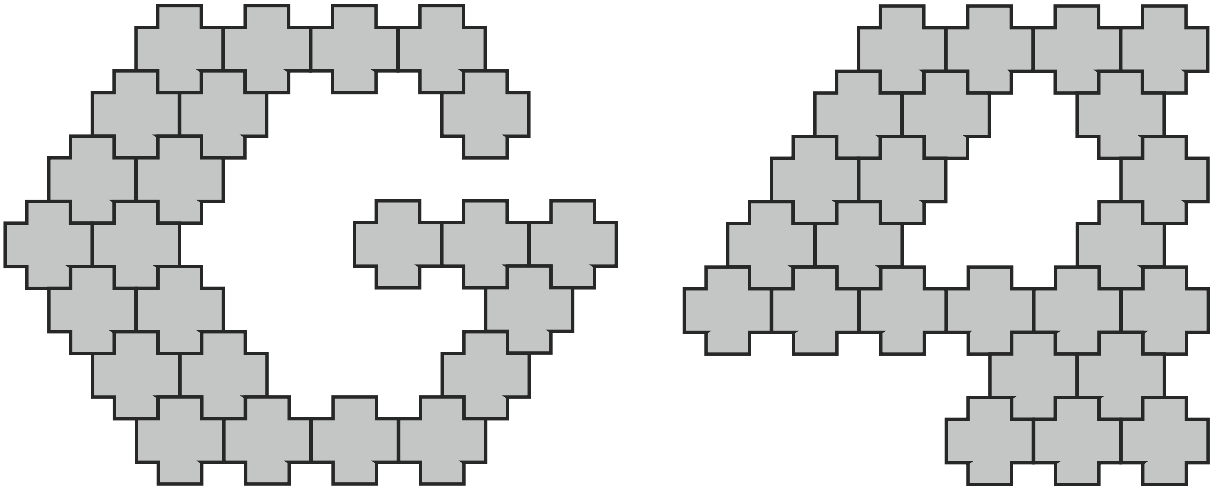
Mixed matching: examples



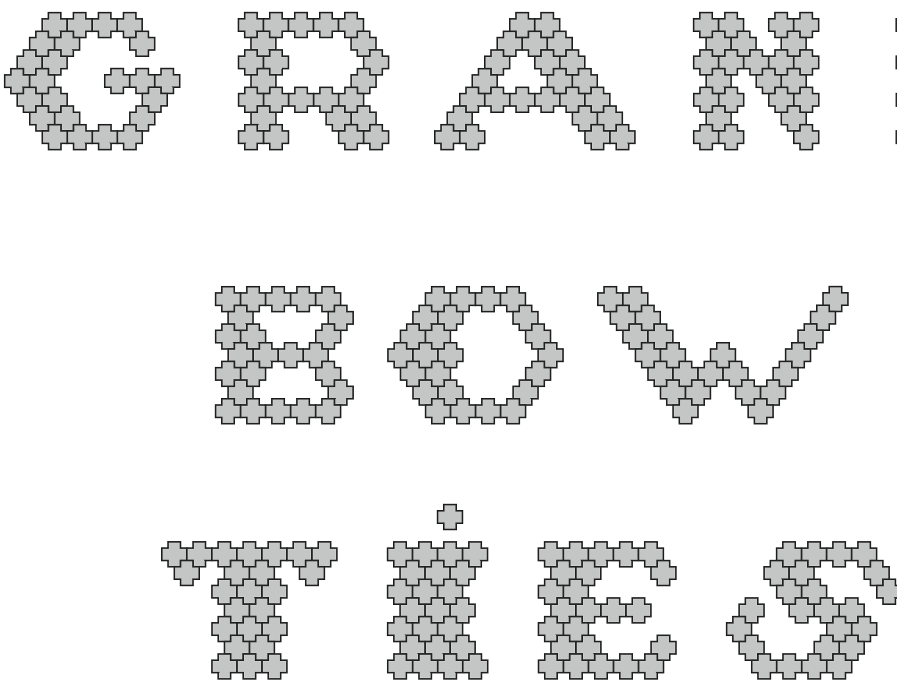


Example 9
(Medium - Symmetry 2)

Step 6: Letters & Digits



Letters & Digits: examples

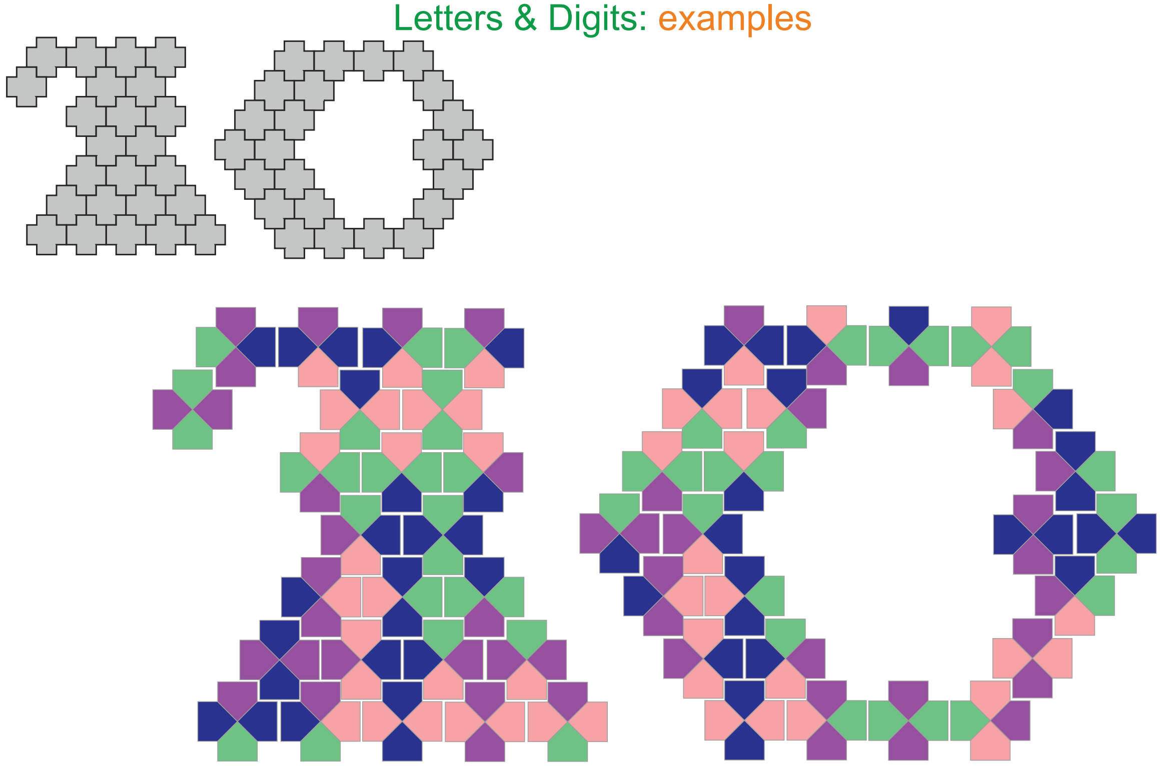


Letters & Digits: examples

Example 10
(Medium - Letters)

G4G10

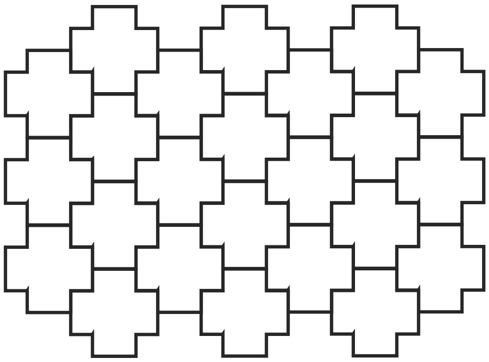
Letters & Digits: examples

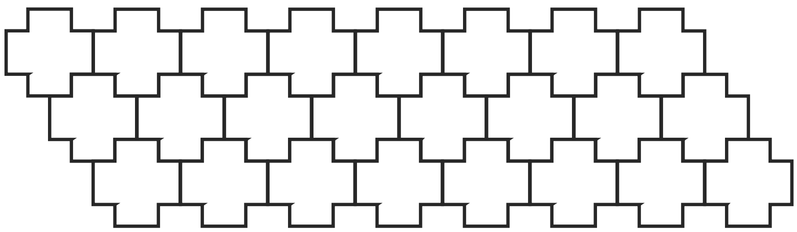


Example 11
(Medium - Number 10)

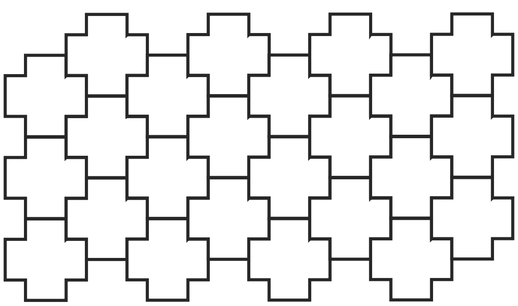
G4G10

Step 7: Compact figures

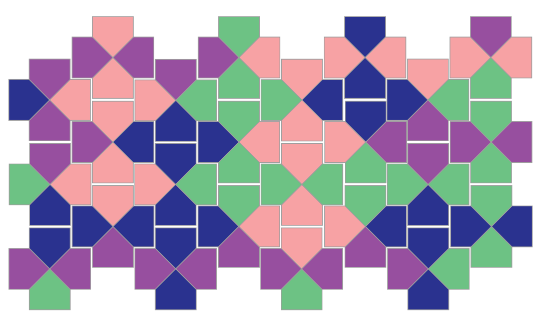




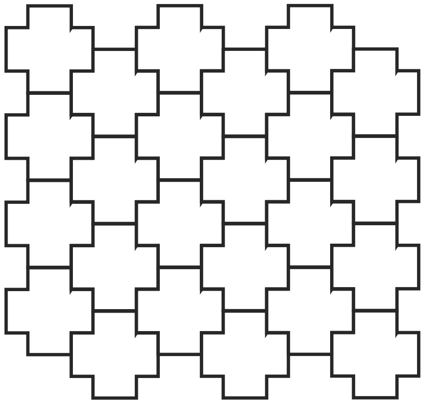
Compact figures: examples



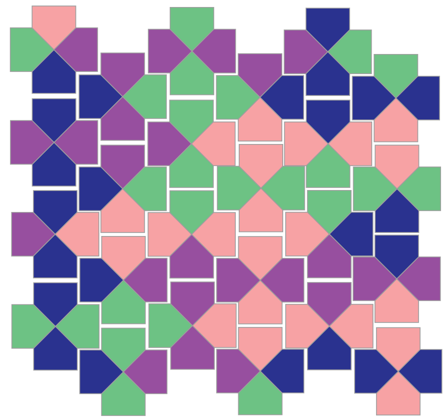
Example 12
(Challenging figure 3x8)



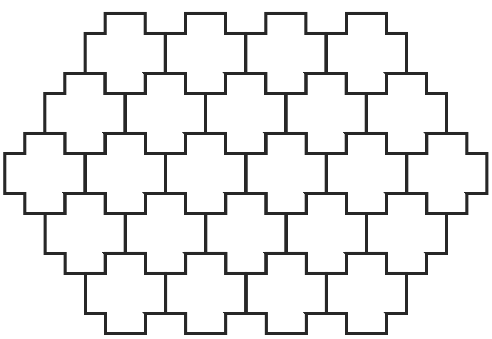
Compact figures: examples



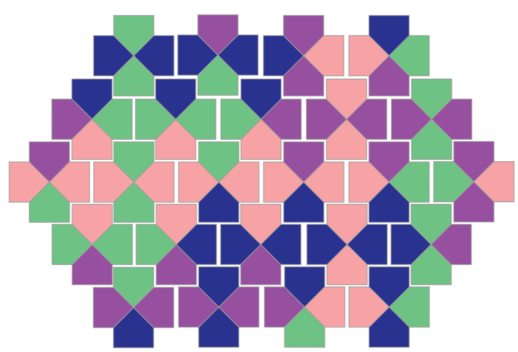
Example 13
(Challenging figure 4x6)



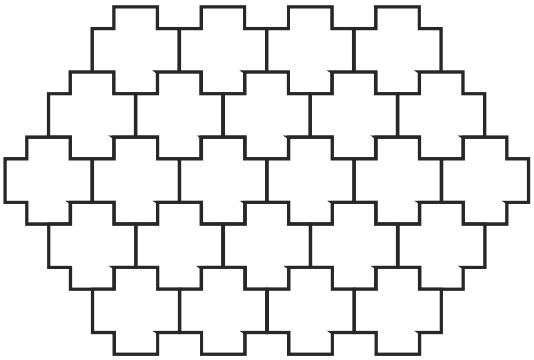
Compact figures: examples



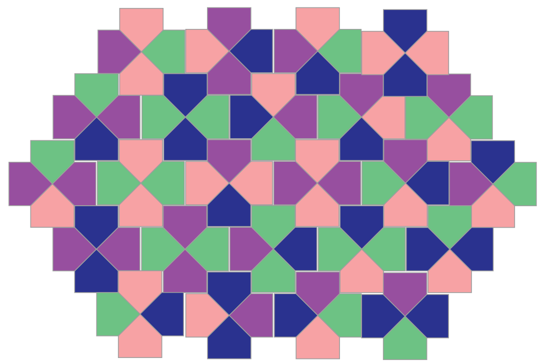
Example 14
(Challenging figure 4-5-6)



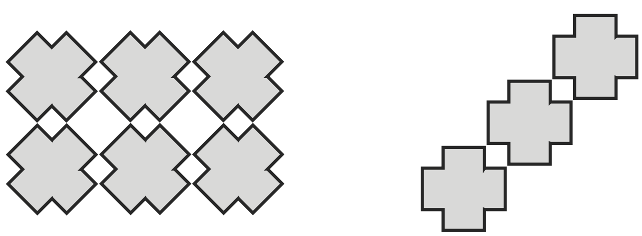
Compact figures: examples



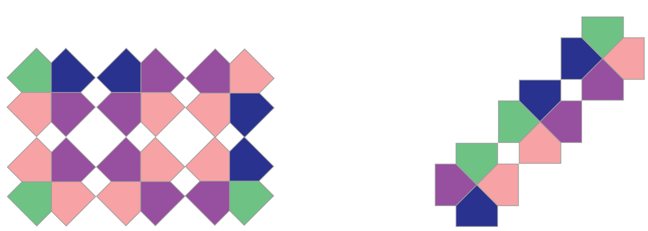
Example 15
(Challenging figure 4-5-6)
Non-match



Step 8: Vertex touch



A

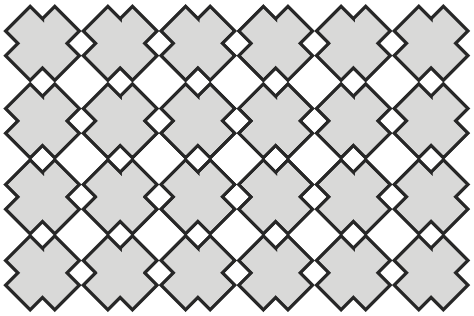


Color
vertex-touch

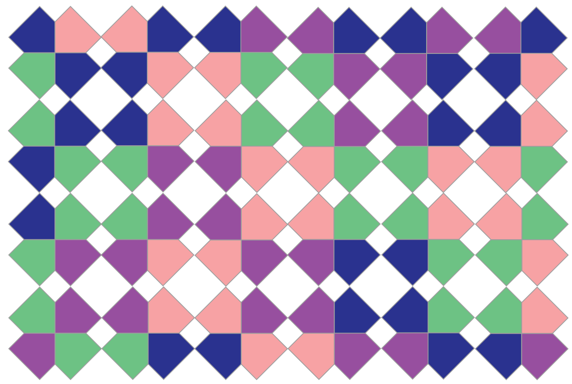
B
Alternative

Non-matching

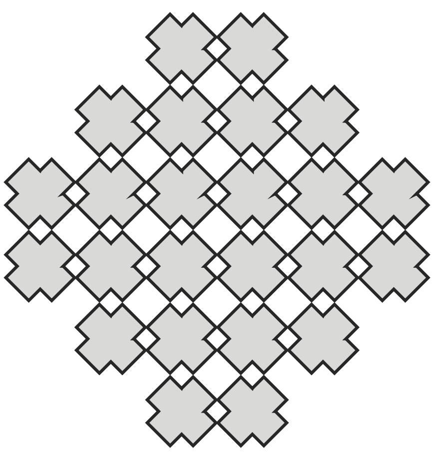
Vertex touch: examples



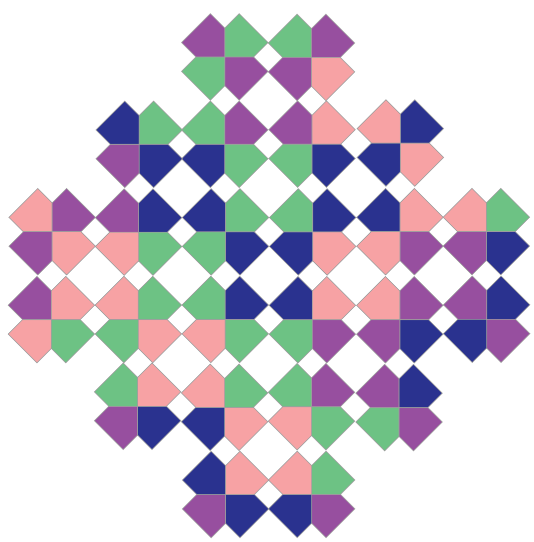
Example 16
(Vertex figure 1)



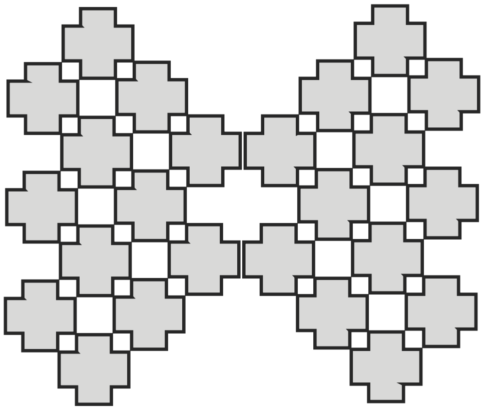
Vertex touch: examples



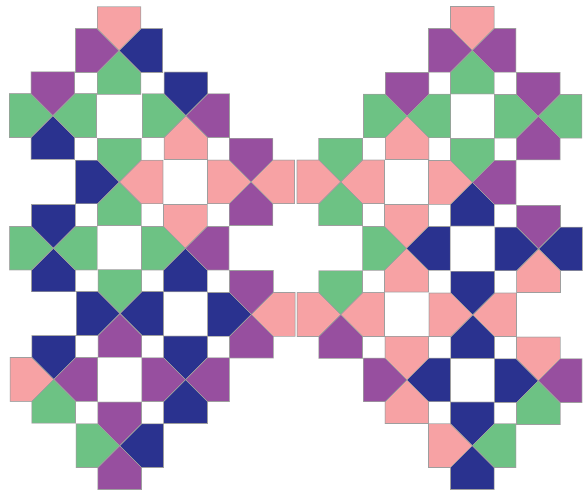
Example 17
(Vertex figure 2)



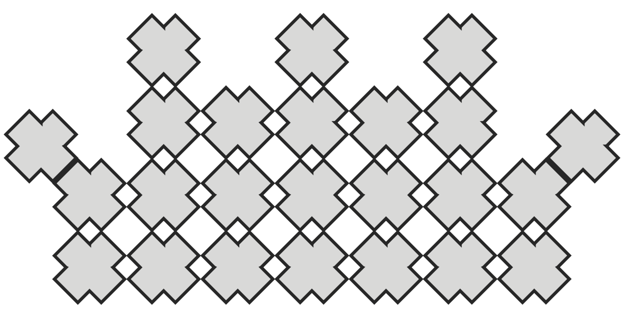
Vertex touch: examples



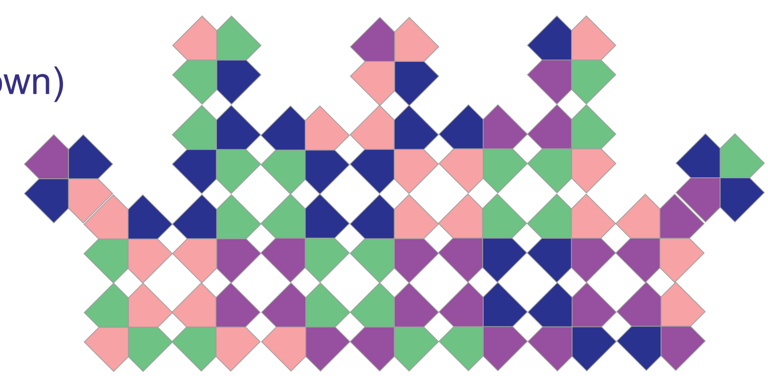
Example 18
(Vertex figure 3)



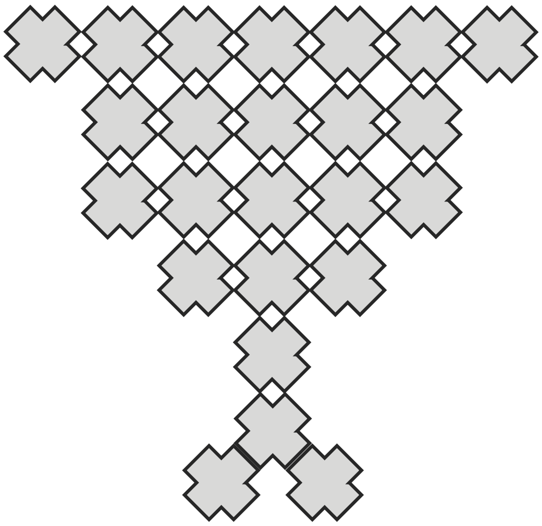
Vertex touch: examples



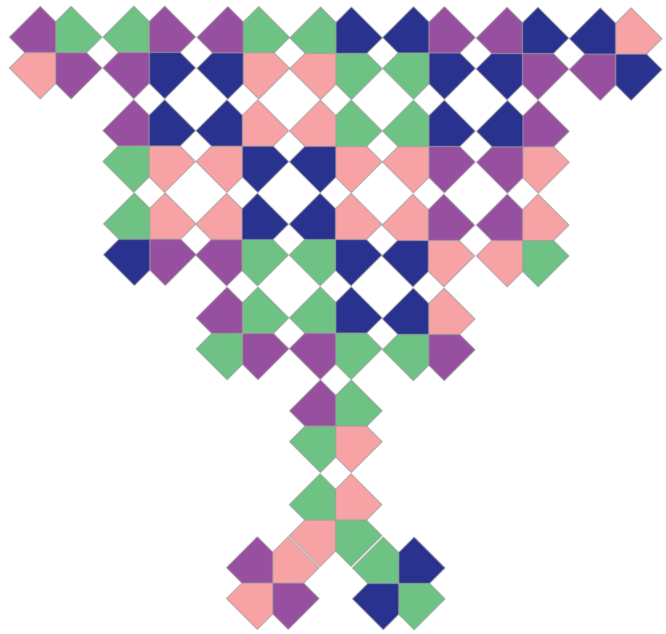
Example 19
(Vertex figure: Crown)



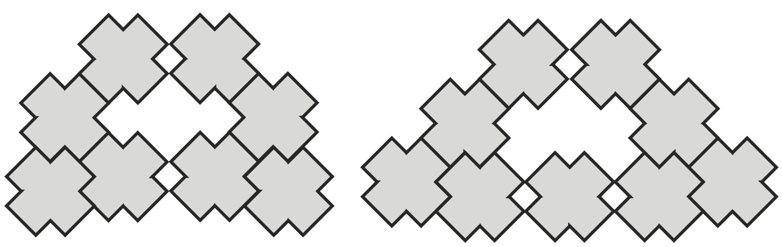
Vertex touch: examples



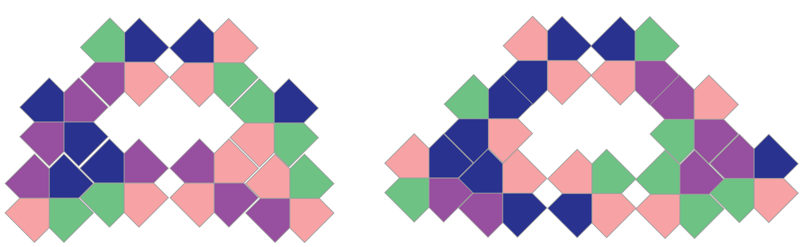
Example 20
(Vertex figure: Glass)



Step 9: Hybrid contacts

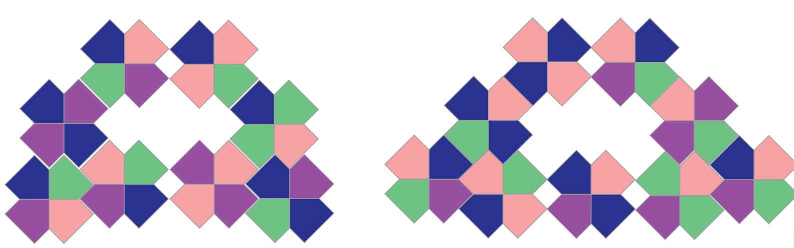


A



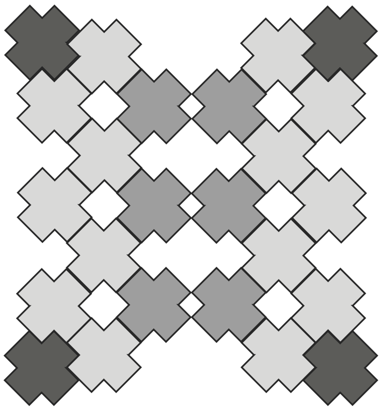
Color
edge-matching

B
Alternative

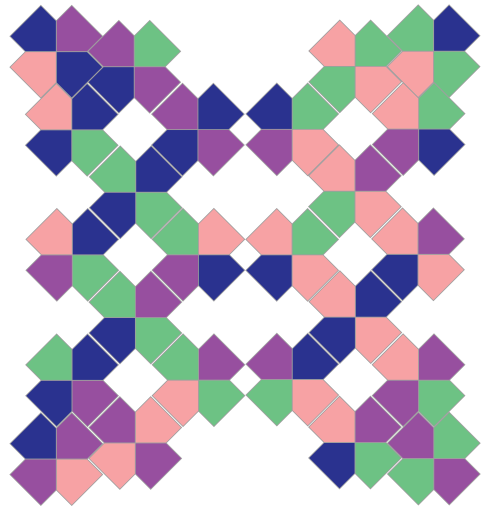


Non-matching

Hybrid contacts: examples



Example 21
(Combined figure 1)

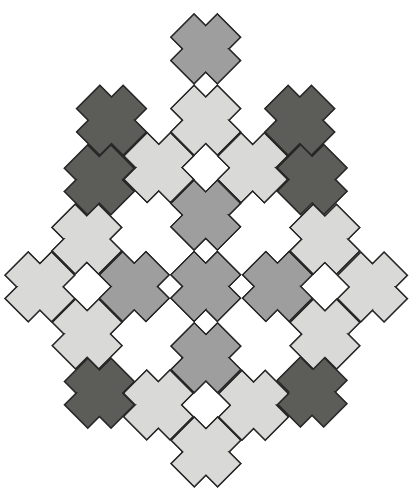


End-to-end

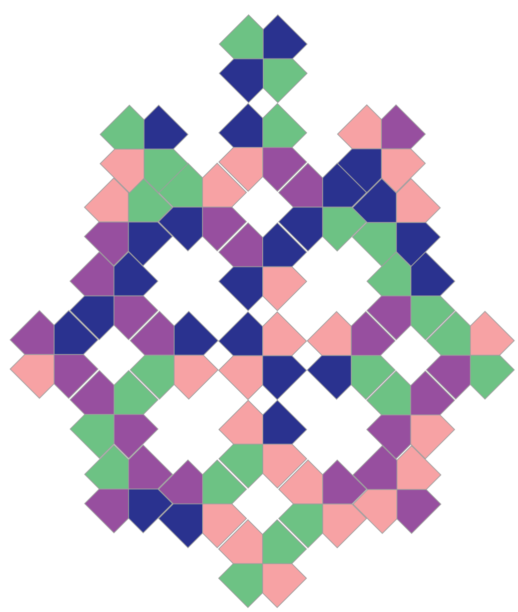
Vertex

Corner-to-corner

Hybrid contacts: examples



Example 22
(Combined figure 2)

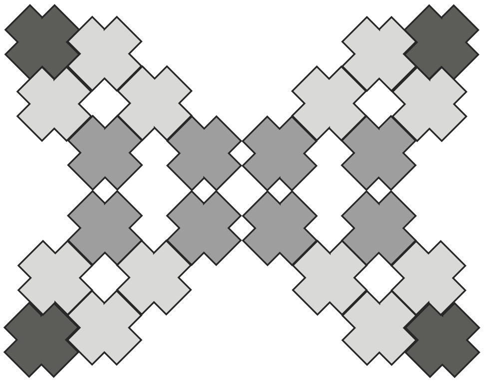


End-to-end

Vertex

Corner-to-corner

Hybrid contacts: examples

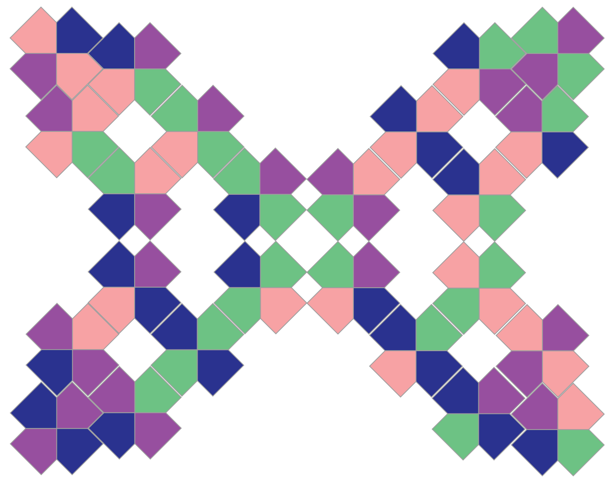


Example 23
(Combined figure 3)

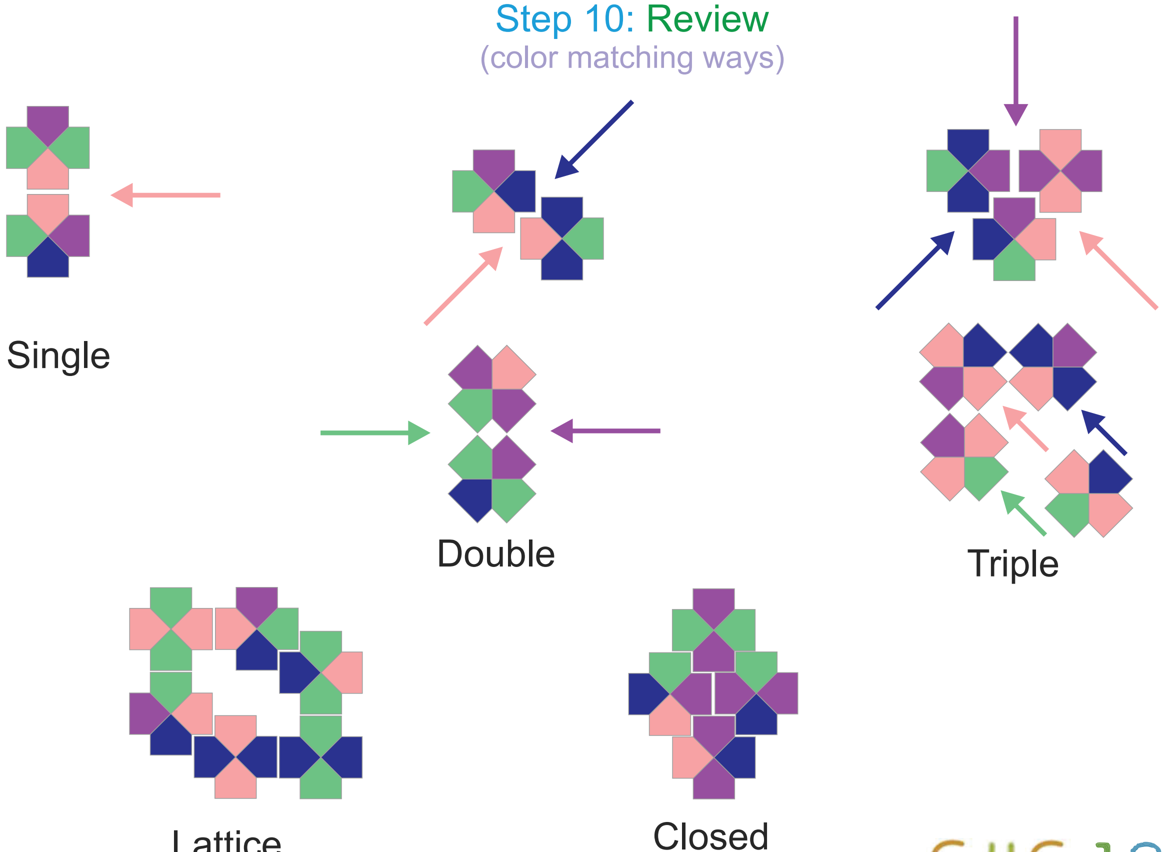
End-to-end

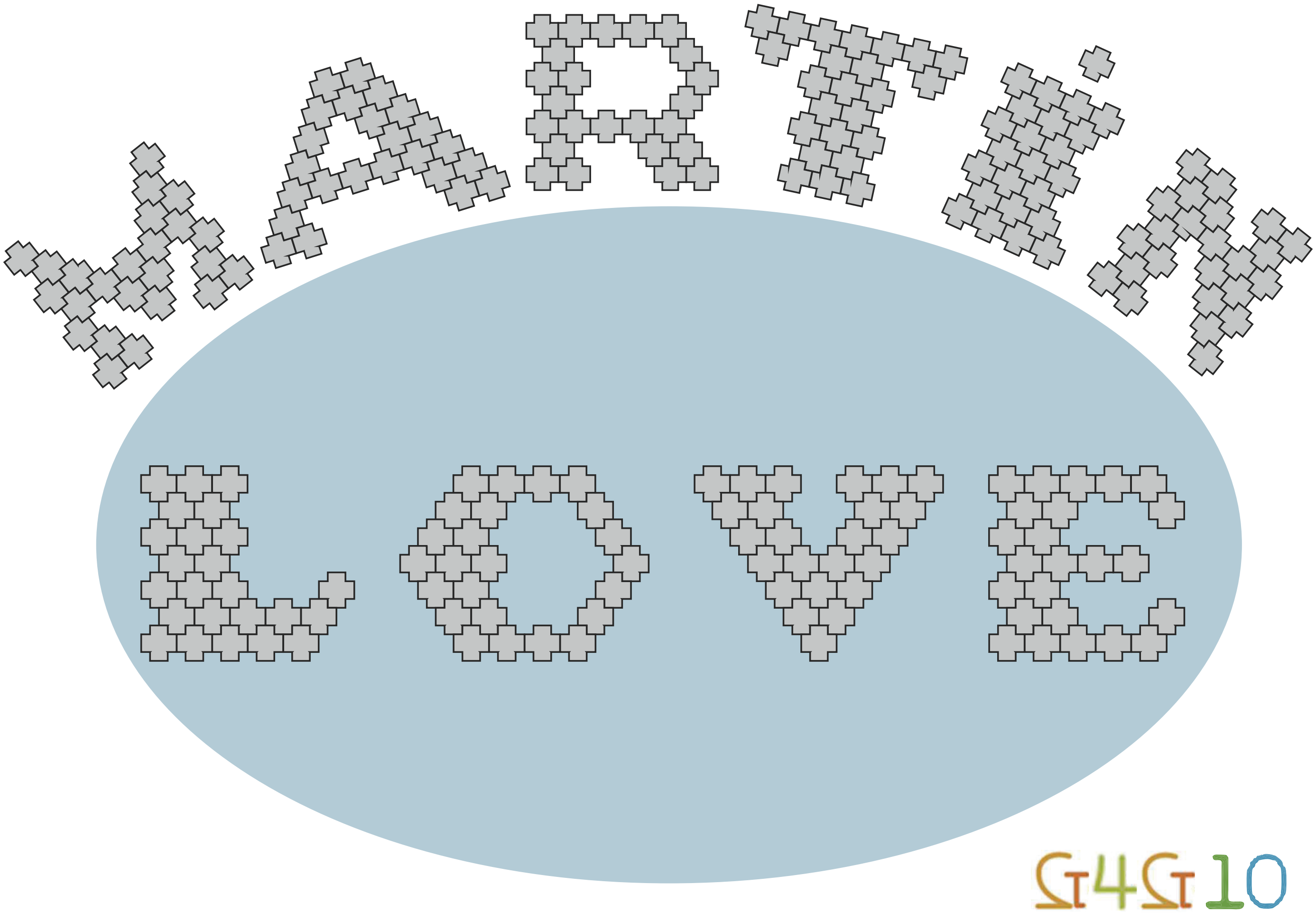
Vertex

Corner-to-corner



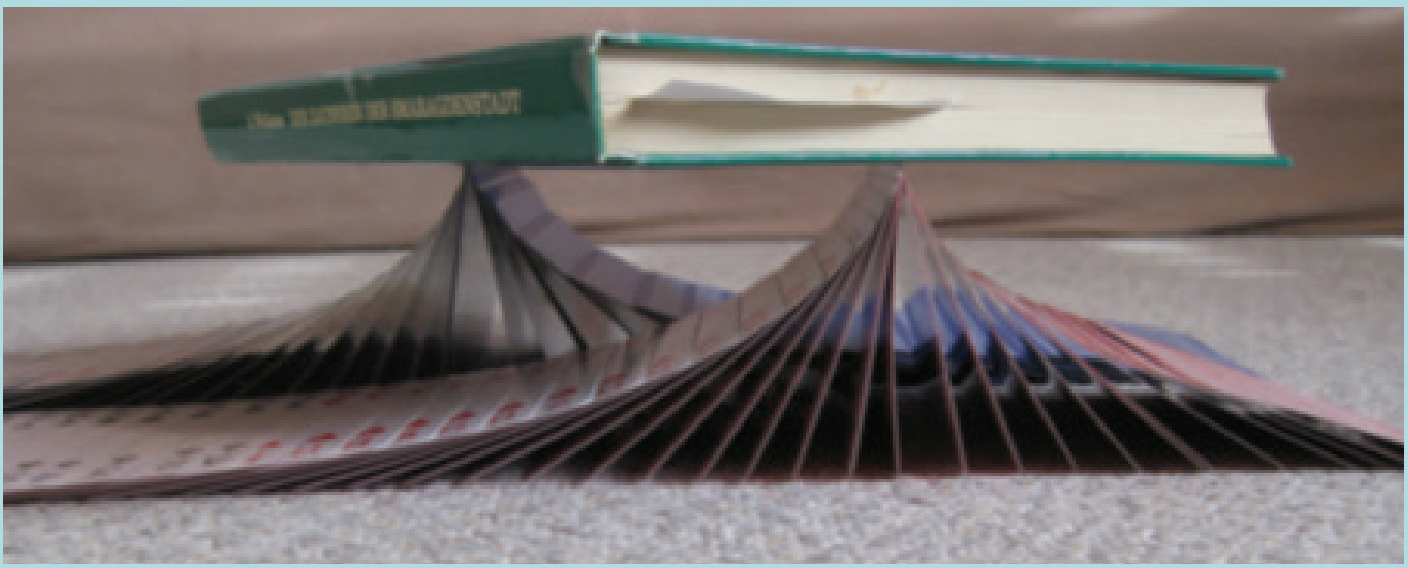
Step 10: Review
(color matching ways)




MARTIN
LOVE
G4G10


MARTIN
LOVE
G4G10

GAMES



2x Ribbon Spread Turnover = 1 x Tractrix Racer | Burkard Polster | Page 135

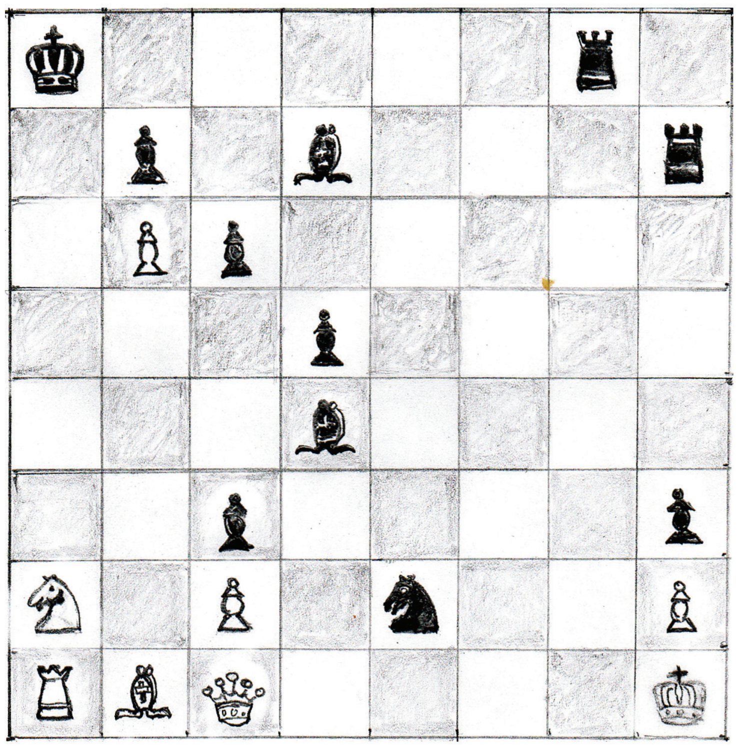
Author: Jerry Butters

Another Chess Mystery of Sherlock Holmes

Holmes had been very despondent for some weeks, and was looking even more gaunt than usual. For lack of any other idea, I practically forced him out of bed and made him accompany me to the chess club. Holmes was rather fond of peculiar chess puzzles, and I hoped that we might run across a game of interest to him. You may have read about some of these puzzles in the book "The Chess Mysteries of Sherlock Holmes," by Raymond Smullyan. Holmes had no interest in actually playing the game, but he took pleasure in trying to deduce all the logical possibilities regarding both past and future moves related to unusual chess positions.

When we arrived at the club, there were several tables laid out with chess pieces, but one caught our attention immediately, because a game seemed to be in progress, yet the table was abandoned. We sat down as follows:

Holmes



Watson

[white pieces: pawns on b6, c2, and h2; rook on a1, knight on a2, bishop on b1, queen on c1, king on h1; black pieces: pawns on b7, c6, d5, c3, h3; rooks on f8, g7; bishops on d4, d7; knight on e2; king on a8.]
I studied the position carefully, but Holmes appeared distracted and seemed to pay no attention. Soon I exclaimed, "Holmes, I declare that I can checkmate you in three moves." Holmes sighed and peered at the board for a minute. "Watson, are you entirely sure of your claim?" "Why, yes," I said. "Let me show you." Playing White, and with Holmes playing Black, the game proceeded as follows:

1. Q-a3 1...K-b8
2. Q-a7ch. 2...K-c8
3. Q-a8 mate

“You see, Holmes,” I said triumphantly. “Surely there is nothing you could have done to prevent this checkmate.” “Well, yes, I suppose you are right, Watson,” he replied, “at least from a certain philosophical point of view. But yet I am not entirely convinced.”

Before I could ask Holmes to explain this strange remark, two gentlemen joined us at the table, our old friends Colonel Marston and Sir Reginald Owen. “Ah, Holmes, I haven’t seen you at the club for some time,” said the Colonel in greeting, “and your friend Watson as well. What do think of our game? I do believe I have the upper hand, would you not admit, Sir Reginald, despite the disadvantage of playing with the black pieces? “Yes, I confess that you are right,” he replied, “but I have only been playing seriously for last few moves, and once having fallen behind, there was little I could do to salvage the situation.”

At this point, Holmes interjected: “Marston, I see that you and Sir Reginald have been playing one of your highly unorthodox games.”

“Why, yes, I do admit that we have not been playing in a conventional manner. Perhaps you deduced that from the unusual placement of Sir Reginald’s bishop. Sometimes we play just to explore the many possibilities inherent in the game, without regard to who wins or loses, but we always adhere strictly to the rules.”

Holmes continued: “But I quite agree with you, Marston, that you are in control of the position. As it is clear to see, you can force a checkmate in only three moves.”

“But Holmes,” I whispered hurriedly, did you not hear that Marston is Black? It is White that can checkmate in three, not Black. It appeared to me that Holmes’ depression must be deeper than ever to cause him uncharacteristically to pay so little attention to detail. Unfortunately, the Colonel overheard my rebuke, for he joined in by saying, “I do believe that Watson is correct, for even though it is my turn to move, I can see no way that I can force checkmate is less than 5 moves.” And, indeed, neither could Sir Reginald nor I.

To my chagrin, Holmes did not concede his error. “You may not see the solution, said Holmes, but it is plain to see right there in front of you.” “In that case, replied Colonel Marston, surely you would accept a wager. If you convince us you are right, I will pay you one pound, but if you are wrong, you must come to the club tomorrow and take us all out to dinner.” As he finished speaking, he gave a slight wink in my direction. Surely the Colonel had recognized my friend’s deteriorated condition and was trying to help me draw him back into a more normal manner of living.

Taking no notice, Holmes responded to the challenge. “Very well, I accept, but I do not wish to take unfair advantage of you. First we must be very clear about the terms of our wager. I win if I can demonstrate that Black can play in such a way that the game will end before he needs to play four moves. It might end in three moves, and it might end sooner. In any case, the result will be checkmate.”

“Nothing could be clearer,” agreed the Colonel. You may take the black pieces, then, and I will take White, and let us see if you can finish the game in less than four moves. “It is your move.” “But wait, you agree not to resign before the four moves are up?”

“Most certainly,” said Holmes, “both players must continue to play for four moves, as long as it is possible.”

Holmes moved his hand towards the rook on his bottom rank, hesitated for a moment, and then placed it back in his lap. “There is a slight problem,” he said. I am not certain how to play.”

"You may have all the time you wish," said the Colonel, "as I have nothing else to do this afternoon." "No, you do not understand," said Holmes. It is not a question of time. It is just that I do not know for certain how I should play. "In that case," said Sir Reginald "surely you must concede defeat. If you do not know how to play, and if time will not help you, then there is no alternative."

"I most assuredly do not concede defeat," said Holmes. "I can prove that Black can play and force mate in three moves, and yet I cannot yet be certain exactly how it can be done."

At this point, I could not restrain myself from exclaiming, "But Holmes, surely that is impossible. I have heard that in the realm of higher mathematics, it can sometimes be possible to prove that a solution to an equation is possible even though one can not solve for the exact result. But surely nothing of the sort could occur in a simple game. Why, there are only a finite number of possibilities, and surely we could check out all of them."

"That is true, Watson," said Holmes, "but nonetheless your conclusion does not follow. Perhaps it would help explain the situation if I let you finish your game, and we could continue this discussion later."

All three of us stared at Holmes without knowing what to say. I will admit that he looked more like his old self. He was sitting up straighter and looked at each of us in turn with a twinkle in his eye, and yet his proposal was so unreasonable that I wondered if this time he was really going insane. Did he really hope that by observing our play we would somehow discover a solution that he had missed, and none of us could see? Or was he simply stalling for time?

Nonetheless, without even sitting down, the Colonel and Sir Reginald proceeded to finish their game, as follows:

	1.... NxQ
2. NxN check	2.... K-b8
3. R-a8 check	3.... KxR
4. N-e2	4.... R-f1check
5. N-g1	5.... Rxg1 mate

"I am sorry, Holmes," said the Colonel generously, "but in truth I did my best and I don't see how I could have made the game shorter, given White's various threats."

"Not at all, Marston," replied Holmes, with assurance. "For now I know how to play." Replacing the pieces to their previous position, Holmes began with:

1.... R-b8

"It is your turn, Marston."

"But Holmes, how could you possibly hope to cause a checkmate by retreating your rook into the corner," I cried. And before I had quite finished, Marston added, "Really, Holmes, in fairness I must allow you to retract that move, because by hemming in your own king, you are allowing me to win immediately just by moving my knight. Your king will have nowhere to escape the discovered check from my rook!"

After a moment, Sir Reginald began to laugh. I was embarrassed beyond words. How could he triumph so at Holmes' humiliation? Maybe he was not aware of Holmes' unusual condition, and smarted from previous occasions when Holmes had proven us all wrong.

“But don’t you see, Marston,” said Sir Reginald, you really can’t move your knight! That is, if you did move your knight, you would checkmate Black, and the game would not last for 4 moves. You might win the game, but you would lose your bet!”

“And why would I lose the bet?” demanded Marston. "Holmes said he would checkmate me in under four moves, not let me checkmate him!"

“No, you are wrong,” said Sir Reginald. “Holmes gave you fair warning. He said the game would end in checkmate. He didn’t say anything about who would checkmate the other! If you reflect upon his words, you will realize that he spoke them very carefully and very precisely.”

“In any case, I have not lost the bet, because I have not played yet, and nobody has yet checkmated the other,” said Marston, somewhat louder than necessary. And after thinking for a while, he played, and continued, as follow:

2. Q-e1 2.... R-g1 check
3. QxR 3.... BxQ

“Now you must admit defeat, Holmes,” I said, “because you have played your third move, and you have not even placed White in check.

“Not so,” said Sir Reginald. “For it is still White’s move. And White is still able to checkmate Black. And, more to the point, White must checkmate Black. For the only piece White has left that can move is his knight,...”

4. N-anywhere, mate

“... and he must make a move, because Marston himself insisted that the game could not be cut short by resignation! So Holmes wins the bet.”

And so, after some more discussion, Marston and I had to agree with Sir Reginald.

Holmes had for some time been quiet. But finally he spoke up, and to our astonishment, said: “Really, gentlemen, you are conceding defeat all too easily. For I have not yet proved my case. I have merely demonstrated one half of the necessary argument. Do you not wonder why I waited until you finished your game to play out one variation?”

“Why, yes, I do wonder why you created such an air of mystery,” said Colonel Marston, “and pretended not to know how to play, when all the time you knew full well how to proceed.”

“I will admit,” said Holmes, that I did see foresee that line of play, “but until I saw how you finished your game, I could not be sure that it was the right way to play.” “You see, coming upon your game in the middle placed me at a disadvantage, for I did not know what had proceeded before. In particular,...

“But Holmes,” I interrupted, “I don’t see how the preceding play could possibly be relevant. And even if it were relevant, what possible help would it be to observe a future line of play?”

At this point Marston interposed thoughtfully, “Well, I do see how the preceding line of play could sometimes be relevant. For example, in some positions it is not clear whether a player has the ability to castle or not, because we can’t tell whether the player’s king or rook has been previously moved. But I don’t see anything of this sort in our particular game. Let us hear what Holmes has to say.”

“I really did not intend to be so circuitous,” said Holmes. “But the future often tells us something about the past, just as the past tells us something about the future. Perhaps I should have asked directly about the past moves. For example, I could have asked you, Marston, whether you indeed made a capture with your pawn on h3 as your last move before the position we found when we arrived.”

“Yes, as a matter of fact, I did make such a capture,” said Marston.

Losing patience once again, I burst in and said, “But Holmes, how could you possibly know that? And what does that have to do with the checkmate? You are piling mysteries upon mysteries!”

“One thing at a time, Watson,” Holmes said patiently. “Marston has accused me of prolonging the mystery, but each question of yours diverts me further from the explanation.” Chastened, I resolved not to say anything more until Holmes finished.

“As for the pawn capture, Watson, that is really quite elementary. Sir Reginald assured us that in his last moves he was sincerely trying to play well. But White was the last to play in the position we found. What was White’s last move? Why did Sir Reginald leave his queen where it could be captured by the knight? Indeed, why didn’t he begin the checkmate sequence you found so quickly when we arrived? The only reasonable explanation was that a more important piece was at stake, his king. His king must have been in check, and he must have moved his king out of check. Where could it have come from? Not from g1, because the square g1 is in check from three pieces, the knight on e2, the bishop on d4, and the rook on g7. That is impossible: there is no way black could have moved so as to simultaneously place white in check in three different ways. So, white must have moved his king from g2 to h1. Now, what was black’s move prior to that?”

Sir Reginald was first to reply: “Yes, when white’s king was on g2, it was in double check, from the pawn on h3 and the rook on g7, and the only way this could have happened was for the pawn to have made a capture, moving from its previous position on g4 and exposing the king to check from the rook, as well as administering a check of its own. By why, then, did you need to ask?

“Because that is not the only other possibility,” said Holmes. There is one more. Or to be precise, there are several more, all of a similar nature.”

“Really, Holmes, this is too much,” I protested. There can be no other possibility. Black can’t have moved two pieces at once. His pawn can’t have captured starting from a different square. White’s king can’t have moved from h2, because he can’t share the square with his own pawn!”

“But yet, there is another possibility,” said Holmes. “Perhaps you are forgetting how little we knew when we entered the room. We did not know what moves had gone before. We learned who the players were, who played White, and who played Black,” (and as he said this, if I am not mistaken, he fixed me with his gaze for a moment longer than usual), “but we did not even learn where they were sitting as they played.”

“What could one possibly learn from that?” I cried out.

“Yes, Watson, what could one learn from that? Where do you normally sit when you play chess?”

“I sit on the white side, if I am playing white, and on the black side, if I am playing black.”

"Quite so."

"For example, I am now sitting on the white side, and you are sitting on the black side."

"Yes, we know that now. But we didn't know that before."

"How can you say that. Isn't it obvious merely from the position that the white pieces are on my side, and the black pieces are on your side?

"Not necessarily," said Colonel Marston. "After all, some of the black pieces: the knight and bishop, and two pawns are on your side now, and one of the white pawns is on Holmes' side. It is evident that the pieces are able to move about. How do you know where the pieces were at the beginning of the game? As Holmes said, you didn't see the beginning of the game."

"But my side must be the white side," I insisted. "Are you trying to tell me that the white pieces could have begun on your side of the board, Holmes, and marched down to my side, while my pieces marched over to your side, and they all politely stayed out of each others' way, except for a few, mostly pawns, that were captured, and that Black has three pawns almost ready to be queened, but has considerately refrained from doing so?

"Exactly, Watson. That is just what I have been thinking. It may seem wildly improbable, but it not impossible. For example, the pawns could have side-stepped one another by capturing and changing files."

"But it is truly impossible, Holmes, because the final position reached by Marston and Sir Reginald would not be a checkmate if the white pieces began at your side of the board. In that case, the white pawn on h2 could move backwards, so to speak, to capture the rook on g1!"

"Capital, Watson!" cried Holmes. "That is exactly how I reasoned. That is why I did not know how to play until I saw the completion of the game. Only then did I know for certain in which the direction the pawns were allowed to move."

Once this mystery was cleared up, we were able to resolve all of the remaining problems. Marston was first to discover the other half of Holmes' proof. Assuming that white moves from top to bottom in the diagram, there is only one way black can checkmate in 3, as follows:

1.... N-g3 check (the white pawn now can't capture the knight!)
2. K-g2 (the king is not under attack from black's pawn on a3, which is moving up the board!) 2.... R-f2 check
3. K-g1
3.... R-d2, discovered check and mate.

Sir Reginald pointed out why Holmes' first line of play would have failed if White had been moving down the board. After 1...R-b8, and 2...R-g1 check, White could have defended with 3. PxR (promotes to queen), and when Black captures the new queen, White still has the old queen to play (and also the pawn on c7, which could queen on c8) and therefore is not forced to checkmate Black.

And I then chimed in: "Now I understand your other point, Holmes! As you explained, White's previous move, starting from the position we first saw, was to move the king from g2 to h1, but when the white king was on g2, it might have been in check only from the rook on f8, because the black pawn on a3 might have been moving up the board, not downward as I presumed. In that case, black's previous move could have been a move with its rook, ending on g7, to deliver the check."

"But Holmes," added Marston, "why are you so sure that White's previous move was a king move? Perhaps instead white moved the white queen to c1, a possibility you did not cover in your explanation. That move might not have been such a blunder as it seems, because perhaps white captured a black piece that was previously on c1, maybe even Black's queen. And Black's queen might have itself just captured a white piece on c1, perhaps a rook.

"Indeed, you are quite right. That is also possible," conceded Holmes, "but in that case, would not Sir Reginald have captured the black queen on c1 with his knight on a2, thereby safeguarding his own queen and administering a check on your king? Sir Reginald has assured us he was playing his best at the end, and with this move, he could have turned the tables on you, so to speak."

"And now gentleman, I hope you will all join me for dinner tomorrow night, for I can think of nothing better on which to spend the pound I have just won from Marston. I know that you all have been concerned about my health, but I assure you that nothing is wrong with me, and our afternoon together has quite lifted my spirits."

We were all so pleased that we quite forgot to be annoyed with Holmes for making such a mystery out of the simple but far-fetched notion that the original position did not reveal the direction in which the pawns could move. I myself have recounted this story as I have merely due to my desire to convey the events as faithfully and truthfully as possible.

It was only later that I remember the moment we first arrived at the club, when I had displayed my checkmate in three moves for White. What had Holmes meant when he said I was that I was right only in a philosophical sense? Upon reflection, I realized that if White had been moving from top to bottom, then my move 2. Q-a7 check would fail, as Holmes could have responded with 2…KxQ! Or, if I had tried instead, 2. Q-d6 check, followed by 3. Q-c7 check, it would have still met 3… KxQ, since in both cases the pawn on b6, moving down the board, no longer guards the Queen.

But there is indeed another sense in which I was right. Sitting as I was on the "South" side of the board, as long as it was my move, I could indeed win by checkmate in 3 moves. There are two possible cases. In the first case, the white pieces are moving upwards on the board, as I had presumed, and I was playing white, and my checkmate was valid. In the second case, the black pieces are moving upwards on the board, and sitting South, I should have played the black pieces instead of white! But then I still had a forced checkmate in 3, using the moves demonstrated by Marston. But I would have faced the same dilemma as Holmes. Even though I could have truthfully asserted (and, unwittingly, did assert) that I could checkmate Holmes in 3 moves, I would not have known how to do so, because I would not have known whether to play White or Black!

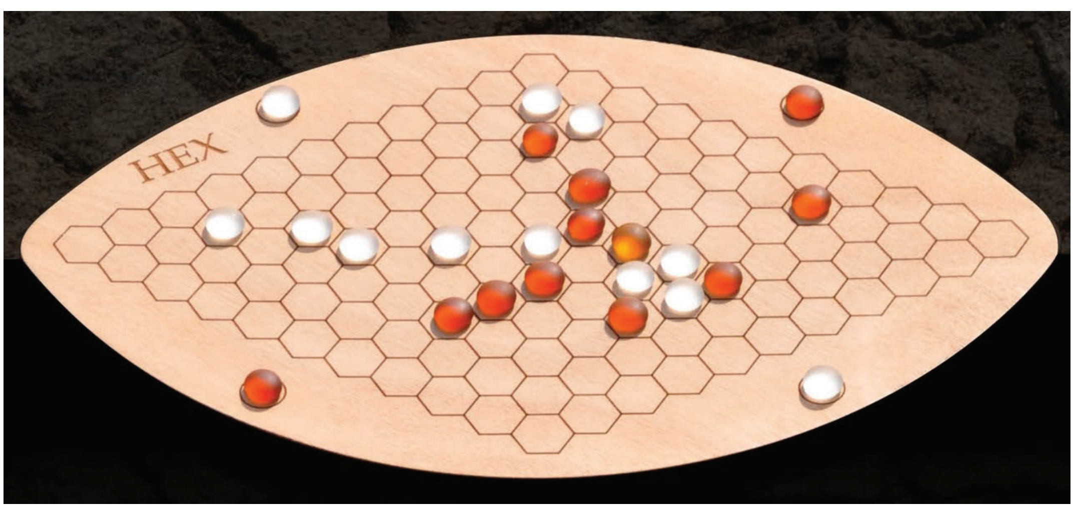
Portuguese Championship of Mathematical Games

Alda Carvalho

pwp.net.ipl.pt/dem.isel/acarvalho

The Portuguese Championship of Mathematical Games is organized yearly in Portugal. In this event, high school students play games with perfect information such as Hex and Dots and Boxes, which were appreciated by Martin Gardner. Last edition had 2500 participants in the finals.

See www.cnjm.com.pt/index.php/pt/ or the site of Ludus Association http://ludicum.org/ .



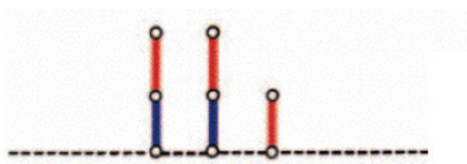
A detail of one of the posters.

White to play and win.

$$\frac{1}{2} + \frac{1}{2} - 1 = 0$$

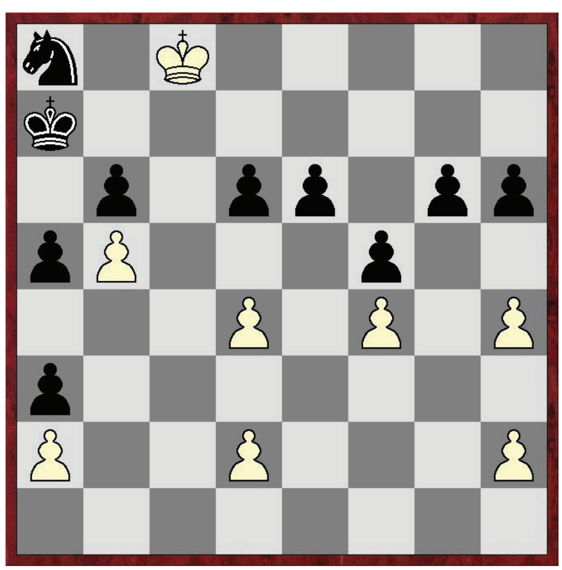
Carlos P. Santos

www.sites.google.com/site/cpshomepage



What do we mean by Half a Move?

Winning Ways for Your Mathematical Plays
E. Berlekamp, J. Conway, R. Guy



Carlos P. Santos, 2011

Factor Subtractor

by

Barry Cipra
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Factor Subtractor is a game played with numbers. It is intended to reinforce basic skills in arithmetic while offering something of interest to professional (or amateur) mathematicians. It can be played on paper, at the blackboard, or even, depending on the players' facility with mental arithmetic, aloud, for example during a car trip.

The games start with one player factoring a large number, such as 100, as the product of two smaller numbers, such as 4x25 or 5x20. Play then proceeds with the players taking turns as follows. When it's your turn, you pick one of the two factors, subtract it from the product, and then factor the resulting difference, again as the product of two numbers. For example, suppose player A starts by factoring 100 = 4x25. The game might proceed as follows:

B: 100-4 = 96 = 8x12
A: 96-12 = 84 = 6x14
B: 84-14 = 70 = 2x35
A: 70-2 = 68 = 4x17

You might have noticed, we haven't said what the object of the game is, i.e., how to win it. But note that the numbers being factored are getting smaller and smaller. So eventually we're going to run out of numbers. And that's the object: To be the player to reach 0. Let's see how this might play out by continuing the game above:

B: 68-17 = 51 = 3x17
A: 51-3 = 48 = 6x8
B: 48-8 = 40 = 4x10
A: 40-10 = 30 = 2x15
B: 30-15 = 15 = 3x5
A: 15-5 = 10 = 2x5
B: 10-2 = 8 = 2x4

A: 8-4 = 4 = 2x2
B: 4-2 = 2 = 1x2
A: 2-2 = 0

so player A wins. It didn't have to end that way. It turns out, each player in this example made a couple of "bad" plays. The last one occurred when player B subtracted 15 from 30 instead of 2. Let's see why this is.

The secret lies in the sequence

4,9,10,14,16,18,22,25,26,28,30,34,36,40,46,48,49,54,55,56,62,63,
65,66,68,74,75,76,80,81,84,88,90,94,96,....

The list goes on and on; these are the one- and two-digit "target" numbers for the subtraction step, for a which a player can pick a factorization that guarantees a win. Notice that 15 is not on the list, but 28 is. Therefore, player B should have played 30-2=28 instead of 30-15=15 after A had offered 30=2x15. Had he chosen the better number to subtract, B needed to factor 28 as 4x7, because neither 28-4=24 nor 28-7=21 is on the list. (The factorization 2x14 would have been a mistake, because it would allow player A to get to either 28-2=26 or 28-14=14, both of which are on the list.)

You may notice that 30 is also on the list. This means that player A actually made a mistake in factoring it as 2x15. The "correct" move would have been 30=3x10, since neither 30-3=27 nor 30-10=20 is on the list. For that matter, player B made a mistake early on, factoring 96, which is on the list, as 8x12 instead of 4x24; doing so allowed player A to get back on the list, whereas neither 92 nor 72 would have been.

So where did this list come from? The short answer is, recursive computation. The list, along with its complementary list of "losing" numbers, is built starting at 1. Obviously 1, along with every prime number p, is a "loser" because you can only factor such a number as p=1xp, which allows your opponent the immediate winning move p-p=0. (Only a fool, or a very kind parent, would opt for the non-winning move p-1 for a prime p.) The first winning number is 4, since its factorization as 2x2 forces your opponent into 4-2=2=1x2. The numbers 6 and 8 are both losers because the factorizations 6=2x3 and 8=2x4 allow your opponent to counter with 6-2=4=2x2 and 8-4=4=2x2, respectively. But 9 is a winner, because 9=3x3 forces your opponent into 9-3=6=2x3. Similarly 10 is winner, because 10=2x5 leaves your opponent either 10-5=5 or 10-2=8, both of which are on the losing list.

Let's do just a couple more numbers: 12 is on the losing list because 12=2x6 allows your opponent to get to the winning number 12-2=10, and 12=3x4 allows the winning move 12=3=9. Similarly for 15: it's a loser because 15=3x5 allows for 15-5=10. But 16 and 18 are on the winning list, with winning moves 16=4x4 and 18=3x6.

In general, once you have a complete list of all winning numbers less than N, you can determine whether or not N goes on the list by looking at its possible factorizations. If there is a factorization N=hxk for which neither N-h nor N-k is on the list, then N goes on the list; otherwise N does not go on the list. To do one more example, let's show why 72 is not on the list. To do so, we have to consider all its factorizations: 2x36, 3x24, 4x18, 6x12, and 8x9. For 2x36, we have 72-36=36, which is on the list. For 3x24, we have 72-24=48; for 4x18, we have 72-4=68 (and also 72-18=54, but all you need is one of the two); for 6x12 we have 72-6=66; and for 8x9, we have 72-9=63.

As mentioned, the list as presented shows all one- and two-digit winning numbers. The reader may wish to check that it properly omits 97, 98, and 99. (The easy case is 97: it's prime.) But what about 100? As noted, A's opening factorization 100=4x25 was a bad move, because it allowed B to get to 96. (Actually, 100-25=75 would have been a better move. As we saw, B chose the "wrong" factorization for 96. You can check that, for 75, there is no bad factorization.) Is there a different factorization of 100 that could have guaranteed A a win?

Once you get started, it's possible to extend the list indefinitely, with a fairly simple computer program. Matt Richey at St. Olaf College in Northfield, Minnesota, wrote such a program and computed the list up to N=200,000. There is no readily discernible pattern to the list. Indeed, that's what makes it of possible theoretical interest: the sequence (which has yet to appear in the Online Encyclopedia of Integer Sequences, although that's likely to be self-correcting sometime soon) is easy to define, but has no obvious properties that allow one to say that a given (large) number is or isn't on the list, beyond the one obvious "theorem" that the list contains no prime numbers. For example, based on the "early returns" showing 4, 9, 16, 25, and 36 on the list, one might speculate that all squares greater than 1 are winning numbers. The next square, 49, which is also on the list, would seem to confirm that hypothesis. But then you get to 64....

Richey's computation shows there are 366 numbers on the list up to 1000, 4033 up to 10,000, and 42,563 up to 100,000, but it's unclear whether the fraction is leveling off or continuing to climb. It would be worthwhile to confirm (or correct) these calculations and to extend them for another few orders of magnitude.

Just knowing a number is on the list doesn't in itself say which factorization guarantees a win (unless the number is the product of two primes, or the square or cube of a single prime). If there is any pattern here, it has eluded me. Contrast this with the classic game of Nim, in which the winning positions are easy to spot and the winning moves easy to calculate.

Nim is often used as an enrichment activity in math classes, but I believe Factor Subtractor has its own pedagogical advantages. In particular, even if students play at random, they are still getting valuable practice doing arithmetic. And children seem to enjoy playing the game, in part because it provides an obvious motivation -- namely winning -- for doing what might otherwise be a tedious worksheet assignment, and in part (to toss around some educational jargon) because it "empowers" them to choose which computations they do.

I have a couple of data points worth of support for this: My daughter-in-law, Sanae Tomita, has used the game with a class of middle-school children, and Kurt Hedin, a teach at Bandelier Elementary School in Albuquerque, New Mexico, whom I met and showed the game, has taught it to his fourth-grade class. I would be delighted to hear from other teachers who might have occasion to give Factor Subtractor a try with their students.

Borders

a variant of Dots & Boxes

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introduction In this article we propose a variant of the well-known game Dots & Boxes. We investigate two simple positions of the game, and conjecture a criterion for determining which player has the win (in terms of the dimensions of the rectangular board used).

description The game is played on an $a \times b$ grid (the squares being 1×1). A move consists of building a fence along one side of a 1×1 square. If the fence built completes an enclosure of one or more squares, all squares within the enclosure are claimed by that player, except for any squares already claimed by the other player.

differences from Dots & Boxes The game Borders is different from Dots & Boxes in two ways:

- The building of a single fence segment may result in the player claiming multiple squares.
- Claiming a square does not permit the player to build another fence segment.

acknowledgement In an effort to see if this game had already been described, the author found an online reference to a variant of Dots & Boxes played in Poland where multiple squares can be claimed at once, but was unable to find a more complete description of that game. It is possible that it is the same game proposed here, but it need not be, as the Polish variant may retain the rule, of Dots & Boxes, that claiming squares permits the player to build another fence segment. That is, the Polish variant may exhibit the first difference from Dots & Boxes described above, but not the second.

"k-canal" We say that one square is "accessible" from another if they are adjacent, and no fence segment has been built between them. By "k-canal", we mean a sequence of k squares bordered each by exactly two fence segments, each square accessible from the previous one, and the first and last squares of the k-sequence not accessible from any square outside the k-sequence having more than one fence segment bordering it. (Note the difference between the "canal" in Borders and what is termed a "chain" in Dots & Boxes.)

two simple positions

- A $2k$-canal has a value of 0 to the player who plays on it first. The optimal move is to split the canal in half, creating a disjunctive sum of two identical positions, which has value 0.

  If the first player had made any different move on the $2k$-canal, the second player could have fenced off an even number of squares in the middle of the canal, leaving a disjunctive sum of two identical positions (at the two ends of the canal). The first player would then have lost a number of points equal to the number of squares fenced off in the middle of the canal.

- A $(2k + 1)$-canal has a value of -1 to the player who plays on it first. The optimal move is to split the canal in two parts, one of length k and the other of length $k + 1$.

  The second player's best move is then to complete the fourth fence segment bordering the middle square in the chain, gaining him one point, and leaving a disjunctive sum of two identical positions. No other move by the second player would do better than this (for the second player).

  If the first player had made any different move on the $(2k + 1)$-canal, the second player could have fenced off an odd number of squares (more than one) in the middle of the canal, leaving a disjunctive sum of two identical positions (at the two ends of the canal). The first player would then have lost a number of points (more than one) equal to the number of squares fenced off in the middle of the canal.

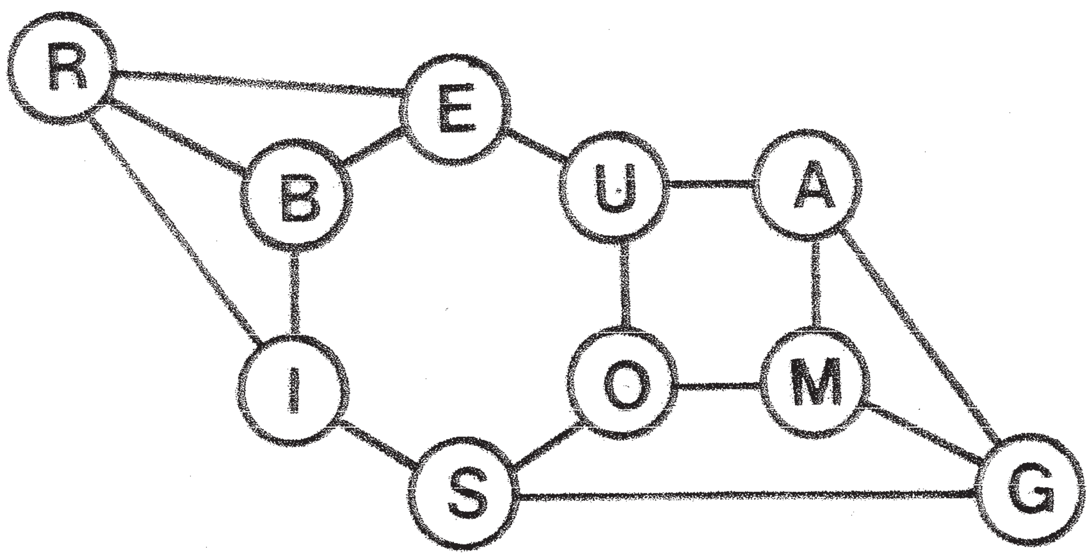
conjecture Suppose the game of Borders is played on an $a \times b$ board. The game is a win for the first player if $a + b$ is odd, and for the second player if $a + b$ is even.

Presented by
Jeremiah Farrell
To Honor
Martin Gardner
At G4GX, Atlanta
March 28 – April 1, 2012

THE SEABORGIUM CHEMICAL TABLE

A Puzzle-Game
by Jeremiah Farrell

The diagram is an example of a connected, cubic graph with its ten nodes labelled with the letters of SEABORGIUM, chemical element 106. Connected means it is in one piece and cubic means that each node has exactly three edges on it.



For our purposes we use instead a new graph constructed from this cubic known to graph theorists (see references B and H) as a line graph. This line graph has 15 nodes and 30 edges. The new nodes represent the 15 edges of the old cubic graph and are labelled, using the letters of the 10 nodes of the cubic an edge connects. I.e., we obtain these nodes.

Ag Am Au Be Bi

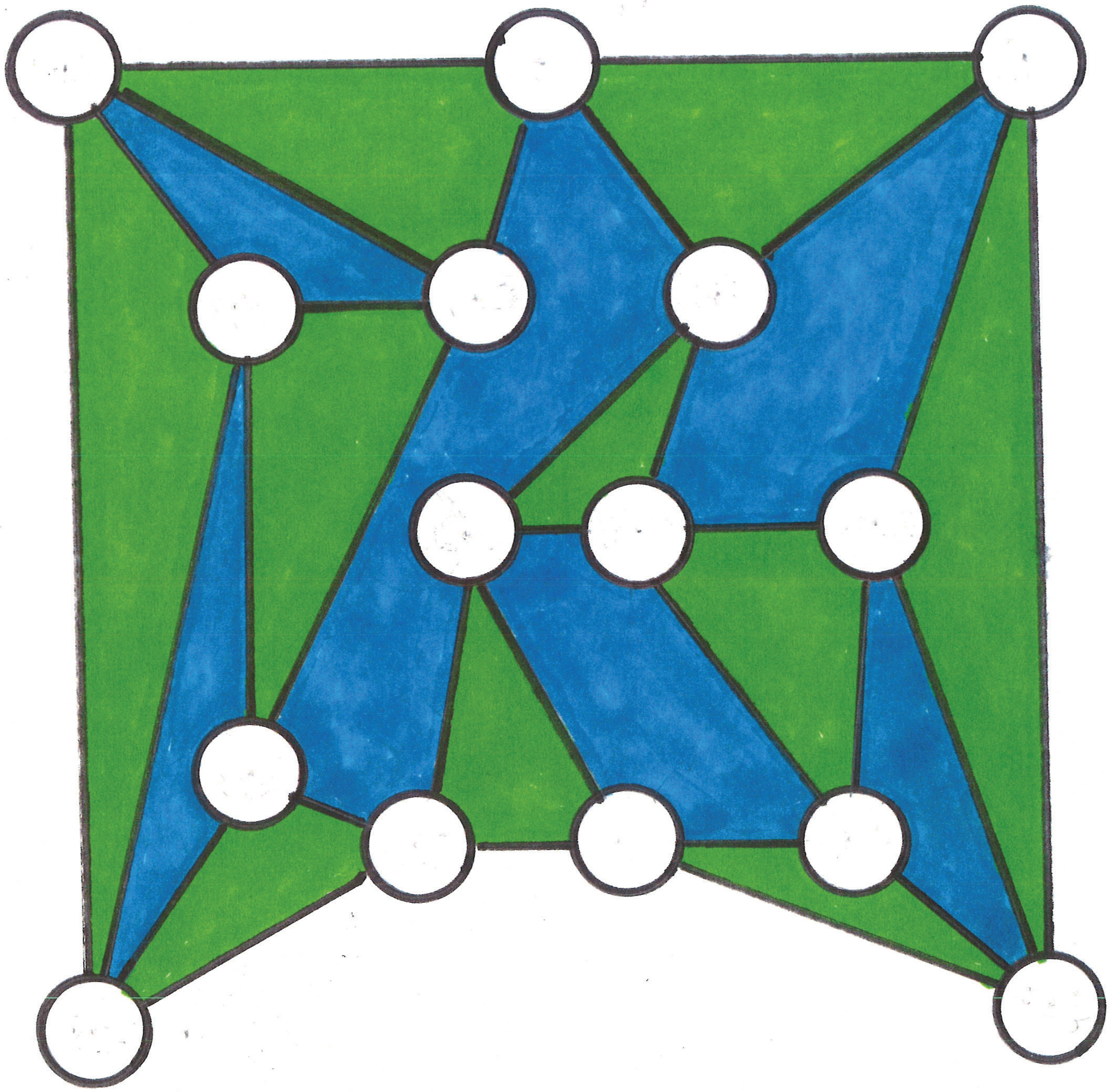
Br Er Eu Ir Mg

Mo Os Sg Si Uuo

Each label is the symbol of a chemical element (Uuo is element 118, Ununpentium). On the next page is a representation of the line graph where the edges connect nodes with a common letter.

The Puzzle: Place the 15 nodes on the line graph so that each node connects to another with a common letter.

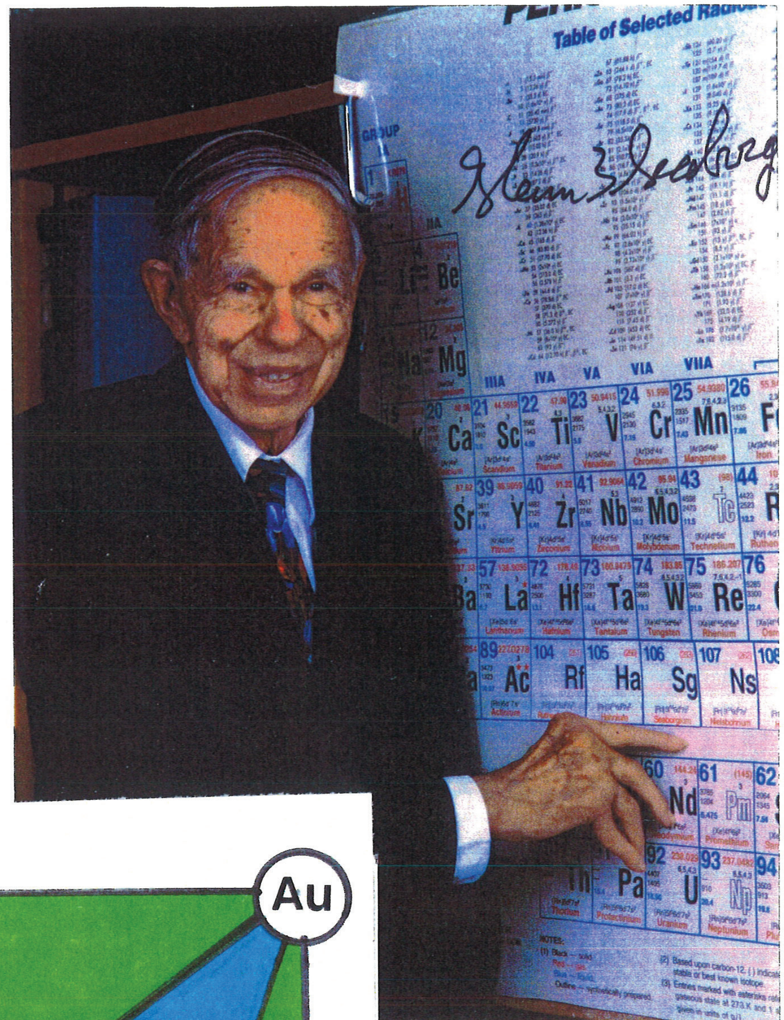
The Game: The players (two or three) select five tokens each and alternately place them on the graph so that abutting tokens have a common letter. Last player to be able to move wins. For two players, the extra five tokens are face-down in a kitty and may be drawn by a player when needed.

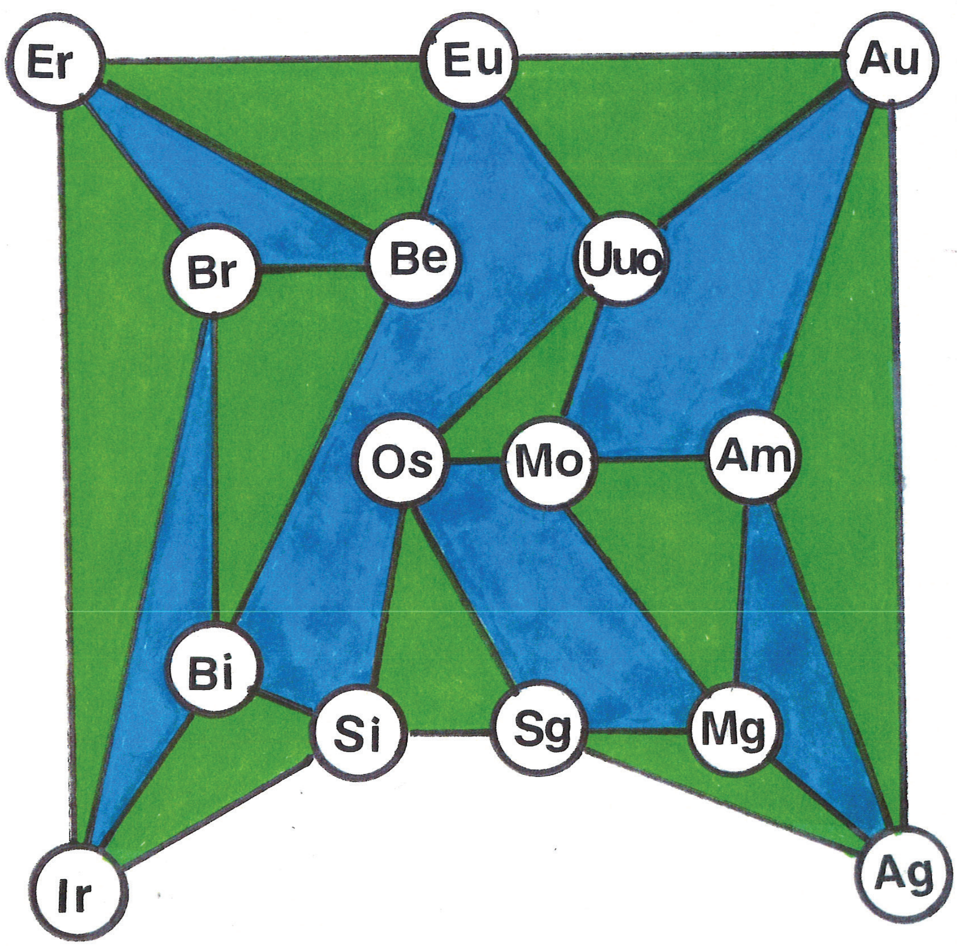


GLENN T. SEABORG
(1912-1999)
1951 Chemistry Nobelist

Dr. Seaborg worked on the Manhattan Project from 1942-1946, during which he was co-discoverer of plutonium and all further transuranium elements through element 102.

The radioactive element 106, Seaborgium, is named after him.





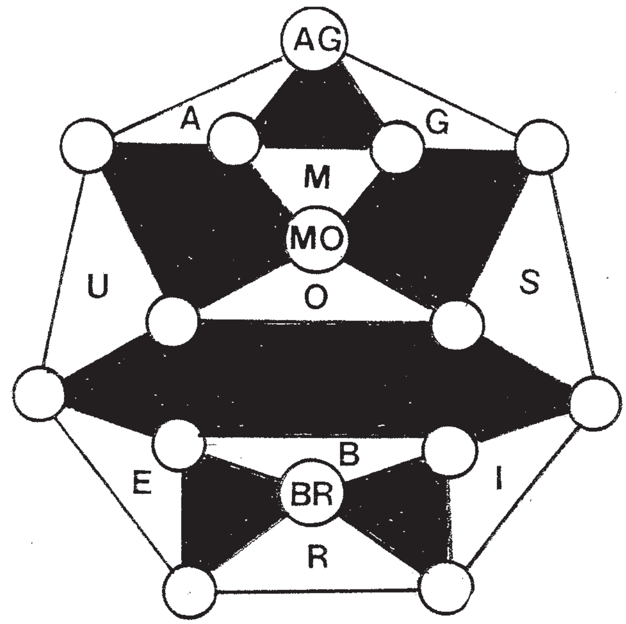
A solution to the Seaborgium Chemical Table.

All entries are chemical symbols using the ten letters of Seaborgium (Sg).

Another solution appears later in this paper.

SEABORGIUM has a second solution that can be discovered if we redraw the original line graph thusly

Notice the line of symmetry AG-MO-BR that could be a reflective axis to obtain the new solution Sg interchanged with Au, etc.



This illustrates the wide variety of playing bounds possible with these graphs. And this is not all! According to Wolfram Math World (http://mathworld.wolfram.com/regular graph.html) there are 19 nonisomorphic cubic 10s. This means that in addition to our SEABORGIUM cubic there are 18 more cubics that can yield 18 other line graphs on none of which our tokens can be successfully placed.

For example, another of the 18 is the cubic we label NIGHTMARES. It is drawn on the interior of its line graph at the end of this article. The line graph nodes are all main entries in The Chambers Dictonary, 11th ed. On the cubic notice that the outer 5 nodes are in order, NIGHT and the inner 5 are MARES. Perhaps knowing this can help in playing the game NIGHTMARES.

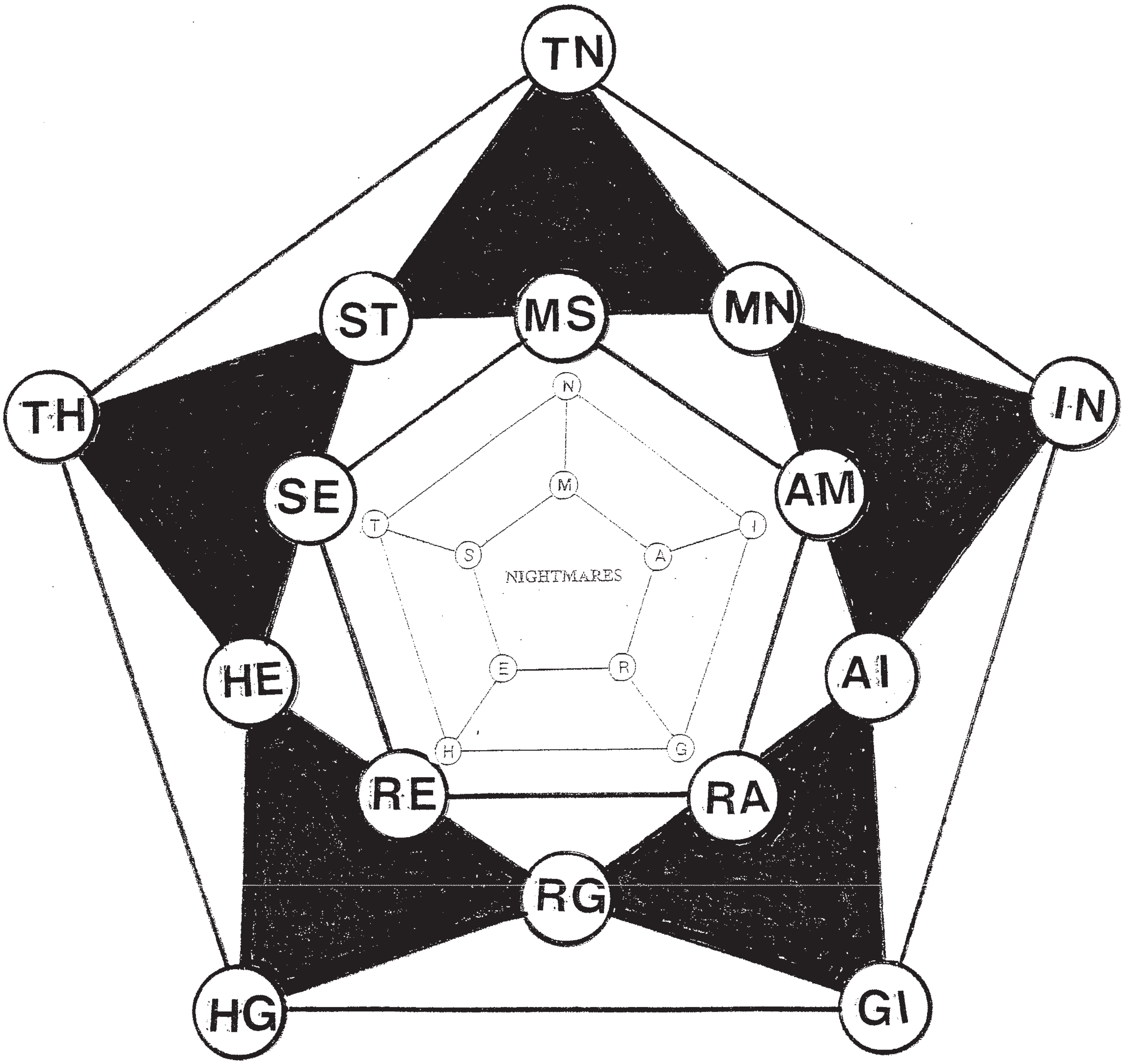
Wolfram claims nonisomorphic cubics to be: 85 on 12 nodes, 509 on 14 nodes, and 4060 on 16 nodes. Examples of each of these three follow: NOTARY PUBLIC, RHAPSODY IN BLUE, and OSCAR THUMPBINDLE. THUMPBINDLE is a regular comtributor to *Word Ways: The Journal of Recreational Linguistics.* For possible game playing note the outer and inner cycles are respectively NOTARY-PUBLIC, EUPHONY-RIBALDS, and BRUNCHES-DIPLOMAT.

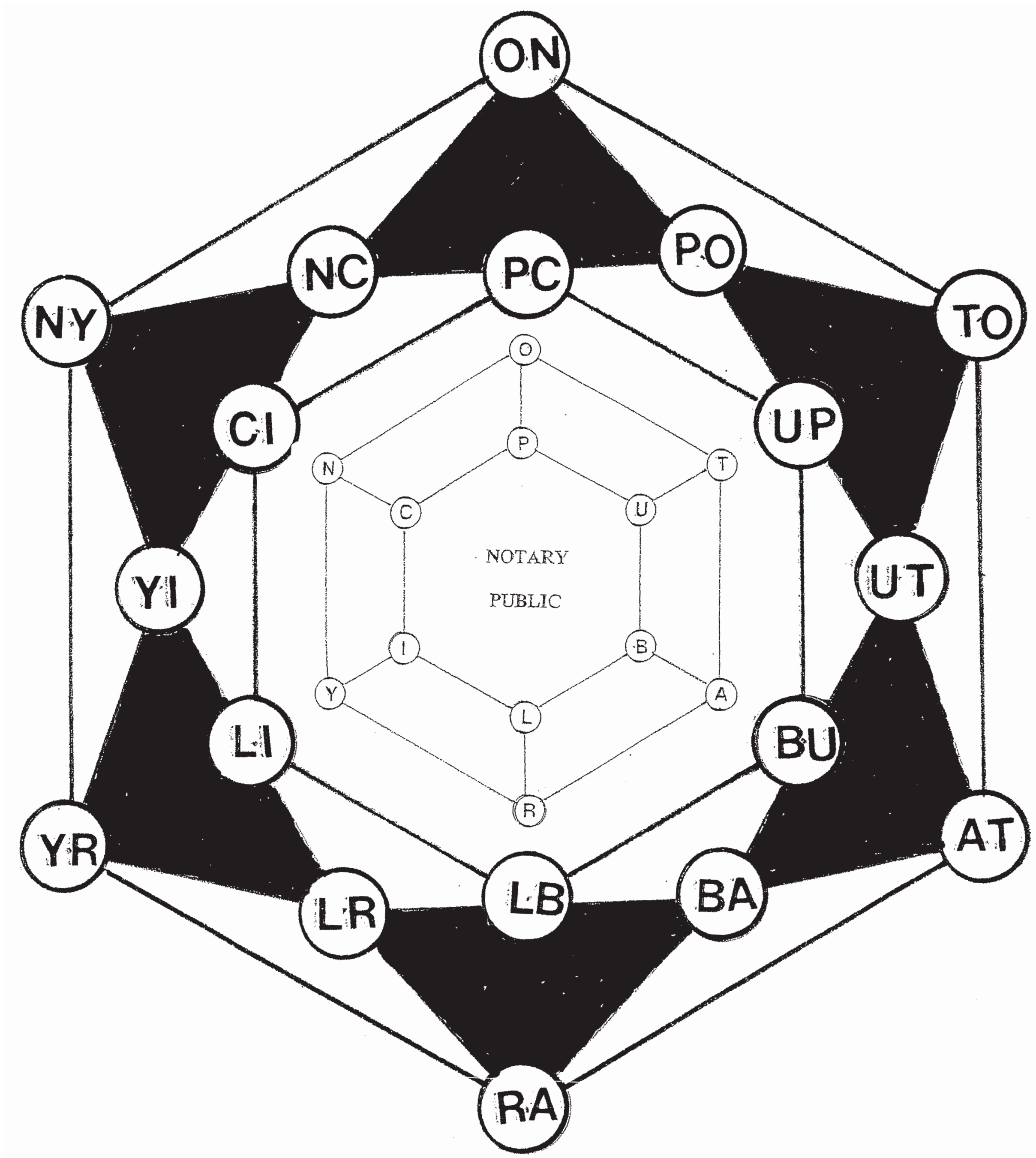
The reader should also be aware that most non-cubic graphs have line graphs and are therefore suitable for puzzles and games.

As a final activity, we offer the following word puzzle. Place the letters of MOUSTERIAN (of an early Palaeolithic culture) on the NIGHTMARES cubic so that on the line graph, each triangle contains one of the ten letters and each of the dark 4-gons anagrams into a word as well as the two 5-gons (inner and outer). Each of the 15 nodes should be dictionary entries as well. A hint to our answer appears below.

References
:
(B) Beineke, L.W. "Derived Graphs and Digraphs". In *Beitrage sur Graphentheorie*, ed. H. Sachs et.al., Leipzig, Germany: Teubner 1968.
(H) Harary, F. *Graph Theory*. Reading, MA: Addison-Wesley, 1994.

Solution Hint: We used MINUS, ORATE, MIRÓ, SUET, SOME, UNIT, and RAIN.

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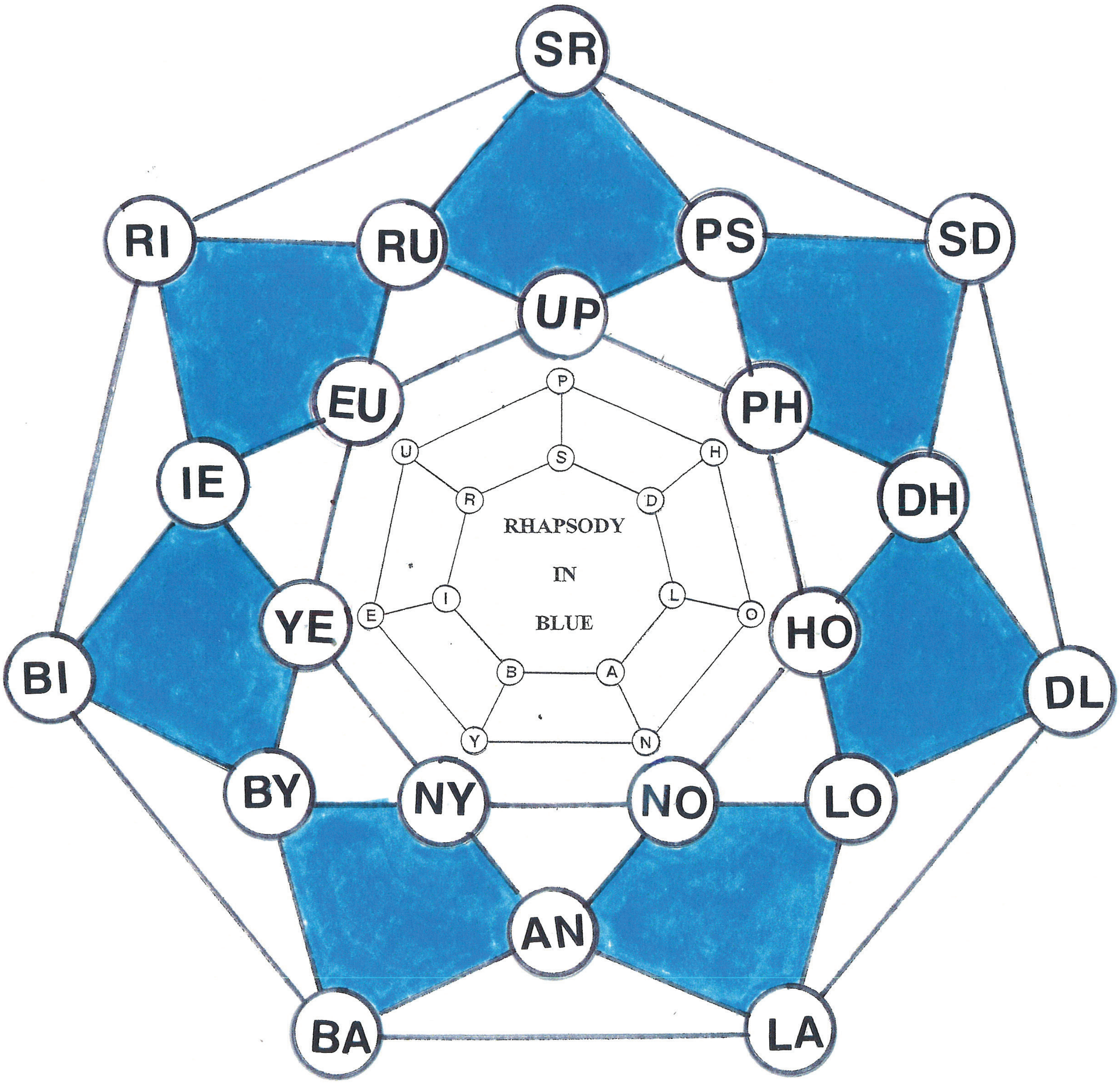
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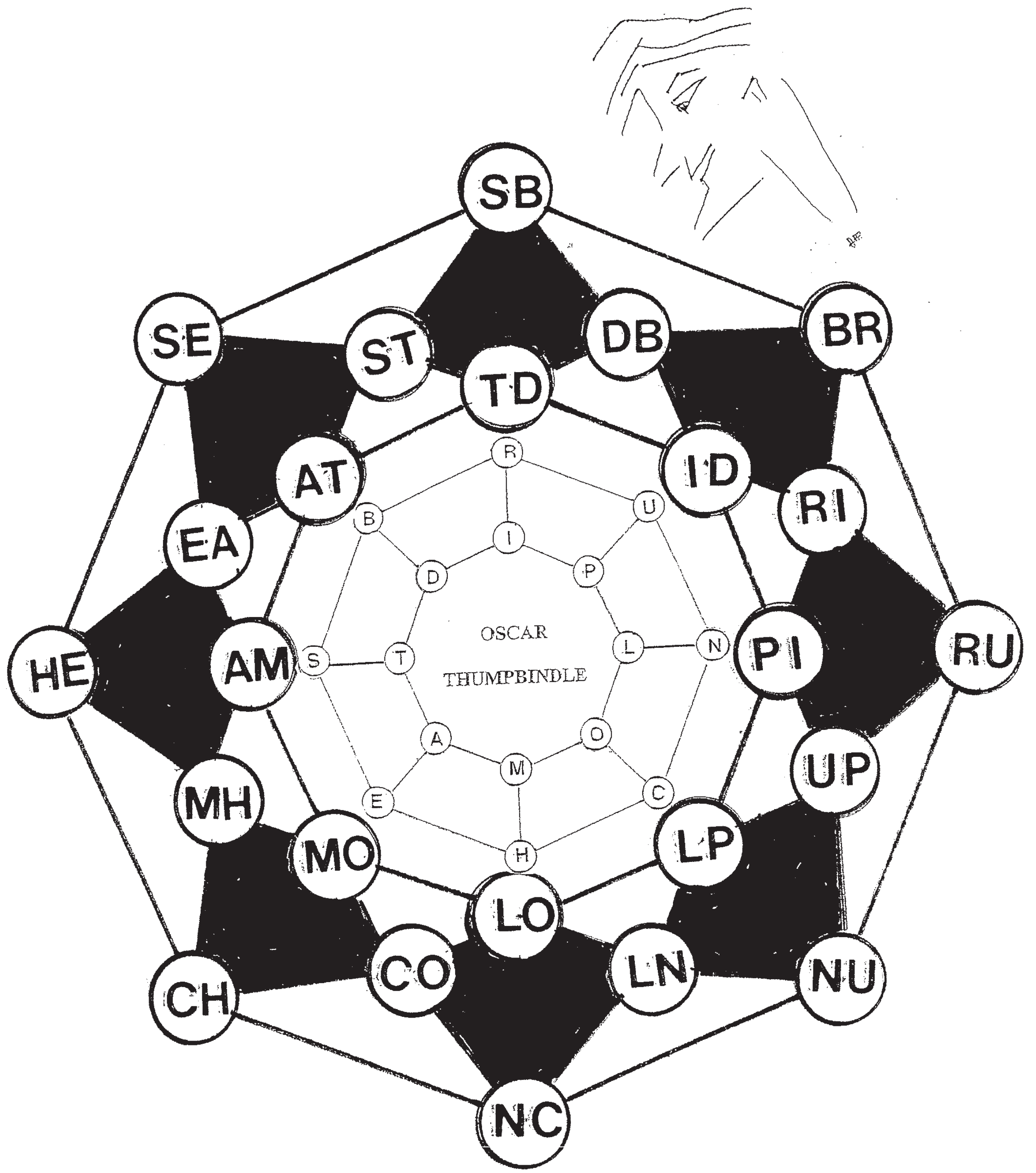
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Non-Euclidean Board Games

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Introduction

Most traditional grid-based board games are played in Euclidean space. It is a fun and interesting exercise to try and play them in non-Euclidean spaces. In this paper, we will explore playing the game of Chess in non-Euclidean space, but the concepts used to extend Chess to non-standard geometries can be extended to many other board games. A personal favorite game for these purposes is non-Euclidean Carcassonne.

Boards with Unusual Macrostructure

The simplest way to transform a standard board into a non-Euclidean board is to connect the sides of the board together in some way. Connecting the squares on adjacent sides to each other creates a board with an inherently spherical topology. In contrast, connecting the squares on opposite sides can lead to a board with a toroidal structure. A twist in the way the squares are connected can lead to a board with the topology of a klein bottle.

While these macrostructures are interesting, they are not inherently well suited for Chess. In particular, none of these boards have edges. Thus, it is impossible for a Pawn to reach the edge of the board and become a Queen. In fact, it's not even clear that we weren't playing on boards with these macrostructures all along, and were just playing a subsection of space that was locally equivalent to a subsection of a Euclidean space.

The Chutes and Ladders Connection

How can we create a non-Euclidean board that does not lose the edge constraints of the original Chess board? One way is to connect random squares to each other by the edges. We will refer to these kinds of connections as "chutes" in homage to a classic board game with these kinds of connections - Chutes and Ladders.

We will use the chutes defined by the board in Figure 1 to explain basic movement on a board with chutes, however one can easily add their own chutes to any standardly gridded board to make it non-Euclidean.

Chutes and ladders are functionally the same. The side of the tile going into the "top" of each chute or ladder connects to the side of the tile going out of the "bottom" of that chute or ladder.1 For example, a pawn moving forward two squares out of tile 84 would move out the left side of tile 28 and into tile 29. Note that the pawn is now moving sideways, and can be Queen-ed on its next move onto tile 30. We recommend playing with pieces with some obvious indication of directionality on them, so that you don't lose track of the traveling direction of your pawns.

In order for the board to be connected, we must also map the tiles that come out of the tops and bottoms of each chute and ladder to each other. Thus, the bottom of tile 20 must match the bottom of tile 23 and the left side of tile 18 must match the top of tile 59.

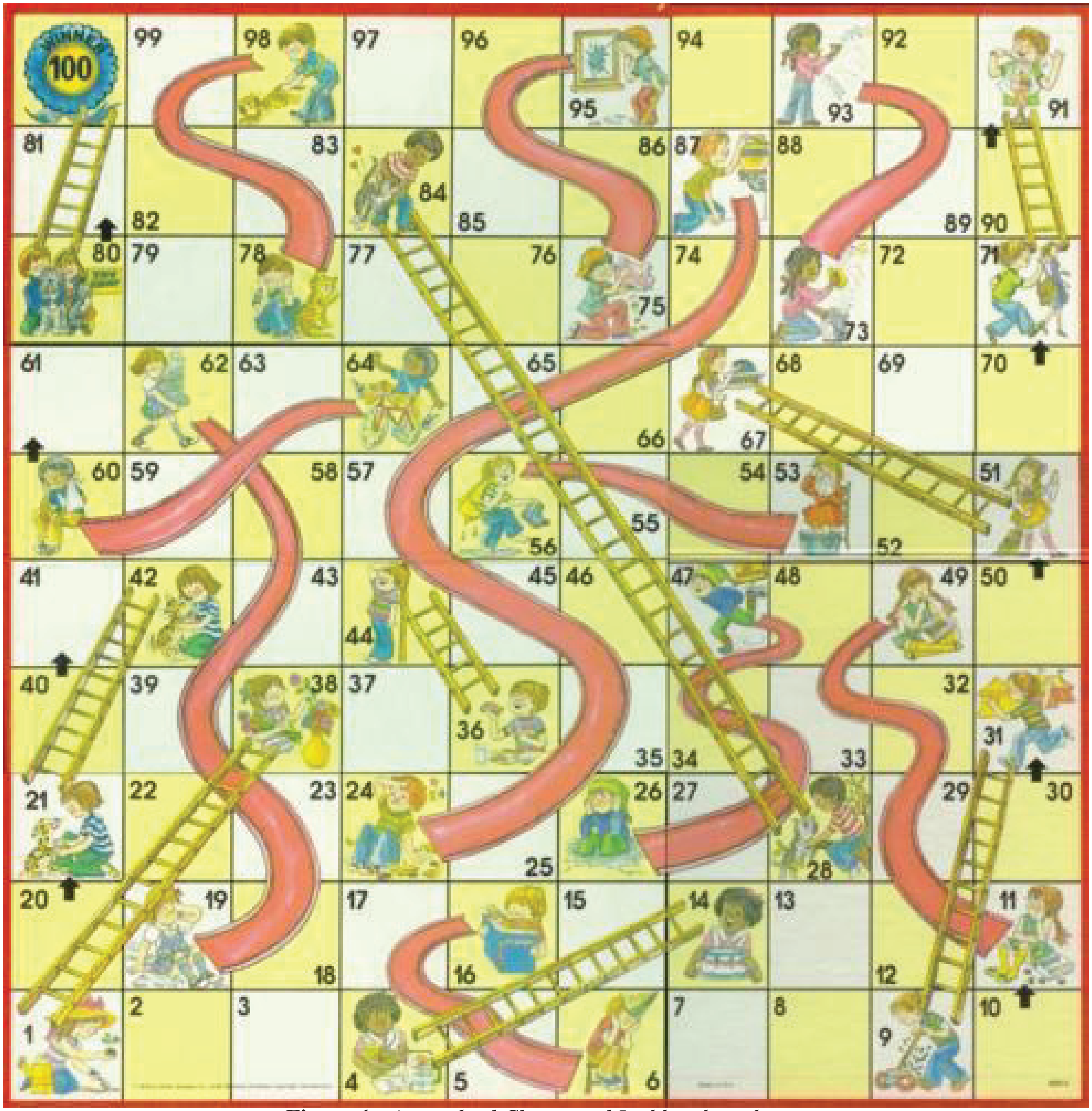

Figure 1 : *A standard Chutes and Ladders board*

The Turn-Left Metric

At this point, it is pretty clear how to travel across the sides of the tiles, but what it means to travel diagonally is less well defined. We have chosen to use a "turn-left" metric to define diagonal moves. Let us imagine a bishop moving diagonally out of the top right corner of square 4. In a standard Euclidean board, it would end up in square 16 after moving one square. This is equivalent to moving into square 5, then turning left and moving into square 16.

On our non-Euclidean board, we find the corner by moving in the direction of square 5, which brings us out square 14 without rotation and turning left to end up in square 27. Since we haven't rotated, if the bishop wants to continue moving along the same diagonal it may. For its second square of movement, it can travel in the direction of square 28 causing it to enter square 77 from the top. This time, when it turns left, it enters square 76. Additionally, it has rotated $-\pi/2$ radians, thus, to continue along the same diagonal it will want to move out of the bottom right corner of square 76, and comparably straight-forwardly into square 66. Following similar logic, the next squares that the bishop can visit are 54, 26, 36, and 37.

Extensions

There are many other fascinating games that you can try playing along similar lines. Players might take turns moving, then adding a new chute to the board. Roice Nelson has suggested playing chess or checkers on a hyperbolic checkerboard. Are there any other interesting boards that you can imagine playing chess on?

A Puzzle

Can you design a chess board with a single chute such that White as a mate in one?

Let's play a concentration game with a few cards

Hirokazu Iwasawa*

I believe that many of you will want to play a concentration game with a few cards after you read this article—because it shows nice strategies to win the game with a high probability. In fact, when you play a two-player concentration game described below, the probability of your winning will be more than twice that of the opponent's if she is a normal clever person with a perfect memory, whether you are the first player or the second player!

The game

You can apply the central idea of this article to a concentration game with a different number of cards. But, for convenience, we concentrate on the following concentration game:

> **Number of players:** Two.
>
> **Cards used:** Four pairs of cards (e.g. ♢ A, J, Q, K; ♡ A, J, Q, K).
>
> **Rule:** The usual rule as follows. Shuffle the cards and lay them on the table, face down, in a pattern (e.g. 2 cards × 4 cards). In turn each player turns over two cards (one at a time). If they are of the same rank and color, that player wins the pair and plays again. If they don't match, they are turned face down again and it becomes the other player's turn. The game ends when the last pair has been picked up. The winner is the person with more pairs, and there may be a tie.

Normal players

To consider strategies, suppose that

> your opponent has a perfect memory and is such a *normal* person that she never gives up a chance to make a pair in any turn of hers (more specifically, she never chooses a known card unless she is sure she can then make a pair using that card).

At least in my observation, we are actually allowed to suppose this for a regular person when we play a concentration game with a few cards.

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We don't need any advanced knowledge to compute the probabilities of winning/losing this game. The calculation is, however, quite cumbersome because we need to consider separately so many cases although there are only eight cards. So I show here only the results of calculation:

If both players are *normal* in our sense,

The probability of the first player's winning $= 3/5 = 60\%$,
The probability of the second player's winning $= 4/15 \risingdotseq 26.7\%$,
The probability of a tie $= 2/15 \risingdotseq 13.3\%$.

It means that the probability of the first player's winning is 2.25 times that of the opponent's. Therefore, this game may look very unfair to the second player. But the truth is that there are nice strategies for the second player.

The best strategy against a *normal* opponent

If you are the first player, being *normal* is good enough as we've seen it. Indeed, it is the best strategy against a *normal* opponent.

How about when you are the second player? The following is the best strategy against a *normal* opponent:

If the very first two cards opened by the first player don't match, you turn over the same two cards.

If you take this strategy and the opponent is still *normal*,

The probability of your winning $= 62/105 \risingdotseq 59.0\%$,
The probability of the opponent's winning $= 29/105 \risingdotseq 27.6\%$,
The probability of a tie $= 2/15 \risingdotseq 13.3\%$.

It means that the probability of your winning is more than 2.1 times that of the opponent's. Wow, whether you are the first player or not, you win, in a long run, more than twice as many as you lose.

The practically best strategy

You may say, "OK. But, as the suggested best strategy gives the opponent a strange impression, she may become cautious and copy your play to disable your strategy."

Well, you're so clever as I thought. But don't worry. You can take the following quieter strategy, which can be taken also after your opponent copies your first play:

Choose a known card (if possible) for a second card in your each turn if there remains a pair neither of which has appeared.

If you take this strategy and the opponent remains *normal* as I hope,

$$\text{The probability of your winning} = 10/21 \fallingdotseq 47.6\%,$$
$$\text{The probability of the opponent's winning} = 43/105 \fallingdotseq 41.0\%,$$
$$\text{The probability of a tie} = 4/35 \fallingdotseq 11.4\%.$$

So you still have advantage over the opponent!

What happens if both players do the best?

By the way, what is the first player's best strategy when the opponent plays best? A trick appears, for example, in the following situation:

> There still remain all the eight cards. Three of them were turned over before and no two of the three match.

When you encounter this situation in your turn, don't you turn over a new card? —That's not good! You should turn over two of the known cards, mathematics says.

So, at such a tricky point, the game stops in effect as neither player will turn over any new card if they play best. And it's not rare—it occurs at the rate of $4/7 (\fallingdotseq 57.1\%)$.

However, I think it rarely happens actually unless, say, both players have read this article. So, don't hesitate to play this concentration game with a regular person. —Good luck!

Appendix: The best strategy for the standard two-player concentration game

If you have a very good memory, you may want to use more cards. Here is the best strategy for up to 26 pairs (the full deck) optimizing (not the probability of your winning but) the expected number of cards you get.

Let (m, n) express a situation in which there remain m pairs on the table and there are n known cards among them.

The best strategy:

1. When your turn comes,
   (a) if there is a pair among the known cards, take them;
   (b) otherwise, choose one of the known cards if and only if the situation (m, n) is such that $n \geq 3$ and $m - n = 1$ or the situation (m, n) is equal to either of
   $(4, 2), (5, 3), (7, 2), (7, 3), (8, 4), (9, 2), (9, 5), (10, 3), (10, 6),$
   $(11, 4), (12, 2), (13, 3), (15, 3), (17, 2), (18, 3), (20, 2).$

2. After you choose your first card,
   (a) if there is a known card that matches your first card, choose it;
   (b) otherwise, choose a new card if and only if the situation (m, n) is such that $n \leq 1$ or $m = n$ or the situation (m, n) is equal to either of
   $(5, 2), (6, 3), (7, 4), (8, 5), (8, 2), (9, 3), (10, 2), (11, 3), (13, 2),$
   $(14, 3), (16, 2), (17, 3), (18, 2), (21, 2), (22, 20), (23, 21), (24, 22),$
   $(24, 2), (25, 23), (26, 24).$

If you take this strategy against a normal person, the probabilities of your winning, losing and a tie are 0.70458, 0.23895 and 0.05647, respectively, when you're the first player; and they're 0.70452, 0.23907 and 0.05640 when the second player.

You might feel the best strategy is too complicated for practice. Then you may simplify 1(b) to:

> otherwise, choose a new card unless the situation (m, n) is such that $n \geq 3$ and $m - n = 1$

and 2(b) to:

> otherwise, choose a known card (if possible) unless the situation (m, n) is such that $m = n$.

If you take this simpler strategy against a normal person, the probabilities are 0.70395, 0.23950 and 0.05656 when you're the first player; and they're 0.70390, 0.23962 and 0.05649 when the second player. Practically speaking, the effect is almost the same as the best strategy.

What is the best strategy in the sense that both players do the best? There is a nice paper on this topic.[1] I've confirmed their results also in my calculation up to the 26 pairs' case. In my representation of the best strategy in this sense, 1(a) and 2(a) are the same as above but 1(b) is:

> otherwise, choose a new card unless the situation (m, n) is such that $m - n$ is odd and $n \geq 2(m + 1)/3$

and 2(b) is:

> otherwise, choose a known card (if possible) unless the situation (m, n) is such that $m - n$ is odd or $(m, n) = (6, 2)$.

If you wish, you may take this strategy also against a normal person. Then the probabilities are 0.64140, 0.29478 and 0.06382 when you're the first player; they're 0.63960, 0.29635 and 0.06405 when the second player.

[1] U. Zwick, M.S. Paterson, "The Memory Game," *Theoretical Computer Science*, Vol.110, No.1, 1993.

Computer Shogi－The History and the Techniques

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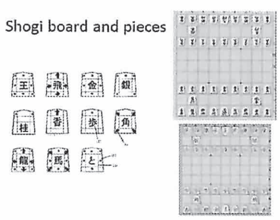
Introduction

- Shogi(将棋) is a game with the same origin as chess and xiangqi , played in Japan.
- Computer shogi studies started in the 1970s.
- Nowadays, it has reached professional players' strength.
- Many technologies, peculiar to computer Shogi, have developed as well as application of chess knowledge, such as: end-game search, realization probability search, evaluation function learning, etc.
- I will talk about this history of this computer Shogi with the technical development.

Outline

- What is shogi?
- History of computer Shogi
- Techniques and Algorithms
  - －Many techniques
  - －Realization probability sea
  - －Bonanza method
  - －Endgame search
- How strong and systems
- Next match and future

Shogi board and pieces



What is Shogi?

- It has the same origin as western chess and Xianqi
  - －the origin is in India
  - －various chess games are distributed in Eurasia
- It is disputed whether first Shogi came from south or west
  - －my opinion is from south
  - －Shogi and Xianqi are brothers
- The significant rule:
  - －captured pieces can be used by dropping it on a vacant square
  - －dropping moves
- Shape of pieces
  - －No difference between first and second player's pieces
  - －for the reuse
  - －Correctly there is a little difference in King
- 王 for upper grade, older, or second player
- 玉 for lower grade, younger, or first player

Special rules

- prohibited moves
  - －double pawn(二歩)
  - －non-movable piece
  - －dropping-pawn checkmate(打歩詰)
  - －continuous check perpetual move(連続王手千日手)
- draws
  - －perpetual move(千日手)
  - －mutual migration of kings to opponents' territory(入玉)

Shogi People

- population: 12 million -- 10% of Japanese
  - －Go has 4 million
- 157 professional players
  - －Go has 450 professionals
- organization
  - －Japanese Shogi Association(日本将棋連盟) the union of professional players
  - －many local amateur groups
  - －also groups of endgame Shogi problem

Computer Chess: Before the History

- Artificial brain machine was an imaginary goal for scientists for long ago. Playing chess was understood as its example.
- People already thought computer chess in 1950, when first computers started working:
  - －Shannon's and Turing's algorithms
  - －match between paper algorithm document and a novice woman
- In 1960s and 1970s, computer chess became strong, when most of fundamental techniques were invented such as αβ search, transposition table, etc.
- Deep blue defeated Gary Kasparov in 1997

History of Computer Shogi

- 1970s first computer Shogi programs
  - －including Takenobu Takizawa's(inFORTRAN) and Kotani's(in LISP) *
- 1980 first Shogi matches among programs
  - －Takizawa's, Kotani's and Osaka Univ.'s
- 1980～ Commercial Computer Shogis were made on PC(PC98,FM-7,MSX etc.) and FAMICOM, around 10 kyu
- 1986 CSA(Computer Shogi Association) was established
- 1986-1987 Tournament in Yomiuri Weekly Magazin
  - －8 comercial softwares

写真1 週刊読売コンピュータ将棋十段戦の第1回のページ.

- 1990 amateur 2-3kyu
- 1990～ Computer Shogi Championship tournament
- 1990～ various activity started
  - －conferences: GPCC,GPW,CG
  - －1990 first Computer Shogi Book
  - －1994 Translation of Computer Chess Book (D. Levi and M. Newborn)

- 1990～ Computer Shogi Championship tournament
  - －annual, 21 times
  - －around 50 participants, 3days
- 1997 longest endgame problem "Microcosmos" was solved
  - －solution has 1519 plies
- 1999 df-pn algorithm (proof-number search for checkmate)
- 2000 realization probability search (flexible game tree depth)
- 2005 amateur 5dan or more
- 2006 BONANZA method (tuning of huge parameter set in position)
- 2007 in the range of professionals
- 2009 BONANZA source code made open
- 2010 a female professional defeated
- 2012 former Grandmaster defeated

An epsode of starting CSA in 1986

- A pair of Englishmen visited me, bringing a prototype Shogi machine, to find a commercial partner. It was a very sophisticated one, and has a Shogi board which recognizes piece moving with magnet sensor, though its CPU was Z80.
- I planned an amusing event to call Kazuo Morita who had made Morita Shogi, which was believed the strongest and well-sold software, and to make a match between them.
- The match was held at Nob Yoshigahara's Studio in Tokyo.
- The result Shogi machine defeated Morita Shogi which used 16-bit CPU 8086.
- We found such communication is much joyful, and then I gathered many people who made or were interested in computer shogi. It was the start.
- I wonder the Shogi machine was sold well or not.

Computer Shogi Data and Algorithms

- Data structure of position and move
- αβ tree search
- Iterative deepening
- Transposition table(TT)
- Move ordering
- Quiescence search or Capture search
- Extension and Pruning
- Fractional Number Extension
- Realization Probability Search
- End game and Proof-Number Search
- Opening
- Evaluation function and BONANZA parameter learning method
- Parallel Processing

Computer Shogi Data Structure

- position data
  - captured pieces
  - bit board often used
- data of a move
  - depends on whether backward position generation is needed or not
  - if needed
- from_xy, to_xy, from_piece, to_piece, captured _piece
- attack table

Game tree search : αβ pruning

- the most fundamental algorithm since 1960s
- effective min-max strategy
- write in Negamax method
- often Window search, Scout method embedded
- pruning methods embedded

Iterative deepening

- main routine calls αβ search iteratively, deepening its search depth by 1 (or 0.5)
- purposes:
  - generate move ordering information for deeper search
- kept in transposition table
  - for thinking time control
- time efficiency is little worse, because shallower searches are shorter enough than the last

Transposition Table

- the mechanism not to search the same position again
  - the possibility is high in Shogi
    A-B-C / C-B-A
    A(promoting)-B(capture) / A(not promoting)-B(capture)
- hash table
  - collision is ignored usually overwriting keeping shallower node etc.
- data in it
  - evaluation value, depth, exact/upper limit/lower limit
  - the best move for the position
  - sometimes the move list
- other TT
  - partial hash: TT without pieces in hand
  - the TT of endgame search is different one, whose values have to be kept correctly

Move Ordering

- necessary for efficient search
- by what?
  - the evaluation value in TT
    - the content saved in the last iteration
  - values in light evaluation function
  - the result of one-ply search
  - various heuristics like killer move
  - combined with move generation in past

Quiescence search or capture search

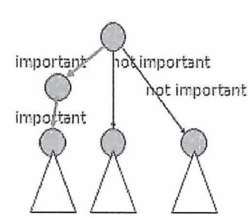
- their meaning is similar
- it is hard to be quiet in Shogi
- search only by :
  - capturing moves
  - escaping moves
  - sometimes
    attacking moves, double attacking moves, check moves
- often exclude pawn capturing moves
- mutually capturing search on the square of the last move

Extension and Pruning

- forward pruning(selected search) was dominated in past
- complete-width search + extension + short pruning + capture search
- extension
  - check extension
- short pruning
  - futility pruning, prob cut, null move pruning

Fractional Number Extension

- delicate, precise extension can be described
- to decrease the depth of search function by fractional number
- Yamashita's 0.5 move extension
  - for top move of move ordering




```
search(pos,alpha,beta,depth){
  if(depth<=0) return eval(  );
  generate moves;
  move ordering;
  for(next positions){
     w= -search(next position,-beta,-ma
                        d - fractional value)
     if(w>max)max=w;
     if(max>=beta)return max;
   }
   return max;
}
```

Realization probability search

- data based tree expansion
- Prof. Yoshimasa Tsuruoka invented and implemented it in Gekisashi System
- Historically investigated what kind of moves should be looked forward in tre
- A kind of fractional number search
- Based on probability of playing for a move
- In practice, the conditional probability:
  p<m> = p(move m is played | m is in category c)
- This value is calculated by many professional play records:
  (the # of the played moves of category c) /
  (the # of the moves of category c in possible move list)
- The probability that the variation $m_1, m_2, \ldots, m_d$ is played is estimated:
  $$p<m_1>p<m_2>\cdots p<m_d>$$
- When this value become equal or less than a constant D, then stops looking ahead

```
search(pos,alpha,beta,depth){
   if(depth<=0) return eval( );
      :
   for(next position by move m){
      w= -search(next position,-beta,-max.
                     d – (-log p<m>));
      :
   }
   return max;
}
```

- example of p<m>
  - p<normal move> = 1/100
  - p<important move> = 1/10 : check, capture
  - p<meaningless move> = 1/10000
  - they consume the depth 1, 0.5 and 2

Opening database

- It gives the best move for completely matched positions in database
- It is made from professional record of play automatically
  - whole covered data: the system knows everything!
- Castle making
  - There is castle making phase after opening database tracking
  - It was an important part in past
  - Evaluation function and search generates castle making, because good evaluation functions were developed now

The evaluation function of Shogi

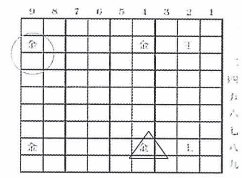
- It gives the advantage/disadvantage grade of a position
- Fundamentally piece value based
- Piece value is dependent upon its location on board and relation with the kings
- Values of pieces in hand are important
- the speed of the evaluation is required:
  - speed and correctness are in trade-off relation
- Hoki made an excellent method of generating the function, known as BONANZA method

BONANZA method: learning of evaluation function parameters

- Kunihito Hoki won WCSC championship in 2006 by this method
- His evaluation function has huge number of parameters, and he succeeded making learning mechanism for them
- parameters
  - values for relation among 2or3 piece locations
  - Especially piece and king relation

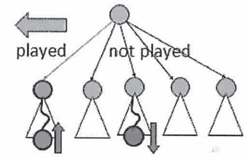
  weight[piece][piece_location][my_king_location]
  14 x 81 x 81

  example: **weight[GOLD][98][28]**



- learning method
  - TD(temporal difference) learnii
  - **learning between brother positions**

data for learning
  - self match data
  - problem collection
  - **professional record of play**



- it learns to increase the value of played position and to decrease that of not played
- unusual phenomenon: the value of Bishop low
- the match against Akira Watanabe who has the title Ryuo in 2007
- Several participants for WCSC championship used its open library and became very strong

On endgame

- chess-like endgame database methods are not used
  - The number of pieces does not decrease
  - Retrograde analysis cannot be used like chess
- only one move checkmate is coded
- instead **Checkmate search** has a big role in Shogi
  - embedded in usual search
  - systems became extremely strong in last stage

Checkmate Search

- an embedded partial search in αβ search
  - in root node and shallow nodes
- not min-max search but AND/OR search
- Possible moves are restricted as:
  - offence side: check moves
  - defense side: defense moves against check
- Return value is Boolean
  - true: checkmate variation is found (offence side's win)
  - false: not found
- Two directions: against first and second kings

Study on Checkmate Search

- Long history since around 1990
  - Simple depth-first search first
  - Best-first searches were made
  - Many method of heuristics were made which select first moves to look ahead
- The algorithm is used to solve Endgame problems(詰将棋) as well as checkmate search
- Masahiro Seo succeeded to make an efficient solving algorithm Seozume in 1998
  - Can solve long move sequence endgame problems
  - Used the concept “Conspiracy Number”
  - renamed as “Proof Number” related with other study
- Afterwards proof number searches were made and succeeded:

—Best-first search
—df-pn (depth-first pn search) by Ayumu Nagai 1999

Proof Number(pn) and Disproof Number(dn)

- Proof number searches are symmetrical on sides
- Proof number for a position:
  —The number of positions to prove it is true
- Disproof number for a position:
  —The number of positions to prove it is false
- Proof number searches act easiest positions first
  —They have lower proof/disproof numbers
- A true position: pn=0,dn=∞ A false position: pn=∞,dn=0
- A new(terminal) position:pn=1,dn=1
- A non-terminal position:
  —pn=minimum of its child positions' dn's
  —dn=sum of its child positions' pn's

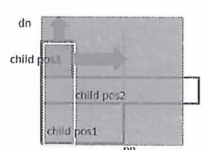
Outline of df-pn

- depth-first search
- iterate recursive call for the easiest child positions
- until target df or pn is found
- it is called by the code:

```
pn=∞;
dn=∞;
search(position, &pn &dn);
```

- the result is given by the return values:
  dn=∞ : positive solution
  pn=∞ : negative solution

```
search(position p, *pn, *dn){
 if(p is in TT and pnTT>=pn or
dnTT>=dn){pn=pnTT;dn=dnTT;return;}
 if(p has no child) ){pn=∞;dn=0;save it to TT;return;}
 generate children of p;
 repeat{
    sum_p= the sum of their pns;
    min_d= the minimum of their dns;
    if(min_d>=pn or sum_d>=dn)
        {pn=min_d;dn=sum_d;save it to TT;return;}
    recursive call to the child whose dn gives min_d;
}
```



- recursive call tries to add pn or dn of position 3, if it has the smallest dn
- it works in a loop till it reaches to the target dn (the arrow upward),
- or to next minimal dn of children (the arrow rightward)

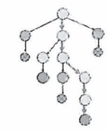
Endgame Problems(Tsume Shogi)

- Most of enjoyed endgame problems are Tsume Shogi
- there are many enthusiasts, and often felt as art
- long history since 400 years ago
- Tsume Shogi has been defined logically
  —Possible moves are the same as checkmate search
  —The solution is unique

Definition of Tsume Shogi

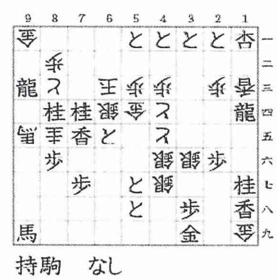
1.endgame position : a position in defense turn where there is no defense move
2.solvable position
 —endgame position
 —a position in offence turn such that at least one child is solvable
 —a position in defense turn such that all children are solvable
3.Tsume variation : a move sequence for a solvable position such that
 —null sequence (endgame position)
 —the move to a solvable position + Tsume variation after it
   (a position in offence turn)
 —the longest one of "a move + Tsume variation after it"
   (a position of defense turn)

- Tsume Shogi is defined as a solvable position such that:
  —all positions in offence turn on its Tsume Variation have only one move to a solvable child position.
- The solution is the Tsume variation.

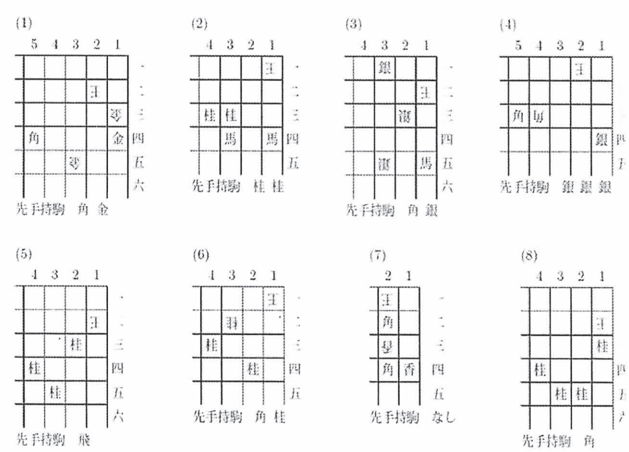


Ability of Checkmate Search to Tsume Shogi

- After developed enough, almost all famous Tsume Shogi problems were solved including Microcosmos (below) by Koji Hashimoto which has the longest solution: 1519 plies
- In hundreds of historically famous problems, tens of errors were newly found as "no solution" and "unexpected solution".
- Tsume Shogi generation was also studied and succeeded to made short ones



Machine Generated Tsume Shogis



Parallel Processing

- Thread programming on multi core machines is widely used
- YBWC
  1.calculate the first child
  2.then parallel processing for the second child to the last
- opinion combining
  —the first idea is 3Hirn
  —act on independent machines
  —Akara system defeated a female professional in 2010
- Parallel machines
  —usual multi-core PC, Playstation, FPG, PC cluster

How strong computer shogi systems are

- I estimated the rating of computer Shogi in the distribution of professional players
  - Matches between professional players and computer Shogi ws decreased
- Shogi Association prohibited easy matches in 2005
- Permission and enough sponsorship are needed
  - Estimation by circumstantial evidence
  - **Computer Shogi exceeded the average of professionals in 2009 at latest**
- Computer Shogi grows about 100 per year in rating

ELO Rating of Humans and Computers(short games)

Grandmaster Habu	**3179**
4 top rank players	3071－3166
professional av.+2σ	**3058**
estimated computer(2011)	**2900**
estimated Yonenaga(retired)	**2800**
professional av.	**2696**
top amateur	2592
amateur(match with pro)	2494
estimated computer(2007)	**2500**
female pro. av.	2382
professional av.-2σ	2334

January 14 Match

- We had a epoch making match of human vs. computer
- Jan. 14, 2012
- Human player
  - Kunio Yonenaga!
- Computer Shogi
  - Bonkuras, by H. Ito
- Sponsor : Dwango
- Time limit
  - 3 hours total
  - 1 minutes per a ply after consumed

(from Ascii Weekly)

Kunio Yonenaga

- The president of Japanese Shogi Association
- Joyful, self-confident, verbose person
  "My elder brothers entered University of Tokyo because they have bad brains. I became a Shogi player because I have a good brain."
- He got 19 titles, which is the fifth best in history, including
  - Meijin(Grandmaster) 1 period
  - 10 Dan 2 periods
  - Kisei 22 periods
- got the name Eternal Kisei
- 68 years old
- retired as professional player
- challenging to recover the state before retire
- training
  - against aging
  - 4 fours per day
- Professional players estimated his current rank
  - 30th to 40th among professionals
  - without opening
- said "the last big match in my life"

Bonkuras

- Computer Shogi: Bonkuras
  - by Hideki Ito
  - Bonanza method based
  - 3 pc combined
- The winner of the championship in 2011

	Program	1	2	3	4	5	6	7	Win	Lost	Tie	SB	MD	Rank
1	Gekisashi	5○	6△	7○	8×	3×	4×	2○	3	3	1	11.0	4.0	4
2	Shuso	8○	7○	6○	4×	5×	3○	1×	4	3	0	10.5	5.0	3
3	GPS	7×	5×	8○	6×	1○	2×	4○	3	4	0	10.5	3.5	6
4	Bonanza	6○	8○	5×	2○	7○	1○	3×	5	2	0	15.0	9.0	2
5	Bonkuras	1×	3○	4○	7○	2○	8×	6○	5	2	0	17.5	10.5	1
6	Ponanza	4×	1△	2×	3○	8○	7○	5×	3	3	1	7.0	2.0	5
7	YSS	3○	2×	1×	5×	4×	6×	8○	2	5	0	5.0	0.0	8
8	Blunder	2×	4×	3×	1○	6×	5○	7×	2	5	0	8.5	0.0	7

- The result of 2011 championship
  final league, round robin matches,
  25 minutes limit

The result

- estimated computer rating 2900
- estimated Yonenaga's rating 2800
  - for now, though 3100 or more in his peak
- human advantage of long limit time(3 hours)
- My estimation was even: 2900=2800+100
- Yonenaga made a good fight
  made unusual opening, made long term planning
  avoided piece exchange, occupied the center
- But then .....Yonenaga was defeated.

Future

- We are planning 5 vs. 5 matches next year
  Top five computers
  Five various rank young professionals
  (perhaps Yonenaga included)
- Afterward it should be the time of the match against the Grandmaster because It seems very near that computer exceeds humani n 3 years
- But Sponsor is wanted
  Yonenaga requests 800 million yen($ 10 million)
  to keep Grandmaster Habu in one year training
- Human side research on Computer Shogi is not enough yet
  - Possibility of several year delay of the day by human systematic study
  - As for chess delayed from 1994 to 1997
- to make a hybrid strong player
  - cooperation: strong human player plays using machine
- to make long solution Tsume Shogi
  - today only under-30-ply problems have been made
  - challengeable
- to solve Shogi (to know which wins in initial position)
  - totally impossible
  - 3x3 and 3x4 mini Shogis have been solved including Doubutsu Shogi(Animal Shogi)
  - Even solving 55-Shogi (a 5x5 mini Shogi) is still too hard

Time flies like an arrow
and Fruit flies like an banana.
Shogi is a fruit fly in computer science,
mathematics, and artificial intelligence.

Some Shogi Problems

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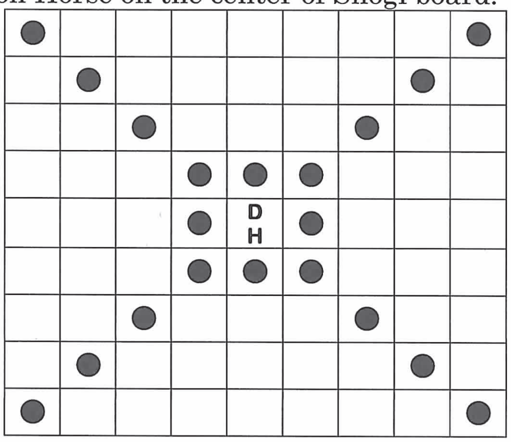
Tokyo University of Agric. & Tech.

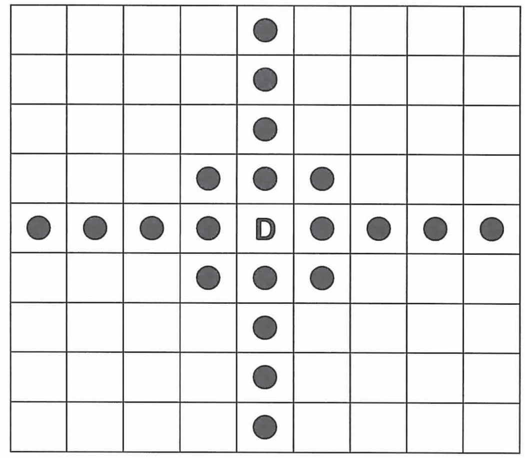
Peaceful Dragon Horses

Arrange 15 pieces of Dragon Horse (promoted bishop) on Shogi board which can move like bishop or king in chess such that no DH attacks other. The marks ● below show the squares of Dragon Horse on the center of Shogi board.



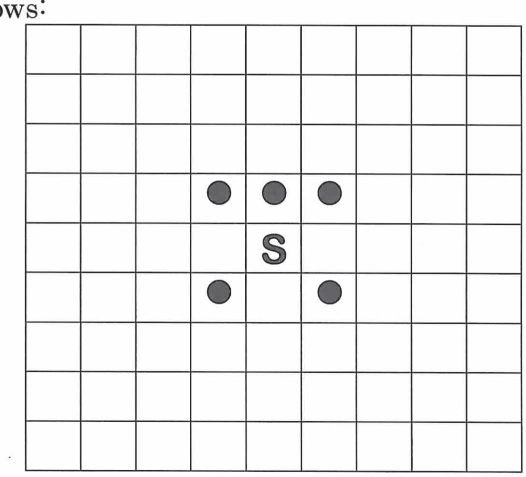
Dragon Covering

Arrange 6 pieces of Dragon (promoted rook) on Shogi board which can move like rook or king in chess such that all vacant squares are attacked by them. The marks ● below show the squares of Dragon on the center of Shogi board.



The Tour of Silver

Make a longest looping path by the piece Silver in Shogi board, where any square can be visited only once like the knight tour in chess. Then prove it is the longest. Silver can move to the marks ● as follows:



Academy of Recreational Mathematics Japan English Issure Membership Starts

The Story of "THE GRASSHOPPING GAME"

Invented by Ivan Moscovich

On the first day of the International Puzzle Party (IPP) in Antwerpen in 2002, there was a long neverending and boring lecture. I was doodling on a squared page of paper, when out of the blue, an idea for a paper-and-pencil puzzle/game surfaced. It was not the first time that my subconscious was providing me with original ideas.

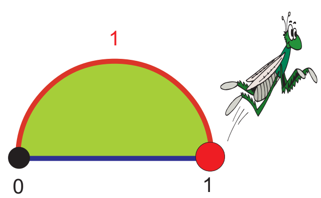
The idea:

Imagine a Grasshopper jumping along a line according to the following rules.

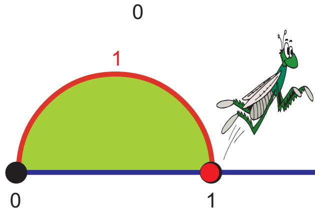
Given a line of integral length 'n', the object of our Grasshopper is to start jumping from point 0, in successive jumps of consecutive lengths : 1-2-3-4-............-n along the line, so as to make as many jumps as possible and finish the n-th jump at the end point of the line, at point 'n'.

If we have a line on which this can be achieved our game ended and we have a solution. I f not, the game has not ended and has no solution.

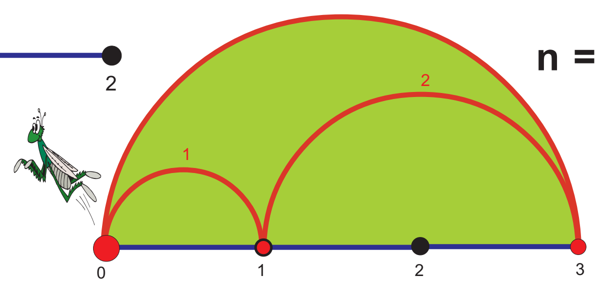
Looks interesting, I decided, and went on doodling systematically to find solutions:



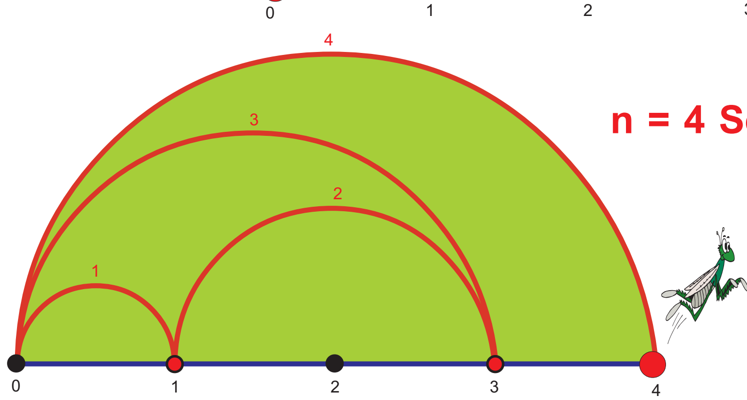
n = 1 First Solution (trivial)



n = 2 No Solution



n = 3 No Solution

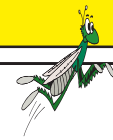


n = 4 Second Solution

THE GIFT EXCHANGE AND PUZZLE CHALLENGE OF IVAN MOSCOVICH

The Story of "THE GRASSHOPPING GAME"

for G4G10 (Gathering for Gardner 10) Atlanta 27 March - April 2, 2012



The Story of "THE GRASSHOPPING GAME"

page 2

By this time I saw there are lines on which the game can be properly ended providing a solution, and other which have no solution at all.. But it also became clear to me that my innocent Grasshopping Game is much more than just a simple paper-and-pencil doodling.

Its solutions are generating an ***infinite number sequence***, and an infinite number of puzzles to solve, with some interesting mathematics behind it.

I wanted to go on doodling to find more solutions, but at this point the boring, but now productive lecture ended.

With this interruption I would like to challenge the audience to find the ***next three solutions*** *to the Grasshopping Game which can now be called the* ***Grasshopping Sequence****. The first person coming to me with the solutions will get as a prize: my next book " The Puzzle -Book" soon to be published.*

After the interruption I sat down and devoted an hour or so, to find the solutions to the first 40 games. I found there are **16** solutions among them. Can you try to do the same?

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

Later that day, I met Dick Hess and asked for his ideas on the general solution of the Grasshopping Sequence. The next day at breakfast I met him again. He politely thanked me for the sleepless night, but assured me, he doesn't give up. Dick joined forces with Benji Fisher, and the next day, the mathematics behind the Grasshopping Sequence was solved, providing me with the solutions for any line length, this time without hard work.

The end of the story of the Grasshopping Game and Sequence:

At the next G4G9 I met Neil Sloane. Somewhat timidly, I told him the story of *Grasshopping*, *my integer number sequence.* He was enthusiastic about it.

The result was, that today, my Grasshopping Puzzle-Game and the Grasshopping Number Sequence occupy a respectable place on the Internet, among the giants of integer sequences, the Pi, the Primes, the Fibonacci and other - in the exciting "On-Line Encyclopedia of Integer Sequences of Neil - of which I am quite proud.

Google it on the Internet, to reveal the general solution and the mathematics behind the Grasshopping Number Sequence.

I.M.at G4G10
Atlanta, 2012

The Secrets of Notakto: Winning at X-only Tic-Tac-Toe

Thane E. Plambeck, Greg Whitehead

March 28, 2012

Abstract

We analyze misere play of "impartial" tic-tac-toe—a game suggested by Bob Koca in which **both** players make X's on the board, and the first player to complete three-in-a-row **loses**. This game was recently discussed on mathoverflow.net in a thread created by Timothy Y. Chow.

1 Introduction

Suppose tic-toe-toe is played on the usual 3x3 board, but where both players make X's on the board. The first player to complete a line of three-in-a-row *loses* the game.

Who should win? The answer for a single 3x3 board is given in a recent mathoverflow.net discussion [Chow]:

> In the 3x3 misere game, the first player wins by playing in the center, and then wherever the second player plays, the first player plays a knight's move away from that.

Kevin Buzzard pointed out that any other first-player move loses:

> The reason any move other than the centre loses for [the first player to move] in the 3x3 game is that [the second player] can respond with a move diametrically opposite [the first player's] initial move. This makes the centre square unplayable, and then player two just plays the "180 degree rotation" strategy which clearly wins.

In this note we generalize these results to give a complete analysis of multi-board impartial tic-tac-toe under the disjunctive misere-play convention.

2 Disjunctive misere play

A *disjunctive* game of 3x3 impartial tic-tac-toe is played not just with one tic-tac-toe board, but more generally with an arbitrary (finite) number of such boards forming the start position. On a player's move, he or she selects a single one of the boards, and makes an X on it (a board that already has a three-in-a-row configuration of X's is considered unavailable for further moves and out of play).

Play ends when every board has a three-in-a-row configuration. The player who completes the last three-in-a-row on the last available board is the loser.

3 The misere quotient of 3x3 impartial tic-tac-toe

We can give a succinct and complete analysis of the best misere play of an arbitrarily complicated disjunctive sum of impartial 3x3 tic-tac-toe positions by introducing a certain 18-element commutative monoid Q given by the presentation

$$Q = \langle\ a, b, c, d \mid a^2 = 1,\ b^3 = b,\ b^2c = c,\ c^3 = ac^2,\ b^2d = d,\ cd = ad,\ d^2 = c^2 \rangle. \tag{1}$$

The monoid Q has eighteen elements

$$Q = \{1, a, b, ab, b^2, ab^2, c, ac, bc, abc, c^2, ac^2, bc^2, abc^2, d, ad, bd, abd\}, \tag{2}$$

and it is called the **misere quotient** of impartial tic-tac-toe[1].

A complete discussion of the misere quotient theory (and how Q can be calculated from the rules of impartial tic-tac-toe) is outside the scope of this document. General information about misere quotients and their construction can be found in [MQ1], [MQ2], [MQ3], and [MQ4]. One way to think of Q is that it captures the misere analogue of the "nimbers" and "nim addition" that are used in normal play disjunctive impartial game analyses, but localized to the play of this particular impartial game, misere impartial 3x3 tic-tac-toe.

In the remainder of this paper, we simply take Q as given.

4 Outcome determination

Figure 6 (on page 7, after the References) assigns an element of Q to each of the conceivable 102 non-isomorphic positions[2] in 3x3 single-board impartial

[1] For the cognoscenti: Q arises as the misere quotient of the hereditary closure of the sum G of two impartial misere games $G = 4 + \{2+, 0\}$. The game $\{2+, 0\}$ is the misere canonical form of the 3x3 single board start position, and "4" represents the nim-heap of size 4, which also happens to occur as a single-board position in impartial tic-tac-toe. In describing these misere canonical forms, we've used the notation of John Conway's On Numbers and Games, on page 141, Figure 32.

[2] We mean "non-isomorphic" under a reflection or rotation of the board. In making this count, we're including positions that couldn't be reached in actual play because they have too many completed rows of X's, but that doesn't matter since all those elements are assigned the identity element of Q.

tic-tac-toe.

To determine the outcome of a multi-board position (ie, whether the position is an *N-position*—a Next player to move wins in best play, or alternatively, a *P-position*—second player to move wins), one first multiplies the corresponding elements of Q from the dictionary together. The resulting word is then reduced via the relations 1, that we started with above, necessarily eventually arriving at at one of the eighteen words (in the alphabet a, b, c, d) that make up the elements of Q.

If that word ends up being one of the four words in the set P

$$P = \{a, b^2, bc, c^2\}, \tag{3}$$

the position is P-position; otherwise, it's an N-position.

5 Example analysis

To illustrate outcome calculation for Impartial Tic-Tac-Toe, we consider the two-board start position shown in Figure 1.

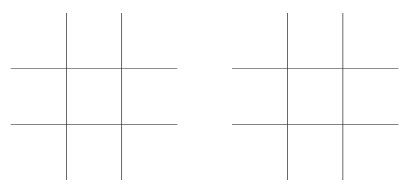


Figure 1: The two-board start position.

Consulting Figure 6, we find that the monoid-value of a single empty board is c. Since we have two such boards in our position, we multiply these two values together and obtain the monoid element

$$c^2 = c \cdot c.$$

Since c^2 is in the set P (equation (3)), the position shown in Figure 1 is a *second* player win. Supposing therefore that we helpfully encourage our opponent to make the first move, and that she moves to the center of one of the boards, we arrive at the position shown in Figure 2.

It so happens that if we mimic our opponent's move on the other board, this happens to be a winning move. We arrive at the position shown in Figure 3, each of whose two boards is of value c^2; multiplying these two together, and simplifying via the relations shown in equation (1), we have

$$\begin{aligned} c^4 &= c^3 \cdot c \\ &= ac^2 \cdot c \end{aligned}$$

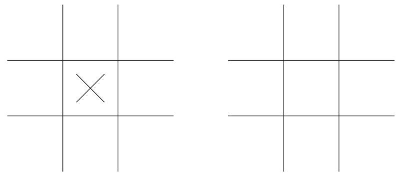


Figure 2: A doomed first move from the two-board start position.

$$\begin{aligned} &= ac^3 \\ &= aac^2 \\ &= c^2, \end{aligned}$$

which is a P-position, as desired.

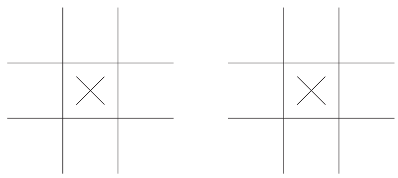


Figure 3: Mimicry works here, but not in general.

So is the general winning stategy of the two-board position simply to copy our opponent's moves on the other board? Far from it: consider what happens if our opponent should decide to complete a line on one of the boards—copying that move on the other board, we'd **lose** rather than win! For example, from the N-position shown in Figure 5, there certainly is a winning move, but it's *not* to the upper-right-hand corner of the board on the right, which loses.

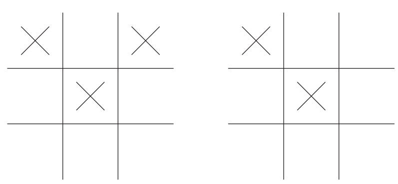


Figure 4: An N-position in which mimickry loses.

We invite our reader to find a correct reply!

6 The iPad game Notakto

Evidently the computation of general outcomes in misere tic-tac-toe is somewhat complicated, involving computations in finite monoid and looking up values from a table of all possible single-board positions.

However, we've found that a human can develop the ability to win from multi-board positions with some practice.

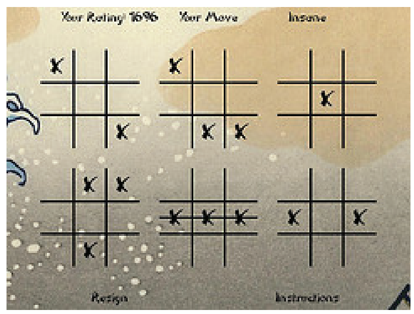


Figure 5: A six-board game of Notakto, in progress.

Notakto "No tac toe" is an iPad game that allows the user to practice playing misere X-only Tic-Tac-Toe against a computer. Impartial misere tic-tac-toe from start positions involving one up to as many as six initial tic-tac-toe boards are supported.

The Notakto iPad application is available for free at `http://www.notakto.com`.

7 Final question

Does the 4x4 game have a finite misere quotient?

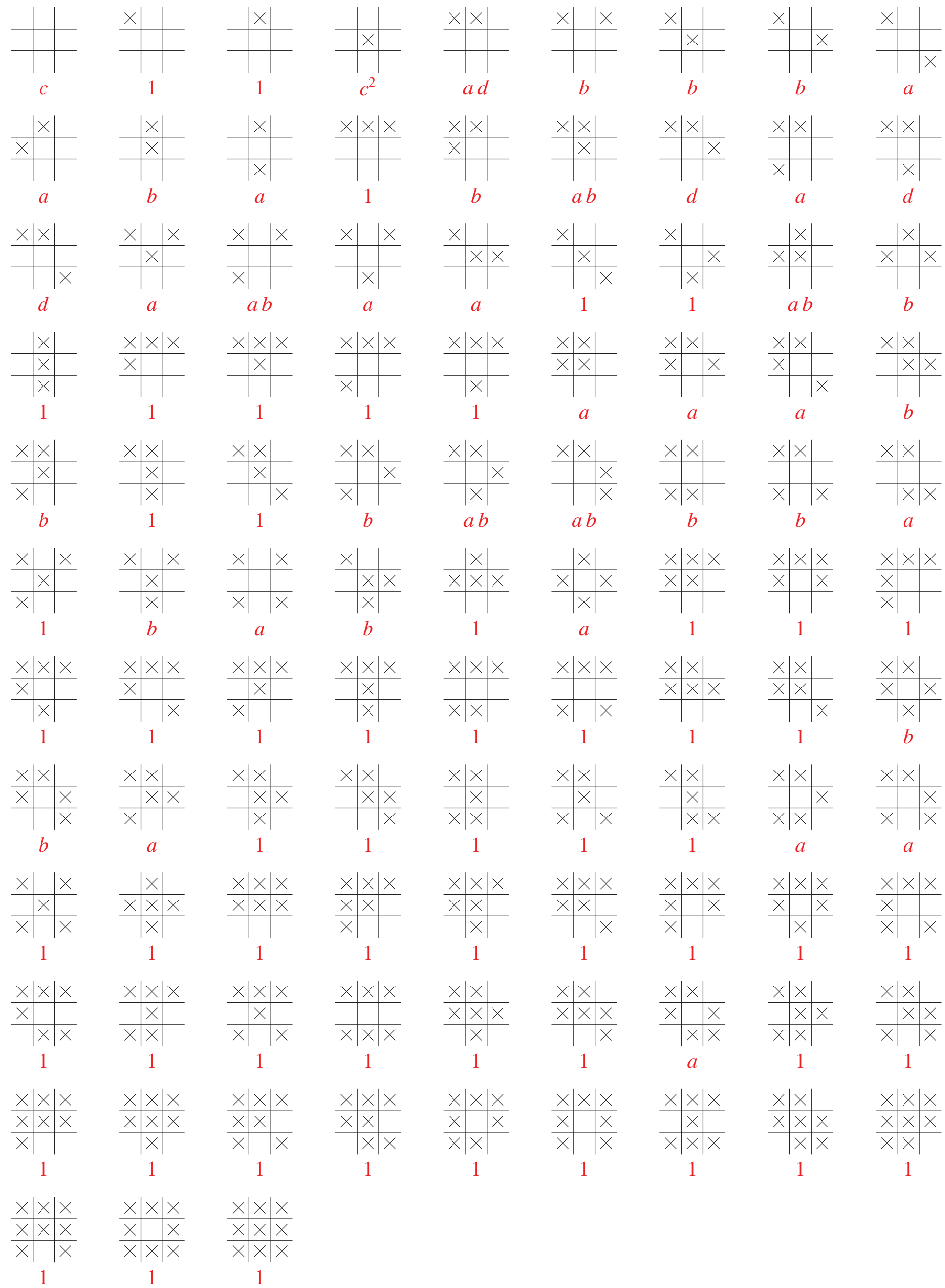


Figure 6: The 102 nonisomorphic ways of arranging zero to nine X's on a tic-tac-toe board, each shown together with its corresponding misere quotient element from Q.

References

[Chow] Timothy Y. Chow, Online mathoverflow.net question, "Neutral tic tac toe", 16 May 2010.
`http://mathoverflow.net/questions/24693/neutral-tic-tac-toe/`

[MQ1] `http://arxiv.org/abs/math/0501315`

[MQ2] `http://arxiv.org/abs/math/0609825`

[MQ3] `http://arxiv.org/abs/0705.2404`

[MQ4] `http://www.miseregames.org`

Quasimino – Playing Dominos on (almost) the Penrose tiles

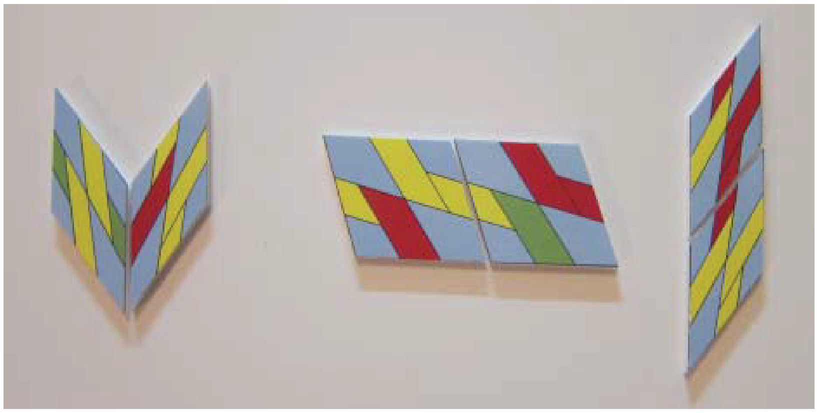
Ayelet Pnueli – Kefgames
Email: ayelet@kefgames.com

Rules of the games:

Quasimino contains a set of 90 diamond shaped tiles (45 regular diamonds and 45 elongated diamonds).
Two tiles can be legally placed next to each other if the stripes that meet on the common edge are of the same color and the two tiles do not form a parallelogram:

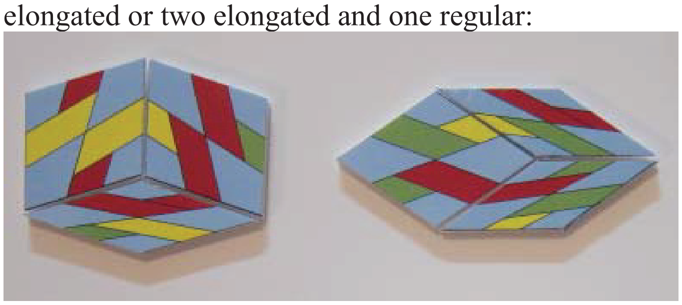
Examples of legal placements:

Examples of illegal placements (color mismatch and parallelograms).

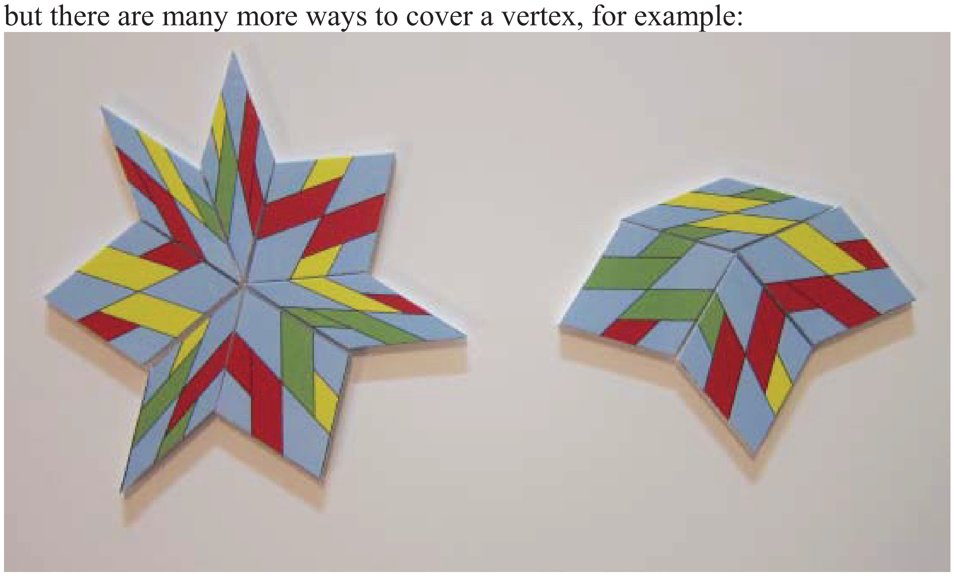


If when placing a tile it touches multiple other tiles, then the placing should be legal with respect to all the touching edges

A vertex (or tile meeting point) is 'covered' if there are tiles surrounding it completely (360 degrees) . Note that the simplest way to 'cover a vertex' is by using three tiles – two regular and one elongated or two elongated and one regular:



but there are many more ways to cover a vertex, for example:



Quasi-Domino

This game is a two dimensional variant of the common Domino game.
Each player starts with a set of tiles and the first player to get rid of all his/her tiles wins.

Before the game all tiles are set face down on the table, mixed together and arranged in a pile.
Each player selects six tiles (three regular and three elongated) from the pile. Additionally, two tiles, one regular and one elongated are placed 'face up' on the table.
The first player that can arrange a legal shape (not necessarily covering a vertex) from a tile in his/her hand and the two open tiles shouts Quasi and starts the game by making that shape. After that, players take turns playing in a clockwise direction.

Each player in his/her turn must place a tile on the table in a way that is legal and touches at least one of the existing tiles along an edge. Note that unlike regular Domino you typically have many edges to which a new tile can be connected.

If, in your turn, you cannot legally place a tile on the table you must take an extra one from the pile.
If by placing a tile on the table you covered a vertex (see above) you get to play an extra turn (or win if this was your last tile).
If by placing a tile on the table you do not cover any vertex you must take one tile from the pile, (so the number of tiles in your hand remains unchanged).
The first player to get rid of all his/her tiles (by covering enough Vertices) wins.

Notes:

1. When taking a tile from the pile you can choose to take a regular or an elongated tile (as long as there are such tiles available in the pile). Each type may be the better choice at different stages of the game.
2. Observing the 'parallelogram rule', is sometimes not easy for beginners. As a result it is possible that during the game players discover that the shape on the table contains an illegal parallelogram. If this is discovered when the player placed his/her tile – he/she should take it back and try another move. If this is discovered after other players have played, the game continues as if that move was legal.

Extra skill rule: To add to the skill level of the game, players can agree that it is possible to rearrange tiles on the table provided that all rearranged tiles touched the rest of the shape on the table with at most one edge and after rearranging them, all tiles that were moved now participate in a covered vertex.

Quasi-Match

Quasi-Match is played with one set of (90) Quasimino tiles plus a set of tokens in different colors (one color per player).
The aim of the game is to 'own' as many 'covered vertices' (see above) as possible.
Before the game all tiles are set face down on the table, mixed together and arranged in a pile.
Each player selects four tiles (two regular and two elongated) from the pile. Additionally two tiles one regular and one elongated are placed 'face up' on the table.
The first player that can arrange a legal shape (not necessarily cover a vertex) from a tile in his/her hand and the two open tiles shouts 'Quasi' and starts the game by making that arrangement. After that, players take turns playing in a clockwise direction. Each player in his/her turn connects a tile to the shape on the table and takes one tile from the pile (thus keeping the number of tiles in hand four).

A player who cannot legally place a tile on the table says so and is skipped over. The next player

gets to play two turns.
A player who, by placing a tile on the table, covered one or more vertices, now owns them, he/she denotes this by placing tokens of his/her color on these vertices. He/She also gets another turn.

The game ends when all tiles from the pile are exhausted and all tiles from the hands of the players have been played. The player with the most tokens on the table wins. Alternatively players may agree in advance on a number (for example, six or ten), such that the first player to reach this number of tokens wins.

Notes:

1. When taking a tile from the pile you can choose to take a regular or an elongated tile (as long as there are such tiles available in the pile). Each type may be the better choice at different stages of the game.
2. Observing the ‘parallelogram rule, is sometimes not easy for beginners. As a result it is possible that during the game players discover that the shape on the table contains an illegal parallelogram. If this is discovered when the player placed his/her tile – he/she should take it back and try another move. If this is discovered after other players have played, the game continues as if that move was legal.

Extra skill: To add to the skill level of the game, players can agree that it is possible to rearrange tiles on the table provided that all rearranged tiles touched the rest of the shape on the table with at most one edge and after rearranging them, all tiles that were moved now participate in a covered vertex.

FastQuasi!

In this version of Quasimino each player selects 14 face down Hexamino tiles (7 regular and 7 elongated). When the game starts each player turns his/her tiles face up and his/her aim is to build from these tiles a shape or shapes with as many covered vertices (see above) as possible as fast as possible.
The first player to finish his/her shape or shapes shouts Quasi! And all players must stop. Each player then counts the number of covered vertices he/she has and the player with the highest count wins.
In order to shout ‘Quasi!’ a player must have all his./her tiles arranged legally into one or more shapes such that each shape has at least three tiles.

Acknowledgement

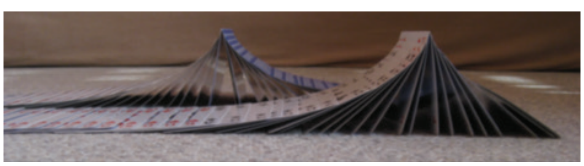
The tiles in Quasimino are based on work by the mathematician and physicist Roger Penrose who used them to show an aperiodic tiling of the plane (in fact they are known as the P3 or Rhombus version of his now famous Penrose tiling).

2 x ribbon spread turnover = 1 x tractrix racer

Burkard Polster, Monash University, Australia, www.qedcat.com

1. Ribbon spread two decks of cards

2. Partially turn over both spreads



3. Balance a book on the two peaks

A couple of observations:

The halfway stage of a ribbon spread turnover is a discrete version of a tractrix.

You get a similar discrete tractrix by tumbling a row of dominos simultaneously from both sides.

So, in a way, turning over a spread of cards is domino tumbling and the reverse of domino tumbling happening at the same time.

The book makes it possible to turn over spreads of cards that are several meters long.

The sound produced by the cards making contact with the book is very similar to the sound you get when you tumble dominos.

I hope that this is not an old hat.

In any case, have fun playing with this!

4. Give the book a little push in the direction of the spreads and it will race along the track turning over the two spreads in the process. Here is a movie http://www.youtube.com/watch?v=pEEcafsT6ec&feature=youtu.be

AL-JABAR

A Mathematical Game of Strategy

Robert P. Schneider and Cyrus Hettle
University of Kentucky

Concepts

The game of *Al-Jabar* is based on concepts of color-mixing familiar to most of us from childhood, and on ideas from abstract algebra, a branch of higher mathematics. Once you are familiar with the rules of the game, your intuitive notions of color lead to interesting and often counter-intuitive color combinations created in gameplay.

Because *Al-Jabar* requires some preliminary understanding of the color-mixing mechanic before playing the game, these rules are organized somewhat differently than most rulebooks. This first section details the "arithmetic" of adding (the mathematical equivalent of mixing) colors. While the mathematics involved uses some elements of group theory, a foundational topic in abstract algebra, understanding this "arithmetic" is not difficult and requires no mathematical background. The second section explains the process of play, and how this arithmetic of colors is used in the game. A third section develops the game's mathematical theory and gives several extensions and variations of the game's rules.

Gameplay consists of manipulating game pieces in the three primary colors red, blue and yellow, which we denote in writing by R, B, and Y respectively; the three secondary colors green, orange and purple, which we denote by G, O, and P; the color white, denoted by W; and clear pieces, denoted by C, which are considered to be "empty" as they do not contain any color.

We refer to a game piece by its color, e.g. a red piece is referred to as "red," or R.

We use the symbol "+" to denote a combination, or grouping together, of colored game pieces, and call such a combination a "sum of colors." Any such grouping of colors will have a single color as its final result, or "sum." We use the symbol "=" to mean that two sets of pieces are equal, or interchangeable, according to the rules of the game; that is, the sets have the same sum. The order of a set of colors does not affect its sum; the pieces can be placed however you like.

Keep in mind as you read on that these equations just stand for clusters of pieces. Try to see pictures of colorful pieces, not black-and-white symbols, in your mind. Try to imagine a red piece when you read "R," a blue and a green piece when you read "B + G," and so on. In mathematics, symbols are usually just a black-and-white way to write something much prettier.

Here are four of the defining rules in *Al-Jabar*, from which the entire game follows:

$$P = R + B$$

indicates that purple is the sum of red and blue, i.e. a red and a blue may be exchanged for a purple during gameplay, and vice versa;

$$O = R + Y$$

indicates that orange is the sum of red and yellow;

$$G = B + Y$$

indicates that green is the sum of blue and yellow; and a less obvious rule

$$W = R + B + Y$$

indicates that white is the sum of red, blue and yellow, which reminds us of the fact that white light contains all the colors of the spectrum—in fact, we see in the above equation that the three secondary colors $R + B$, $R + Y$ and $B + Y$ are also contained in the sum W.

In addition, there are two rules related to the clear pieces. Here we use red as an example color, but the same rules apply to every color, including clear itself:

$$R + C = R$$

indicates that a sum of colors is not changed by adding or removing a clear; and a special rule

$$R + R = C$$

indicates that two pieces of the same color (referred to as a "double") are interchangeable with a clear in gameplay. It follows from the above two rules that if we have a sum containing a double, like $R + B + B$, then

$$R + B + B = R + C$$

as the two blues are equal to a clear. But $R + C = R$ so we find that

$$R + B + B = R,$$

which indicates that a sum of colors is not changed by adding or removing a double—the doubles are effectively "cancelled" from the sum. It also follows from these rules that if we replace R and B with C in the above equations,

$$C + C = C$$
$$C + C + C = C$$
etc.

We note that all groups of pieces having the same sum are interchangeable in *Al-Jabar*. For instance,

$$Y + O = Y + R + Y = R + Y + Y$$
$$= R + C = R,$$

as orange may be replaced by

$$R + Y$$

and then the double $Y + Y$ may be cancelled from the sum.

But it is also true that

$$B + P = B + R + B = R + C = R,$$

and even

$$G + W = B + Y + W = B + Y + R + B + Y$$
$$= R + C + C = R,$$

which uses the same rules, but takes an extra step as both G and W are replaced by primary colors.

All of these different combinations have a sum of R, so they are equal to each other, and interchangeable in gameplay:

$$Y + O = B + P = G + W = R.$$

In fact, every color in the game can be represented as the sum of two other colors in many different ways, and all these combinations which add up to the same color are interchangeable.

Every color can also be represented in many different ways as the sum of three other colors; for example

$$Y + P + G = Y + R + B + B + Y$$
$$= R + C + C = R$$

and

$$O + P + W = R + Y + R + B + R + B + Y$$
$$= R + C + C + C = R$$

are interchangeable with all of the above combinations having sum R.

An easy technique for working out the sum of a set of colors is this:

1. Cancel the doubles from the set;

2. Replace each secondary color, or white, with the sum of the appropriate primary colors;

3. Cancel the doubles from this larger set of colors;

4. Replace the remaining colors with a single piece, if possible, or repeat these steps until only one piece remains (possibly a clear piece).

The color of this piece is the sum of the original set, as each step simplifies the set but does not affect its sum.

As you become familiar with these rules and concepts, it is often possible to skip multiple steps in your mind, and you will begin to see many possibilities for different combinations at once.

Before playing, you should be familiar with these important combinations, and prove for yourself that they are true by the rules of the game:

$$R + O = Y,\ Y + O = R,$$
$$B + P = R,\ R + P = B,$$
$$B + G = Y,\ Y + G = B.$$

These show that a secondary color plus one of the primary colors composing it equals the other primary color composing it.

You should know, and prove for yourself, that

$$G + O = P,\ O + P = G,\ P + G = O,$$

i.e. that the sum of two secondary colors is equal to the other secondary color.

You should know, and prove for yourself, that adding any two equal or interchangeable sets equals clear; for example

$$R + B = P, \text{ and so } R + B + P = C.$$

You should experiment with sums involving white—it is the most versatile color in gameplay, as it contains all of the other colors.

Play around with the colors. See what happens if you add two or three colors together; see what combinations are equal to C; take a handful of pieces at random and find its sum. Soon you will discover your own combinations, and develop your own tricks.

Rules of Play

1. *Al-Jabar* is played by 2 to 4 people. The object of the game is to finish with the fewest game pieces in one's hand, as detailed below.

2. One player is the dealer. The dealer draws from a bag of 70 game pieces (10 each of the colors white, red, yellow, blue, orange, green, and purple), and places 30 clear pieces in a location accessible to all players.

> *Note: Later in the game, it may happen that the clear pieces run out due to rule 6. In this event, players may remove clear pieces from the center and place them in the general supply, taking care to leave a*

few in the center. If there are still an insufficient number, substitutes may be used, as the number of clears provided is not intended to be a limit.

3. Each player is dealt 13 game pieces, drawn at random from the bag, which remain visible to all throughout the game.

4. To initiate gameplay, one colored game piece, drawn at random from the bag, and one clear piece are placed on the central game surface (called the "Center") by the dealer.

5. Beginning with the player to the left of dealer and proceeding clockwise, each player takes a turn by exchanging any combination of 1, 2 or 3 pieces from his or her hand for a set of 1, 2 or 3 pieces from the Center having an equal sum of colors.

The exception to this rule is the combination of 4 pieces R + B + Y + W, which may be exchanged for a clear piece. This action is called the "Spectrum move."

Note: Thus the shortest that a game may last is 5 moves, for a player may reduce their hand by at most 3 pieces in a turn.

If a player having more than 3 game pieces in hand cannot make a valid move in a given turn, then he or she must draw additional pieces at random from the bag into his or her hand until a move can be made.

6. If a player's turn results in one or more pairs of like colors (such a pair is called a "double") occurring in the Center, then each such double is removed from the Center and discarded (or "cancelled"), to be replaced by a clear piece.

In addition, every other player must draw the same number of clear pieces as are produced by cancellations in this turn.

There are two exceptions to this rule:

(i) Pairs of clear pieces are never cancelled from the Center;

(ii) If a player's turn includes a double in the set of pieces placed from his or her hand to the Center, then the other players are not required to take clear pieces due to cancellations of that color, although clear pieces may still be drawn from cancellations of other colored pairs.

Note: The goals of a player, during his or her turn, are to exchange the largest possible number of pieces from his or her hand for the smallest number of pieces from the Center; and to create as many cancellations in the Center as possible, so as to require the other players to draw clear pieces.

7. A player may draw additional pieces as desired at random from the bag during his or her turn.

Note: If a player finds that his or her hand is composed mostly of a few colors, or requires a certain color for a particularly effective future move, this may be a wise idea.

8. A round of gameplay is complete when every player, starting with the first player, has taken a turn. Either or both of two events may signal that the game is in its final round.

(i) One player announces, immediately after his or her turn, that he or she has reduced his or her hand to one piece;

(ii) One player, having 3 or fewer pieces in hand, is unable to make a move resulting in a decrease of the total number of pieces in his or her hand.

In either case, the players who have not yet taken a turn in the current round are allowed to make their final moves.

When this final round is complete, the player with the fewest remaining pieces in hand is the winner. If two or more players are tied for the fewest number of pieces in hand, they share the victory.

Mathematical Notes

For the interested, mathematically-inclined reader, we outline the algebraic properties of *Al-Jabar*. This section is in no way essential for gameplay. Rather, the following notes are included to aid in analyzing and extending the game rules, which were derived using general formulas, to include sets having any number of "primary" elements, or comprised of game pieces other than colors.

The arithmetic of *Al-Jabar* in the group of the eight colors of the game is isomorphic to the addition of ordered triples in $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, that is, 3-vectors whose elements lie in the congruence classes modulo 2.

The relationship becomes clear if we identify the three primary colors red, yellow, and blue with the ordered triples

R = (1,0,0), Y = (0,1,0), B = (0,0,1)

and define the clear color to be the identity vector

C = (0,0,0).

We identify the other colors in the game with the following ordered triples using component-wise vector addition:

O = R + Y = (1,0,0) + (0,1,0) = (1,1,0)

G = Y + B = (0,1,0) + (0,0,1) = (0,1,1)

P = R + B = (1,0,0) + (0,0,1) = (1,0,1)

W = R + Y + B
= (1,0,0) + (0,1,0) + (0,0,1)= (1,1,1).

The color-addition properties of the game follow immediately from these identities if we sum the vector entries using addition modulo 2. Then the set of colors {R, Y, B, O, G, P, W, C} is a group under the given operation of addition, for it is closed, associative, has an identity element (C), and each element has an inverse (itself).

Certain rules of gameplay were derived from general formulas, the rationale for which involved a mixture of probabilistic and strategic considerations. Using these formulas, the rules of *Al-Jabar* can be generalized to encompass different finite cyclic groups and different numbers of primary elements, i.e. using n-vectors with entries in $\mathbb{Z}_m$, that is, elements of

$$\mathbb{Z}_m \times \mathbb{Z}_m \times \mathbb{Z}_m \times \ldots \times \mathbb{Z}_m \ \ (n \text{ times}).$$

In such a more general setting, there are m "primary" n-vectors of the forms (1,0,0,…,0), (0,1,0,…,0),…, (0,0,…,0,1), and the other nonzero m-vectors comprising the group are generated using component-wise addition modulo m, as above. Also, the analog to the clear game piece is the zero-vector (0,0,0,…,0).

In addition, the following numbered rules from the Rules of Play would be generalized as described here:

2. The initial pool of game pieces used to deal from will be composed of at least

$$Am^n - A$$

pieces, where A is at least as great as m multiplied by the number of players. This pool of pieces will be divided into an equal number A of every game piece color except for the clear or identity-element (0,0,0,…,0). Players will recall that the number of clears is arbitrary and intended to be unlimited during gameplay, so this number will not be affected by the choice of m and n;

3. The number of pieces initially dealt to each player will be

$$m^{n+1} - m - 1;$$

5. On each turn, a player will exchange up to n pieces from his or her hand for up to n marbles from the Center with the same sum. The exception to this is the Spectrum, which will consist of the n primary colors

$$(1,0,0,\dots,0), (0,1,0,\dots,0), \dots, (0,0,\dots,0,1)$$

together with the n-vector

$$(m-1, m-1, m-1, \dots, m-1),$$

which is the generalized analog to the white game piece used in the regular game. It will be seen that these $n+1$ marbles have a sum of $(0,0,0,\dots,0)$ or clear.

A player must draw additional marbles if he or she has more than n pieces in hand and cannot make a move;

6. The cancellation rule will apply to m-tuples (instead of doubles) of identical non-clear colors;

8. The first player to have either 1 piece left, or to be unable to reduce his or her hand to fewer than n pieces, will signal the final round.

Thus for the group

$$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

we have $m = 2, n = 4$ and let $A = 10$. Then each player starts with 29 game pieces dealt from a bag of 10 each of the 15 non-clear colors, may exchange up to 4 pieces on any turn or 5 pieces in the case of a Spectrum move, and will signal the end of the game with 4 or fewer pieces in hand.

Here the Spectrum consists of the colors

$$(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1), (1,1,1,1)$$

and the cancellation rule still applies to doubles in this example, as $m = 2$.

Other cyclic groups may also be seen as sets of colors under our addition, such as

$$\mathbb{Z}_3 \times \mathbb{Z}_3 = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)\}$$

in which every game piece either contains, for example, no red (0), light red (1) or dark red (2) in the first vector entry, and either contains no blue (0), light blue (1) or dark blue (2) in the second entry. Therefore we might respectively classify the nine elements above as the set

{clear, light blue, dark blue, light red, light purple, bluish purple, dark red, reddish purple, dark purple}.

Of course, other colors rather than shades of red and blue may be used, or even appropriately selected non-colored game pieces.

Further generalizations of the game rules may be possible—for instance, using n-vectors in $\mathbb{Z}_{a_1} \times \mathbb{Z}_{a_2} \times \dots \times \mathbb{Z}_{a_n}$ where the subscripts a_i are not all equal—and other games might be produced by other alterations of the rules of play.

Some New Combinatorial Games
(From the Past Ten Years)

Aaron N. Siegel

March 26, 2012

The past ten years have seen exciting theoretical developments in combinatorial game theory. Alongside these advances there has appeared a crop of new and fascinating examples—games that exhibit a rich, varied, and often amusing and bewildering structure. Some of these games are new variations on well-known themes; others test and expand the boundaries and capabilities of the theory.

This note surveys several of the more interesting recent inventions. The first two games in our survey, Mem and Mnem, are impartial games with extremely simple rules, and a structure that bifurcates in a surprising way into regions of order and chaos. The next, Toppling Dominoes, is a partizan game in the classical sense, but one with an unusually clear structure. Finally, Entrepreneurial Chess is an unusual Chess variant, played on an infinitely large board, with repetition allowed. It therefore violates most of the "classical" restrictions on combinatorial games, but it nonetheless has a robust and coherent theory.

None of these games were invented by me. Mnem was introduced by Conway; Toppling Dominoes by Richard Nowakowski; and Entrepreneurial Chess by Berlekamp and Pearson. Some familiarity with combinatorial game theory is assumed; the discussion of Mem and Mnem uses only the impartial theory, while the others use the partizan theory and notation as described in *Winning Ways* [1].

Taken together, these games illustrate the boundless possibilities of combinatorial games. They inhabit a miraculous universe, in which elegant and mysterious mathematics springs from a few simple rules.

Mem and Mnem

Mem and Mnem are deceptively simple impartial games. They were introduced by Conway a few years ago and, despite their straightforward rules and "obvious" structure, it appears difficult to prove anything about them!

Mem is played with a heap of tokens. On her turn, a player must remove k tokens from the heap, provided that k is *at least as large* as the number of tokens removed on the prior turn. If played with multiple heaps, then each heap has its own "memory."

We can represent a heap of size n, with memory k, by a pair of integers n_k. Then the legal moves are given by

$$n_k = \{(n-i)_i : k \le i \le n\}.$$

Obviously n_k is a $\mathscr{P}$-position if $k > n$ (since then there are no legal moves), and an $\mathscr{N}$-position otherwise (since there is a move to 0_n).

Mnem is just the same, except that a player has the additional option of *adding* $< k$ new tokens to the heap (but at least one), instead of removing $\ge k$. When a player exercises this option, the value in "memory" decreases (which can't happen in Mem). Denoting a Mnem position by n_k^*, we have

$$n_k^* = \{(n-i)_i^* : k \le i \le n\} \cup \{(n+i)_i^* : 1 \le i < k\}.$$

A game of Mnem needn't ever end. In fact it's easy to construct sequences of moves that traverse an infinite number of distinct positions. For example:

$$4_1^* \to 0_4^* \to 3_3^* \to 5_2^* \to 6_1^* \to 0_6^* \to 5_5^* \to 9_4^* \to 12_3^* \to 14_2^* \to 15_1^* \to 0_{15}^* \to \cdots$$

Here we remove 4 tokens, then add 3, 2 and 1 tokens, leaving 6; then remove 6 tokens, and add 5, 4, 3, 2 and 1 tokens, leaving 15; then remove 15 tokens, and so on ...

But remarkably, both players have to cooperate for this to happen! For example, every 0_k^* is a $\mathscr{P}$-position: the only legal moves are to positions of the form i_i^* with $i < k$, which second player can immediately revert to 0_i^*. If second player sticks to this strategy, then eventually the position 0_1^* will be reached, which is terminal.

But from n_k^* with $k > n$, the only legal moves are to positions $(n+i)_i^*$ with $i < k$, which second player can revert to 0_{n+i}^*. So every such n_k^* is a $\mathscr{P}$-position, just as in Mem. And, likewise, every n_k^* with $k \le n$ is an $\mathscr{N}$-position, since it has a move to 0_n^*.

Beyond this, it's surprisingly hard to say anything at all about the $\mathscr{G}$-values of either variant. We don't even know how to play 2-pile Mem! Yet the experimental evidence suggests an extraordinary amount of structure.

Figure 1 shows an *intensity plot* of $\mathscr{G}(n_k)$, for all $1 \le n \le 96$ and $1 \le k \le 32$. It's a 32×96 grid of boxes, colored in grayscale; darker shades indicate lower $\mathscr{G}$-values. The black triangle in the lower-left is the space of $\mathscr{P}$-positions that we noted above.

What's striking are the thin triangular bands that extend in quadratic fashion upwards from the triangle of $\mathscr{P}$-positions. Just above the $\mathscr{P}$-region lies a region of positions with $\mathscr{G}$-value exactly 1; then a thinner region of positions with $\mathscr{G}$-value 2; and so on. Amazingly, all of these regions satisfy the following conjecture.

Conjecture 1. *If $k^2 \ge n$, then $\mathscr{G}(n_k) = \lfloor n/k \rfloor$.*

Figure 1: Intensity plot of the $\mathscr{G}$-values of Mem. The value of n_k is plotted at row k, column n, for $1 \leq n \leq 96$ and $1 \leq k \leq 32$. Black = 0, White = 14.

Within the region $k^2 < n$, the $\mathscr{G}$-values appear to have a complex structure, consisting of many interlocking, fractal-like triangles. The interested reader will enjoy computing a larger table of values and observing their striking regularity.

The geometric structure of Mnem seems very similar to Mem, and Conjecture 1 appears true for Mnem as well. However, the fine structure of the $k^2 < n$ region differs.

Here's an indication of how little is understood about these games: we can't even prove the following conjecture, which has been verified computationally for $n, k \leq 10,000$.

Conjecture 2. *Every* Mnem *position has finite* $\mathscr{G}$*-value.*

Toppling Dominoes

We turn now to partizan games. Toppling Dominoes, invented by Richard Nowakowski [3], is played with rows of black and white dominoes such as shown in Figure 2.

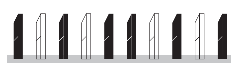
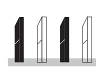


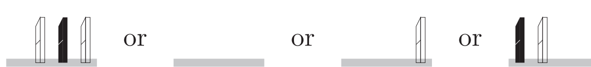
Figure 2: A typical Toppling Dominoes position.

On her turn, Left may select any black domino and "topple" it East or West (her choice). The selected domino is removed along with *all* dominoes from that row in the chosen direction.

For example, from the position



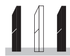
Left can move to



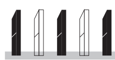
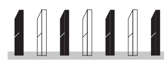
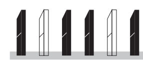
Although Toppling Dominoes is a straightforward, "classical" combinatorial game, it exhibits a remarkable amount of structure. It's easy to see that every *monochromatic* row of dominoes is equal to an integer; for example

 $= 5$

since Left will prefer to topple the black dominoes one at a time. More complicated numbers can be constructed by mixing dominoes of both colors, say

 $= \Big\{$  ,  $\Big|$  $\Big\} = \{0, * \mid 1\} = \frac{1}{2}.$

Here we've omitted options that are duplicates under the obvious east-west symmetry, and used the fact that Left's move to $*$ is reversible. One can similarly show that

 $= \frac{1}{4}$ and  $= \frac{1}{8}$ and  $= \frac{3}{4}.$

(Try it!) In fact *every* dyadic rational number x (whose demoniator is a power of 2) can be expressed as a Toppling Dominoes position, using a simple procedure. Write

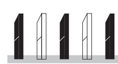
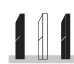
$$x = \frac{m}{2^n}$$

in lowest terms (i.e., with m odd), and let

$$y = \frac{m-1}{2^n} \quad \text{and} \quad z = \frac{m+1}{2^n}.$$

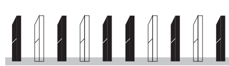
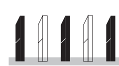
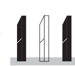
(In the notation of *Winning Ways*, $x = \{y \mid z\}$ in simplest form.) For example, if $x = \frac{3}{8}$, then $y = x - \frac{1}{8} = \frac{1}{4}$ and $z = x + \frac{1}{8} = \frac{1}{2}$.

Then y and z have smaller denominators than x, so we can recursively construct their domino sequences, say Y and Z. In the example $x = \frac{3}{8}$, we have

$Y =$  and $Z =$ 

as noted above.

Now for the trick! Write the sequences for Y and Z side-by-side, insert a black-white pair of dominoes in between them, and amalgamate the whole enlarged sequence into a single position. This gives a new position X, which miraculously has value x! For example,

 $=$  &  & 

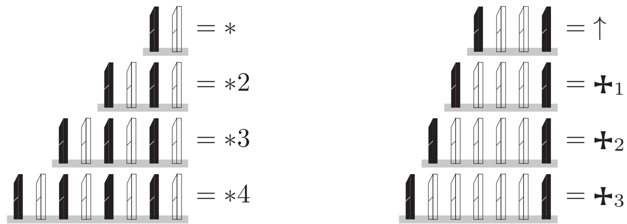
$\frac{3}{8}$ $\qquad$ $\frac{1}{4}$ $\qquad$ $\frac{1}{2}$

showing that the example in Figure 2 has value $\frac{3}{8}$.

What makes this construction truly remarkable is that it's the *only* way to represent numbers in Toppling Dominoes. So every number can be represented *uniquely* as a single row of dominoes! This remarkable theorem is due to

Alex Fink, and it has an equally remarkable proof, the details of which can be found in [3].

Various other familiar values also arise naturally in Toppling Dominoes; here's a sampling:

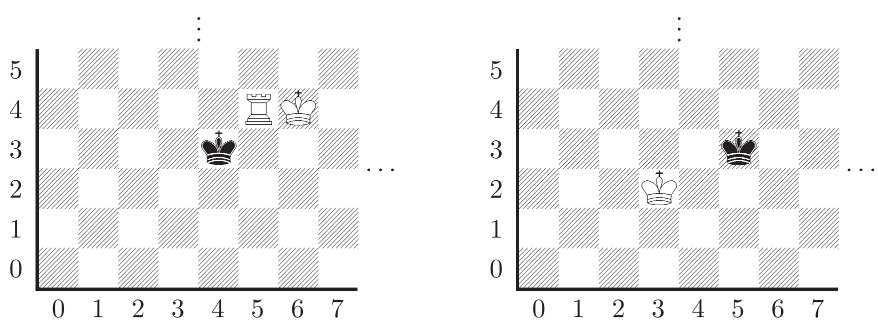


Entrepreneurial Chess

Here's an old Chess problem, first posed by Simon Norton, and later publicized by Guy:

> With initial position WKa1, WRb2, and BKc3 . . . what is the smallest board (if any) that White can win on if Black is given a win if he walks off the North or East edges of the board? [4]

Berlekamp and Pearson invented Entrepreneurial Chess partly in response to this problem [2]. It is played on a quarter-infinite board, such as shown in Figure 3. Right (White) has a king and rook; Left (Black) has just a king; and the pieces move just as in ordinary Chess. However, instead of moving her king, Left may instead choose to *cash out*. If Left cashes out, then the entire position is replaced by an integer n, equal to the sum of the row and column numbers for her king. For example, if Left chose to cash out in Figure 3(a), the position would be replaced by the integer 7.



(a) (b)

Figure 3: Entrepreneurial Chess. (a) A typical position; (b) A pathological position in which the rook has been captured.

ENTREPRENEURIAL CHESS is loopy. From Figure 3(a), Left can do no better than to cash out, while Right is constrained to shuttle his king between the squares bordering his rook. Writing G for this position, we therefore have

$$G = \{7 \mid G\}$$

which according to the theory of loopy games [1, Chapter 11], has value $7+\mathbf{over}$.

Moreover, ENTREPRENEURIAL CHESS exhibits some explicitly transfinite values. Consider the position in Figure 3(b), in which Right's rook has been captured, and Left is free to run indefinitely far in the Northeast direction. Writing H for the value of this position, it's clear that

$$H > n$$

for every integer n, and its value therefore exceeds every *finite* game. However it's confused with certain *transfinite* games, such as the infinite ordinal ω:

$$\omega = \{0, 1, 2, 3, \ldots \mid\}$$

On the sum $H - \omega$, Left will prefer never to cash out and play will continue forever, so the outcome is a draw.

It can be shown that if J is any *finite* game, and $J > n$ for every integer n, then $J > \omega$ as well. Since $H > n$ for every n, but H is confused with ω, it follows that H is explicitly transfinite. In fact one can show that, in terms of the *Winning Ways* theory, H has the remarkable value

$$H = \mathbf{on} \;\&\; \{0, 1, 2, 3, \ldots \mid H\}.$$

Berlekamp and Pearson have undertaken a detailed temperature analysis of ENTREPRENEURIAL CHESS positions. They also solved Norton's original problem: the answer is 8×11.

References

[1] E. R. Berlekamp, J. H. Conway, and R. K. Guy. *Winning Ways for Your Mathematical Plays*. A. K. Peters, Ltd., Natick, MA, second edition, 2001.

[2] E. R. Berlekamp and M. Pearson. Entrepreneurial chess. Unpublished manuscript, 2003.

[3] A. Fink, R. J. Nowakowski, A. N. Siegel, and D. Wolfe. Toppling conjectures. To appear in *Games of No Chance 4*.

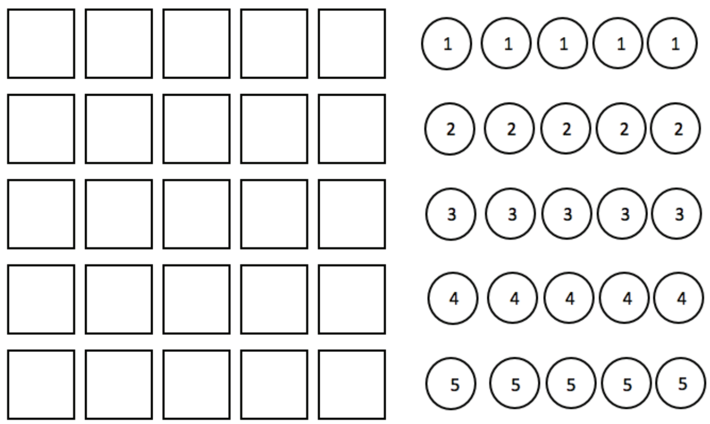
[4] R. K. Guy. Unsolved problems in combinatorial games. In R. K. Guy, editor, *Combinatorial Games*, number 43 in Proceedings of Symposia in Applied Mathematics. American Mathematical Society, Providence, RI, 1991.

4G4G4G10 – Four Games for G4G10

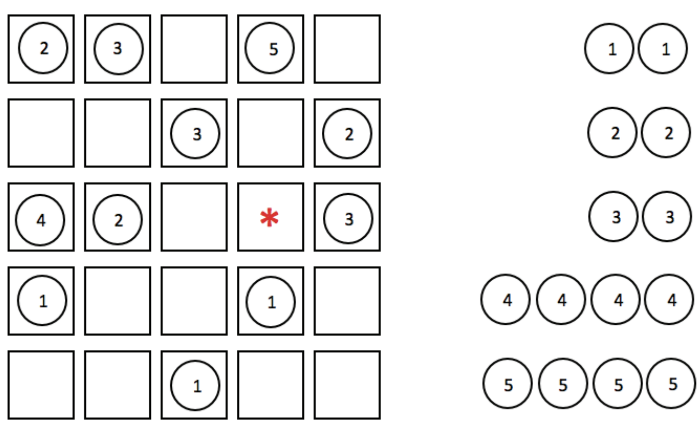
Jorge Nuno Silva

Latin Erdős

In this game each move consists in placing a disc with a number in a free square, provided such a placement does not give rise to any repetition of numbers in any row or column.



Example:



In the marked cell no number can be played.

Two players alternate. The first that, on his turn, is not able to play, for not having any legal move at his disposal, looses.
It can happen that the board gets completely filled with numbered pieces. In this case the winner is the player that conquered more columns.

A column is conquered by the player that, on his move, first gets an increasing sequence of length 3 in that column. In this context, increasing means that the numbers get larger from the player to his adversary.
Example:

B
4
2
3
*
*
A

If player A places 1 in one of the squares marked with *, he wins that column (sequence 1-3-4); if the player B places in one of those cells the number 5, he wins the column (sequence 2-3-5).
It can happen that a column contains one increasing sequence of length 3 for each player, as in the example

4
2
3
1
5

(1-2-4, 2-3-5)

thus, it is important to know who got his first, or whose turn it was when they both got it (whoever makes the move owns the column).
This game can be played in three ways:
I (Beginner) – With random factor. In this version the pieces should have their faces turned down and the players should turn one in each turn;
II (Expert) – Complete information. The pieces show their faces at all times and each player, on his turn, chooses freely the piece to place in the board.
III (Gambler) – Mixed. The pieces, with their faces down, are randomly divided by the players (12 for one, 13 for the other). The player with 13 pieces starts.

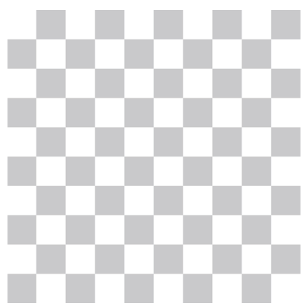
Notes

1) A square filled with numbers according to our rules is called a *Latin Square*. Latin squares were first studied by the Swiss mathematician Leonhard Euler in the 18th century.

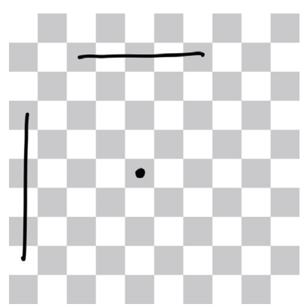
2) In Version II the first player can be sure of always having a legal move at his disposal. He should start by placing 3 in the central cell and, after that, when his adversary plays *x*, he should play *6-x* in the cell that is symmetric with respect to the central square.
This strategy does not guarantee more columns won at the end…
3) To be sure that each filled column is won by one of the players, we must rely on a mathematical theorem of Erdős and Szekeres from 1935!
[http://www.luduscience.pt/erdos.html]

Nail

There is a checkered 10x10 board on which two players (Black and White) take turns destroying cells according to the following rule.



On his turn, Black chooses a black cell still alive and kills it. He can also kill any number of contiguous living cells in one orthogonal direction from the chosen cell. Three possible moves:



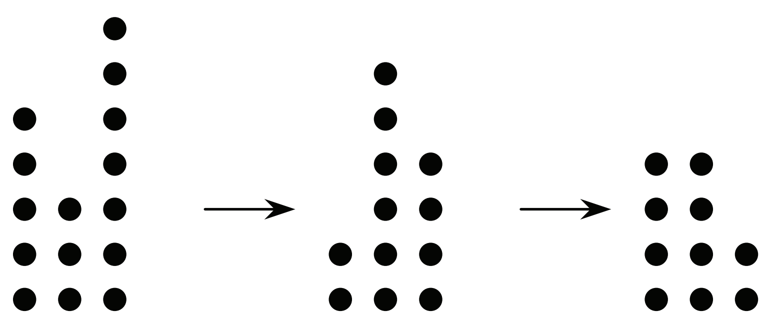
White's moves are similar, but he must start at a white cell. Whoever runs out of legal moves looses. Black starts. White, in his first move, cannot play symmetrically with respect to the center of the board.

LIM

There are three piles of beans.
Two players alternate. Each move consists of choosing two piles, take the same number of beans from each of them and add the same number to the third one. The player that, on his turn, finds two empty piles, loses.
Here is an example of two legal moves:

$$(5,3,7) \rightarrow (2,6,4) \rightarrow (4,4,2)$$

Or, in a diagram:

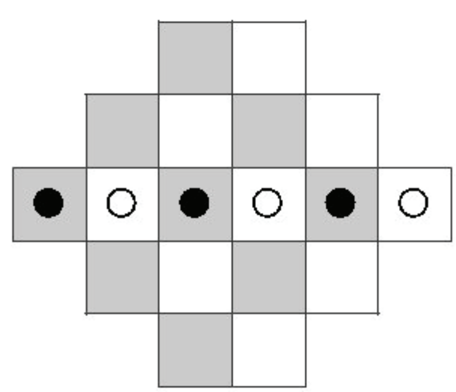


[*Mathematical Games, Abstract Games*. JPN & JNS, ISBN: 9892005074]

Stooges

This game uses a diamond shaped board, like the illustrated below. The cells of the board have two sides, one black, one white.

Two players (Black and White) alternate. Black owns three black pieces, White own three white pieces. The initial position:



On his move, a player must do one of the following:

- Move one of his pieces to an (orthogonally or diagonally) adjacent empty square of his color (white pieces only move on white squares, black pieces on black squares).
- Change the color of a square (from white to black, or from black to white) provided the chosen square is empty and it was not switched in the adversary's last move.

Restriction: If two consecutive moves consist of changing colors of squares, the next move must consist of moving a piece.

The winner is the player that builds a line (orthogonal or diagonal) with his three pieces.
[*Mathematical Games, Abstract Games*. JPN & JNS, ISBN: 9892005074]

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TerseTalk

An Android App for Programming Recreational Maths

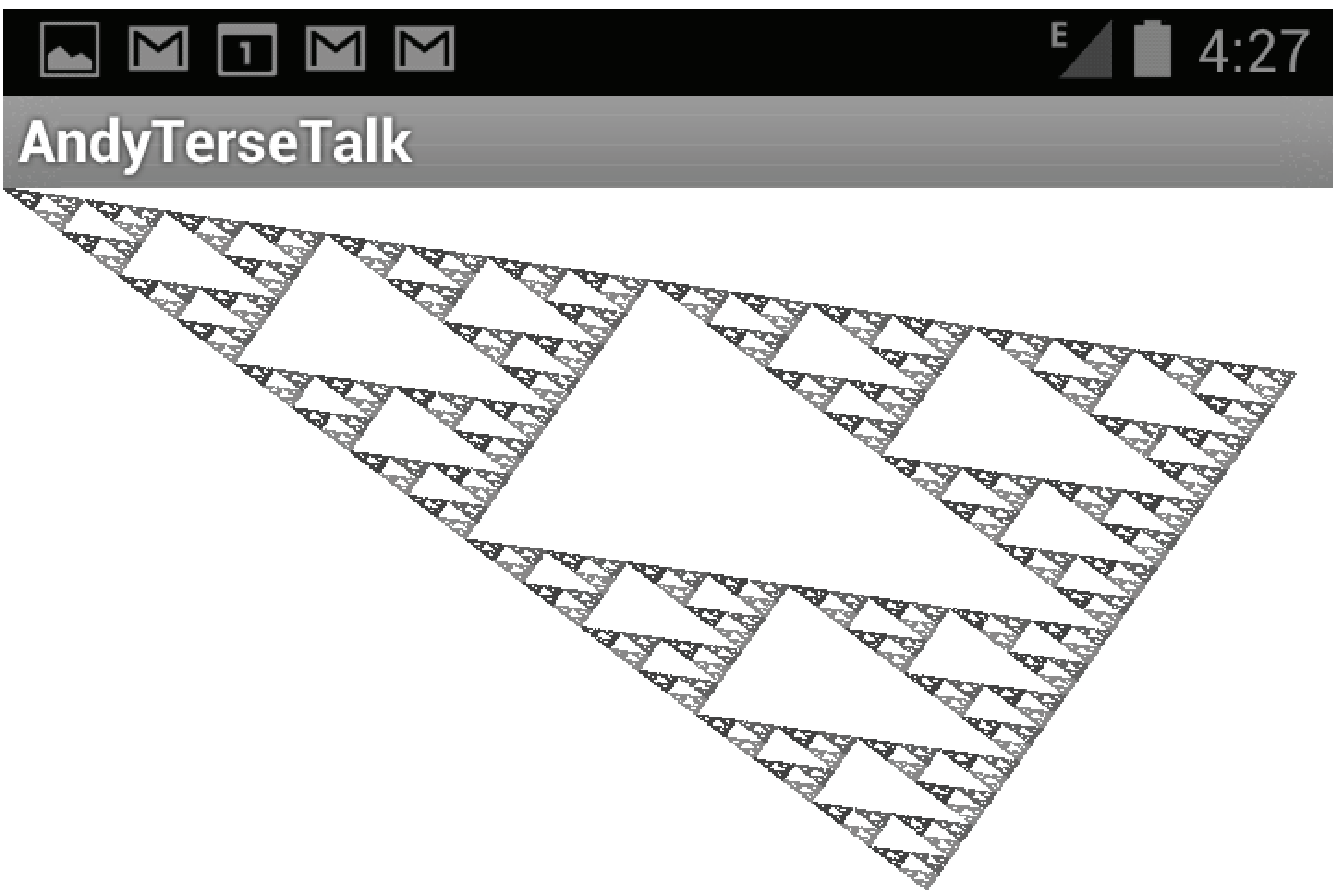
Strick (Henry Strickland) <strick@yak.net> April 2012

http://tersetalk.yak.net/

I program my home computer.
Beam myself into the future.
– Kraftwerk (1981)

TerseTalk is my gift to G4G10.

- It's a programming language that I created.
- It's an android app for phones or tablets.
- It's a complete environment for developing programs and drawing graphics.
- It's inspired by the Smalltalk-80 programming language and environment.
- It's free and open source.
- It's designed to be easy to type on a default Android on-screen keyboard.
- It's designed for experimenting with Gardner's style of Recreational Mathematics.



As a demo, I'll show how to draw Sierpinski's Gasket using the Chaos Game & TerseTalk. (Wikipedia has articles on the following theorem and game and gasket.)

Barnsley's Collage Theorem: If you can completely cover a target T with tiles T_i that are each affine transforms of T, then the functions f_i that map T to each T_i form an Iterated Function System (IFS) whose “strange attractor” is the target.

Barnsley's Chaos Game: Pick a starting point p_0. Generate the sequence of points p_0, p_1, p_2, … by iterating the IFS on p_0, randomly picking one of its member functions at each step, and applying it to the previous point to make the next point. So if the member functions are f_1, f_2, and f_3, and you make random choices 2, 1, 3, 2, …, your sequence will be

p_0, $f_2(p_0)$, $f_1(f_2(p_0))$, $f_3(f_1(f_2(p_0)))$, $f_2(f_3(f_1(f_2(p_0))))$, …

This sequence will converge on the “strange attractor” of the IFS. You might discard the first few points, which might not be near the attractor yet. But plot a finite head of the rest of the sequence, and you'll get an approximate image of the target of the IFS.

Sierpinski's Gasket lives on the plane between three corner points C_1, C_2, and C_3. It's trivially tiled with 3 shunken copies of itself: the three largest sub-triangles T_1, T_2, and T_3, that contain the corners C_1, C_2, and C_3. The affine transform f_i from the entire gasket T to each tile T_i is simply the function “go halfway from where you are, to corner C_i”.

On the following page is the TerseTalk code for a “draw” method that does this. It starts with the origin (0, 0), which happens to be the first corner, and then iterates a step of picking a corner at random and going halfway to that corner. Each point is plotted for 50,000 iterations. So you can see the Gasket emerge as points are individually drawn, the intermediate plot is “posted” to the screen every 200 iterations. It takes around 10 seconds to run on a fast (Samsung Galaxy Nexus) android phone.

Exercises for you to try on your android:

- What do you get if you have more than 3 corners?
- Devise a way to color the gasket, based on the random numbers chosen. (If you look carefuly at the previous page, there are 3 shades of gray, based on the random number chosen 4 steps ago.)
- What happens if your probability distrubtion is not 1/3, 1/3, and 1/3?
- Draw ferns and trees. How can we draw the Menger Sponge (which is 3D)?
- Write a TerseTalk app for the iPad, and convince Apple to allow it.

Visit http://tersetalk.yak.net/ to find the latest on how to create classes and methods for graphical programming in TerseTalk. Also I plan to have a mechanism for sharing your creations with others.

TerseTalk code sample: Statements end with periods. Comments are in "double quotes".
All values are objects, instances of some class, which is represented by a class object.
All work is done by sending messages to objects. The name of the message follows the receiving object.
For instance, "Num rand: 3" sends the "rand:" message to the class object Num with argument "3".
Some messages ("unary") take no argument, like "self white", which returns our copy of white ink.
Syntactically, unary messages bind tightest, then binary operators like "+" and "/", then messages with arguments (like "rand:" and "dot:"), then the list-constructing commas and semicolons.

```
"Draw Sierpinski's Gasket."
```

"0,0" constructs a list with two elements. The variables in the list "x,y=" are assigned successive elements.

```
x,y= 0,0.
```

These are the corners of the triange. The commas construct lists of 2 elements (x, y point values), which are syntactically nested inside the outer list of 3 elements, construted by the semicolons.

```
cs= 0,0; 500,380; 700,100.
```

The DO loop is executed 50000 times. The index variable "i" will range from 0 to 49999.
(All indexing is "0"-based).

```
FOR(i : 50000) DO(
```

The class object for "Num" accepts the "rand:" message, and returns a random integer from 0 to 2.

```
    r= Num rand: 3.
```

"at:" indexes into the list "cs" of 3 corners, to pick a random corner, and its two elements are stored in the two variables "cx" and "cy".

```
    cx,cy= cs at: r.
```

These two lines are the function "go halfway to the chosen corner", by averaging the x and y coordinates of the current point x,y with the corner cx,cy. The division operator "/" is floating point.

```
    x= (x+cx) / 2.
    y= (y+cy) / 2.
```

As a convenience, "self white" returns an object of white Ink.
The Ink is used to draw a single dot on the (by default, black) Screen.

```
    self white dot: (x,y).
```

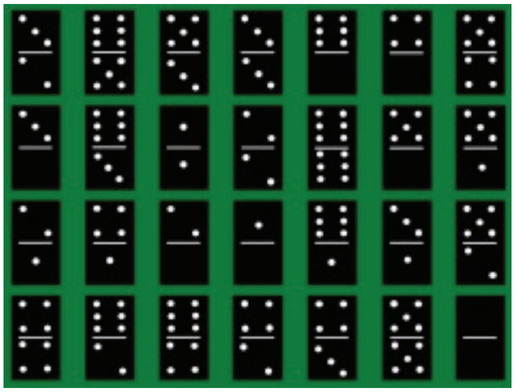
So you can see the Strange Attractor slowly emerge as it is being drawn, we "post" a copy of the current Screen object every 200 iterations. "%" is the modulus operator.

```
    IF(i % 200 == 100)
            THEN(self post).
```

TerseTalk is my own invention. The basic syntax for messages and operators, and the class and object structure, are taken from Smalltalk-80. On top of that, I added "," and ";" for constructing lists. Then I added a new syntax I call "macros", for the `FOR()DO()` and `IF()THEN()` structures. There are other syntactic shortcuts to make it easier to type on Android phones & tablets. See the web site for more.

```
).
"http://tersetalk.yak.net/"
```

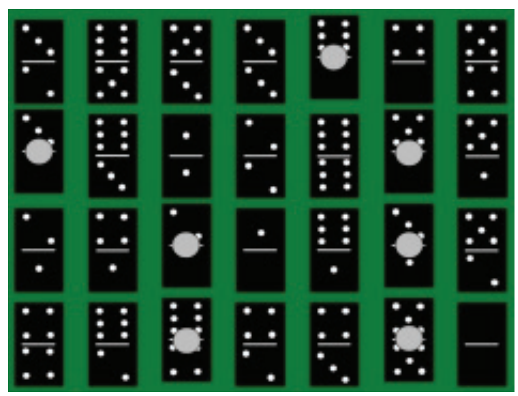
The Domino Effect (An Elementary Look at the Kruskal Count)
Jim Wilder



Pictured here is a random layout of dominoes (28 in all). Choose any domino from the top row. For example, if you take the first domino (3-2), you have an added value of 5. Starting at the next domino (6-5), spell out the number 5 (F-I-V-E), moving one time per each letter, and you will arrive at the 6-0 domino (not the 3-3). Like before, spell out 6 (S-I-X), counting and proceeding the way you read words on a page. You will next arrive at the 3 (T-H-R-E-E). Again, spell out the number value where you arrive. Do this until you arrive to the bottom row and can no longer move without moving off the end of the row. When you arrive to your final domino, take note of where you arrived. Start the process again, but this time, start on another domino in the top row. Again, take note of where you arrive. Continue to repeat the process, each time choosing a different domino. What do you notice? On which domino did you arrive on the first trial? What about the next few trials?

This idea came from various ideas related to the Kruskal Count. Three of my favorite ideas related to this are by Martin Gardner, James Grime, and James Tanton.

This is a diagram of what the "path" should look like if you choose the first domino in the top row of the original layout. Taking coins and laying them at each domino where you arrive might help you to notice a pattern and how you arrive at your final destination each time.



Now, take a regular set of dominoes, and randomly arrange them in a 7 X 4 array. See if your results are similar to what you have found here. What happens if you lay out the dominoes in a 4 X 7 array? What happens if you count the value of the domino instead of spell it? How is the outcome similar or different if you use a deck of cards, or a page from a book?

References:

Grime, J. Maths card trick: Last to be chosen *from YouTube* http://www.youtube.com/watch?v=uRI4XtnJxXo&feature=related, Accessed 28 March 2012

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MAGIC

H-10 MatheMagics | Louis Kauffman | Page 163

“So Many Presentations, So Little Time!”
Submitted by Anthony Barnhart

The Gathering 4 Gardner has to be the most diverse conference in the world. The conference attracts participants from a host of different fields. While there's no question that participants are enthralled for the *entire* conference, every person has one set of talks that they look forward above all others. Let's use a rather unique, magical method to ensure that you'll find a talk that interests you at this year's conference! Imagine that there are **10** talks happening over the course of a day, but you only have time to attend the one that interests you the most. Please gather 12 slips of paper of any size and a pen...I'll give you a moment to do so.

Set two of the slips aside. We won't need them until much later.

You may be interested in talks about *art*...Whether it be performance art like magic or the art of optical illusion. On one of the slips, right the word "**ART**." Leave this piece of paper on the table, with the writing facing up.

You may be more interested in the scientific or mathematical topics at the G4G. These talks are usually about solving problems of some kind, so write "**PROBLEMS**" on another slip and put it on top of "ART."

You may not care for the experimental side of problem solving (nobody’s perfect!), opting instead to focus on mathematical problems. So, write the word "**MATH**" on another slip of paper and add it to the top of the pile.

As a magician and a psychological scientist, I find myself most attracted to the illusions, so write "**ILLUSIONS**" on the next slip and put it on top of the "MATH" slip.

Illusions need not be only sensory. Illusions can also happen solely in your mind. These cognitive illusions often constitute the work of a magician. Write "**MAGIC**" on another slip and add it to the top of the pile.

One trait that's probably shared across the diverse attendees of the G4G is a healthy skepticism that naturally falls out of an awareness of science, mathematics, and illusion. Please write "**SKEPTICISM**" on the next slip and place it on top of the pile.

Skeptics typically don't believe in miracles. They believe that any *miracle* is likely to have a natural cause, and many so-called miracles involve some kind of trickery. Write "**TRICKS**" on a slip and place it on top of "SKEPTICISM."

Although I've suggest a lot of categories for talks at the G4G, you may not have a preference for any of them. You may just be interested in attending talks that are interesting. On the next slip, write "**INTERESTING**" and add it to the pile.

Finally, puzzles of all sorts are ubiquitous at the G4G. On the next slip, write "**PUZZLES**" and add it to the pile.

On the final slip of paper, write "I PREFER TO HEAR ___________ TALKS" and write your favorite category from above in the blank. Keep in mind that some things are naturally plural, like *tricks*, *puzzles*, and *illusions*, while others are singular. Set this slip aside and pick up the other nine. Make two piles as follows: Take the top slip, which has the word "PUZZLES," and set it to your left on the table. Set the next slip to your right. The next one, "TRICKS," goes on top of the first--then another on top of the second--and so forth with the rest of the slips as if dealing hands of cards.

The word "ART" is on one of the slips that ended up on top. Take that pile of slips and drop it on top of the "PROBLEMS" pile. Turn the whole pile over, so that the writing faces down. Take the bottom slip, with "ART" on it, and imagine that the table has a clock drawn on it. Since this is the **10**th *Gathering 4 Gardner*, place this slip, writing side down, at the imaginary 10 o'clock position on the clock.

Take the next slip from the bottom, which is "MATH," and set it writing down at the 11 o'clock position. Continue removing slips from the bottom of the pile and placing them at the appropriate clock positions, moving clockwise. You'll run out of slips at 6 o'clock. Set the "I PREFER" slip at 7 o'clock. Then use the two blank slips to fill the remaining clock positions.

Put your finger on the "I PREFER" slip. Think of the category you most prefer, and, starting with the slip at the 8 o'clock position, spell the name of this category, moving your finger clockwise from one slip to the next for each letter. For example, if you want "MAGIC," you'll spell the "M" on the slip at 8 o'clock, the "A" on the slip at 9 o'clock, and so on. Stop on the last letter of the word.

Keep your finger on the slip you reached. Gather the rest of the slips in any order and put them away in a separate pile. Turn over the slip under your finger and see if it matches the type of talk that you'd most like to see at the G4G. If so, I hope to see you there! Amazing!

Anthony Barnhart
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Adapted from Tamariz, J. & Navarro, G. (2008). Barbecue party. *Verbal Magic* (pp. 73-76). Seattle, WA: Hermetic Press, Inc.

Note from Tamariz and Navarro (2008), p. 76:

> *This trick uses the idea of progressive spelling: the principle of employing words or names, each of which has one more letter than the previous one. The principle dates back at least to the 1920s. Dr. James Elliott may have been the first to employ it. Another principle active here is that of "The Tapping Trick" or "Tapping the Hours," which appears in texts as far back as the late fifteenth century. Early descriptions of it appear in Luca Pacioli's "De viribus quantitatis" (1486-1509) and Horatio Galasso's "Giochi di carte" (1593). The innovation of using pieces of paper with words on them almost certainly belongs to Stewart James, who used it in "The Last Drink," in the August 1962 issue of "New Tops" (Vol. 2, No. 8, p. 18).*

Presented by
Connor Hofmeister
to honor
Martin Gardner
at G4Gx in Atlanta, 2012
March 28 to April 1

TEN CARD MAGIC SPELL

by Jeremiah Farrell and Connor Hofmeister

We introduce our ideas with a simple four card magic spell using the ten, jack, queen and king of clubs which are placed face down on the table and well mixed. The magician secretly writes a prediction down on a slip of paper which is folded and handed to the subject to safeguard. The magician says "We are going to place the cards randomly at the points of the compass NSEW. Please select one for NORTH." The procedure continues until the four cards are arranged NSEW. The subject is then asked to turn over any one card. Suppose it is the jack. "As you know", notes the magician, "the jack can be regarded as the eleventh card and I want you to either count 11 or spell 'jack of clubs' and proceed either clockwise or counterclockwise around the compass." The spectator does so and arrives at a second card and then repeats the instructions as often as he can.

He soon finds that he is "under a spell" and cannot escape from the queen of clubs. This matches the magician's prediction on the paper.

METHOD: The trick is self-working provided the ten is opposite the queen. We maneuver this with slight markings on the backs of the two cards. We will explain later a mathematical theory that does not require the marking of the cards.

<u>THE TEN CARD MAGIC SPELL</u>

The ten cards are stacked as in the diagram. The cards are flaired to show the subject what appears to be a random collection. Also at this time, the "Joker" and "Magic Spell" cards are noted.

The stack is repeatedly cut by the subject or the magician and when satisfied, the magician asks "Shall we have the stack face down or face up?" In either case the top card is noted and placed on the bottom. Using this card, he asks the subject whether he wants to spell or count (the ace can count as either one or eleven) and the magician deals the cards to the bottom, arriving at a new card. The instructions are repeated until the subject finds himself stuck at the "Magic Spell" card. Of course, that card and the Joker can only be spelled.

This effect is self-working and need not require any sleight-of-hand but some skilled magicians may choose to employ some special skulduggery.

We can regard either magic spell trick as a mathematical object called an absorbing Markov chain (Reference 1.). If clockwise or counterclockwise motions are chosen at random, it is possible to arrange the cards themselves at random and still be absorbed at the key card. We have checked every arrangement of four-card and found that the average number of steps until inevitable absorption never exceeds five moves.

This paper relies on "Celestial Magic" in the Nov. 2011 *Word Ways* by the first author. That article is a variation of a spelling trick by Jim Steinmeyer (Reference 2.).

R1: *Introduction to Finite Mathematics*, 2nd ed. by J. Kemeny, J. Snell and G. Thompson, 1966, Prentice-Hall, Inc.

R2. *Subsequent Impuzzibilities* by Jim Steinmeyer, 2008, Hahne.


MAGIC
SPELL
TEN
CARD
MAGIC
SPELL
JOKER

10 MatheMagics for G4G10
In Between Magic and Topology

by Louis H. Kauffman
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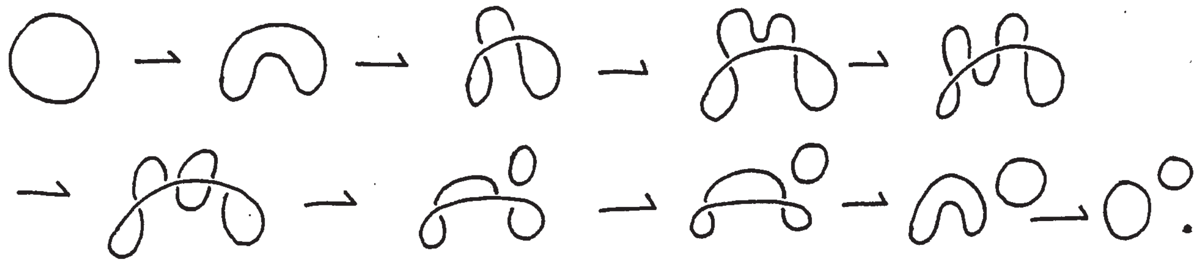
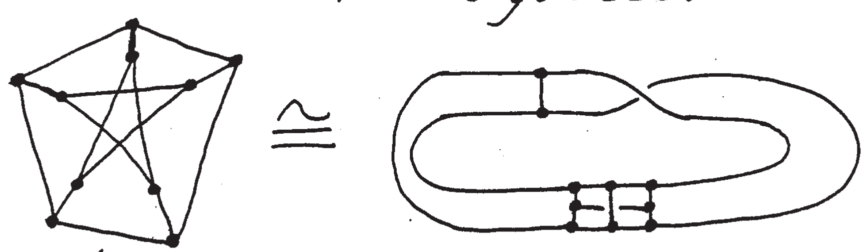
I. A Self-Reproducing Loop (Courtesy of Kurt Reidemeister and Sam Lloyd)



This shows how one loop could become two loops in a series of actions that almost looks topological.

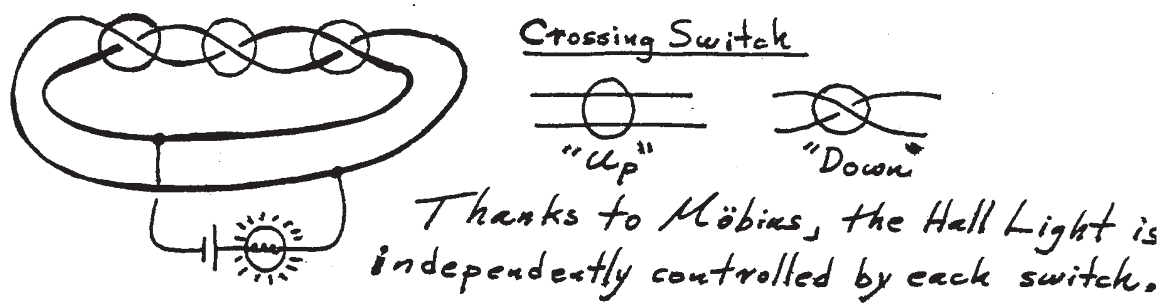
II. "The Snark was a Möbius you see."



The Möbius nature of the Petersen graph "explains" why it is a snark (i.e. not edge colorable in 3 colors with 3 distinct colors at each vertex.)

The famous Petersen graph is on the left in its usual incarnation, but really the Petersen is just another appearance of the Mobius strip.

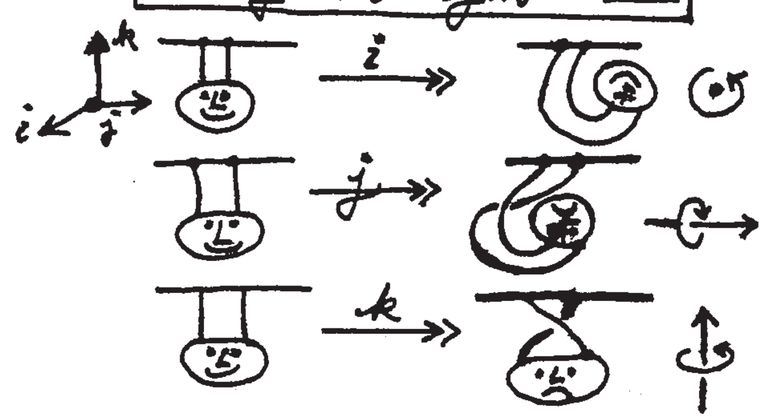
III. The Möbius Circuit



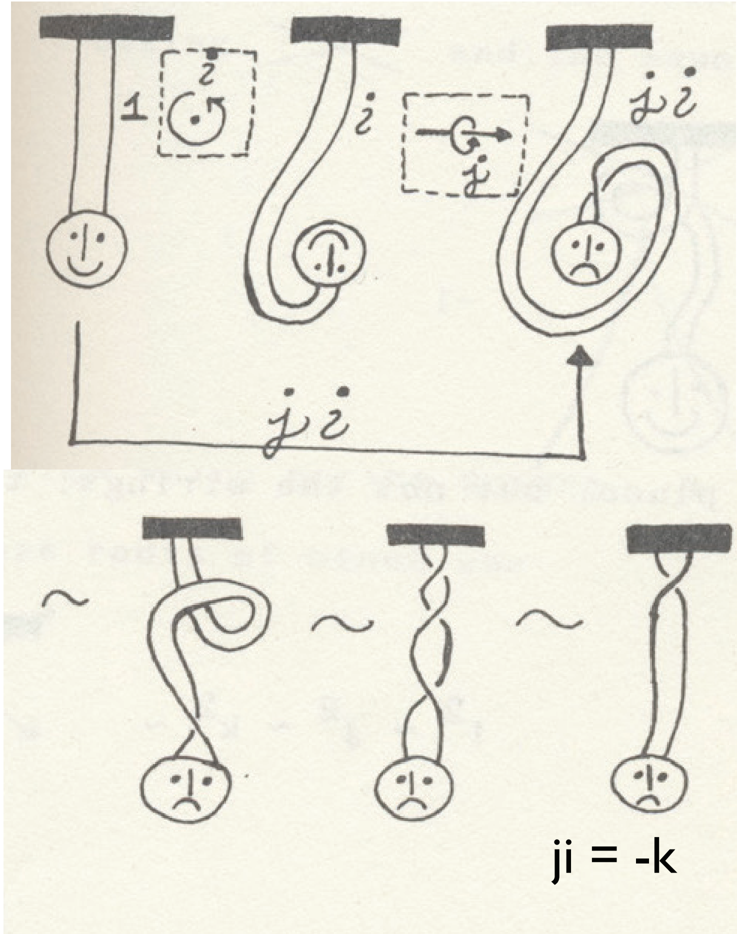
We hope that this is self-explanatory, and that you will go home and use the Mobius band to design a circuit to control the light at your front door from switches in every room in your house.

The Quaternions Personified

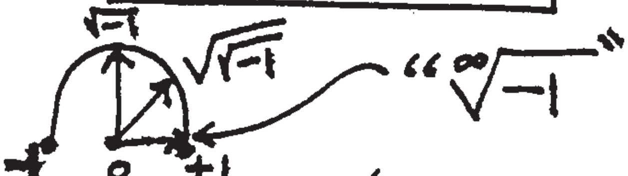
IV. $\boxed{i^2 = j^2 = k^2 = ijk = -1}$



Yes. There they are the quaternions i, j and k. And they can be understood as the topological symmetries of a little face attached by puppet strings to the ceiling.



V. $\boxed{e^{i\pi}+1=0}$



$$\pi = \infty\left(\frac{\sqrt[\infty]{-1}\;-1}{\sqrt{-1}}\right)$$

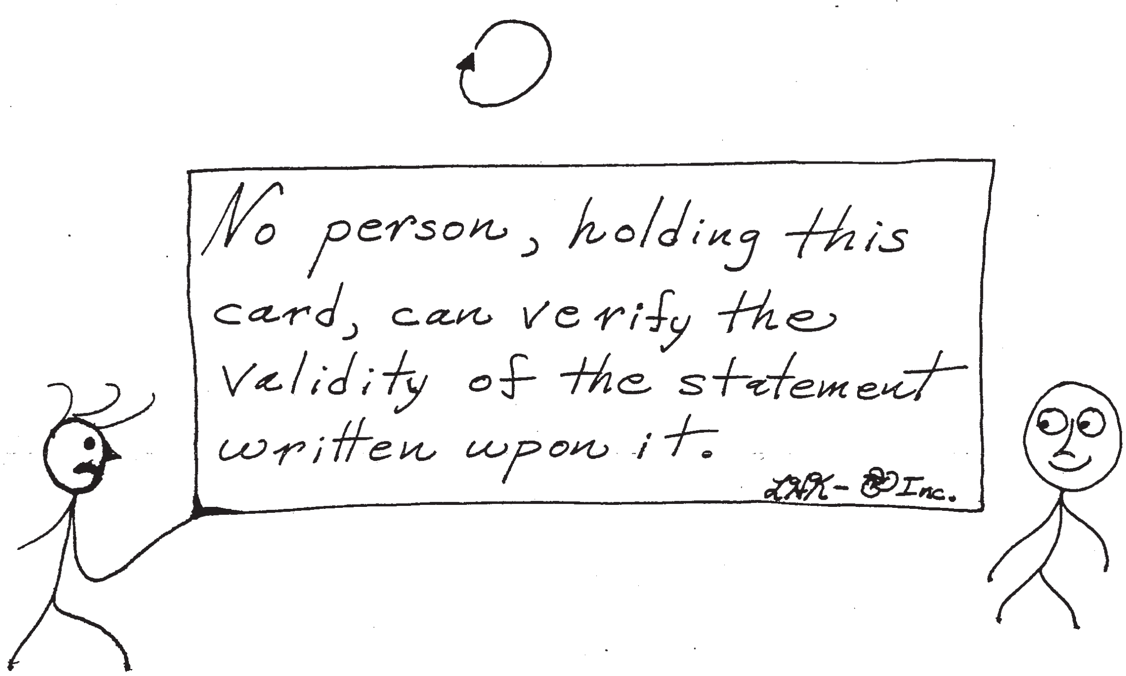
$$e^x = \left(1+\frac{x}{\infty}\right)^\infty \qquad \left[\frac{1}{\infty} \neq 0, \text{ it is } \underline{\text{infinitesimal}}.\right]$$

(Euler)

Here we have Euler's beautiful formula and an iconoclastic formula for Pi that is obtained by solving for Pi in Euler's formula. The formula for Pi is correct!

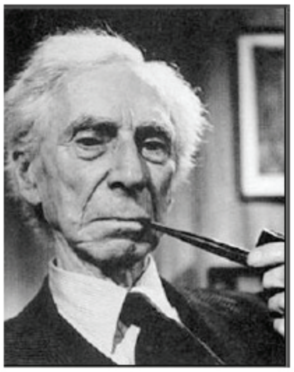
$$\pi = \infty\left(\frac{(-1)^{1/\infty} - 1}{\sqrt{-1}}\right)$$

VI.



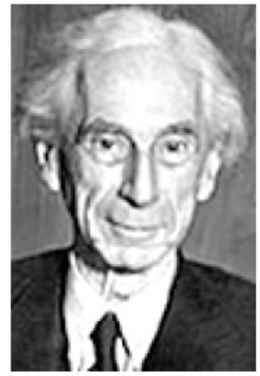
"I'm holding the card.
But if I was not holding
the card, then I could easily
see that anyone who does hold
the card is prevented from
asserting the validity of the
statement on the card. Thus
it certainly is correct — what
is written on the card.
But I am holding the card.
Therefore I am prevented
from doing what I have
just done!"

VII. This Seventh Tale illustrates the
Russell Paradox or lack of it in
Knot Set Theory where a bit of
curve A overcrossing another bit of
curve B means that B is a member of A.
Then a diagram with a curl is a member of itself.
But curls come and go topologically.
Also you will see
Ax to mean "x is a member of A".
So the Russell set is defined by
$Rx = \sim xx$
and the paradox is
$RR = \sim RR$.



$$\boxed{\begin{array}{c} Rx = \sim xx \\ RR = \sim RR \end{array}}$$

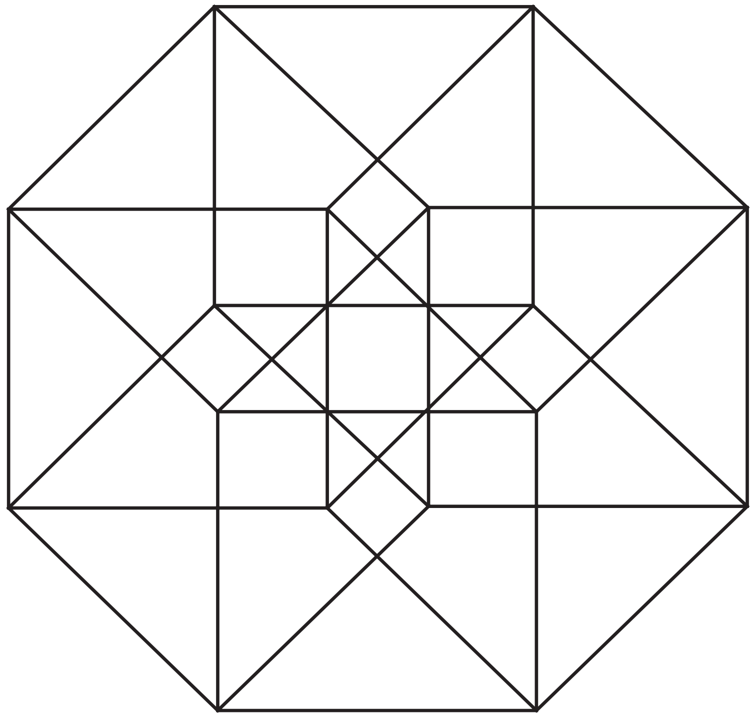
Russell Paradox (K)not.



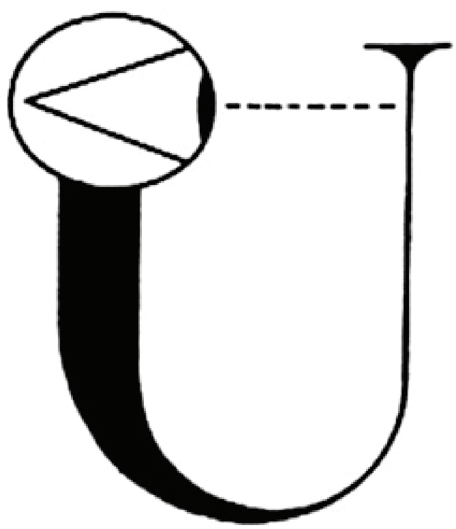
A belongs to A. ⟺ A does not belong to A.

VIII. KLEIN BOTTLE = Union of Two Mobius Strips.

IX. My Favorite Four-Cube

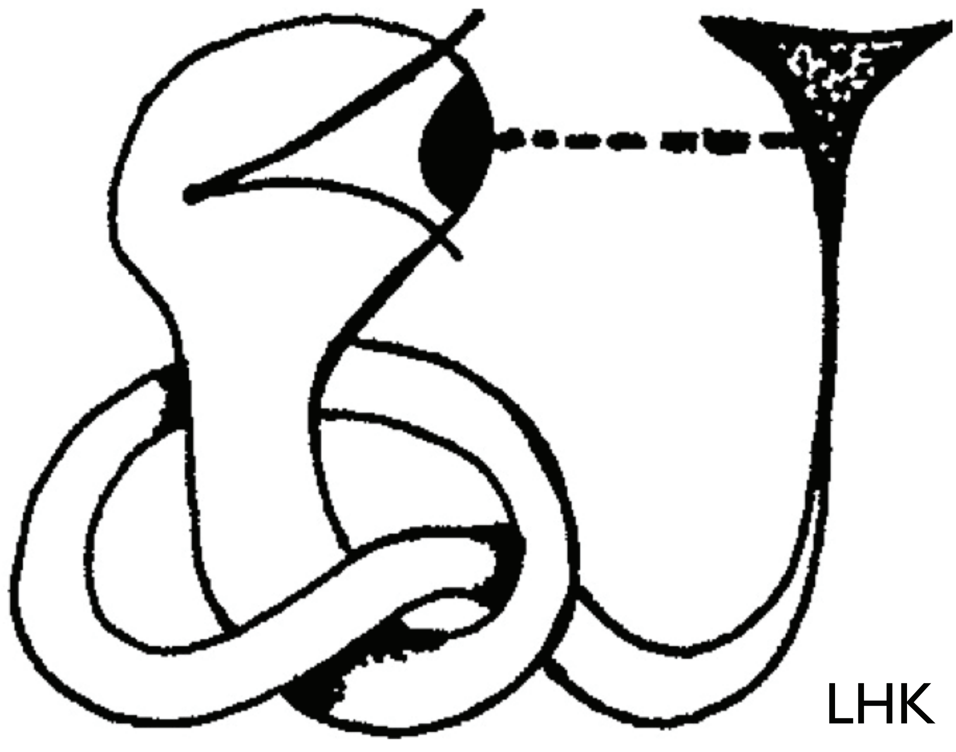


X. The Wheeler Universe and the Knot Wheeler Universe



Here is John Archibald Wheeler's Universe. The letter U looks back to the Big Bang and by observing Itself, brings the Universe into being.

Here is the KnotWheeler Universe, a slight correction to JW's point of view.



LHK

So you want to become a magician…

Doron Levy[1]

You probably saw a magician performing magic at a local event or on television. Perhaps it was at a corporate event, a birthday party, or one of the Gathering for Gardner meetings. Did you ever go back home thinking how nice it would be to be a magician? Studying magic, professionally, is not an easy task. Unlike most other performing arts, there really is no obvious way of studying magic. Contrary to common wisdom, this has nothing to do with secrecy. Definitely not these days, where even the government's top secrets seem to be freely accessible online. It is probably still true that most magicians would love to consider their craft as something one can only learn by belonging to a secret society (a society that would never let *you*, the outsider, join its ranks). The reality couldn't be any different.

There are many publically available resources for studying magic. This does not mean that everything is freely available. Magic is an expensive hobby and an even more expensive profession. Still, almost any secret can be purchased. It is only a matter of price. With books, videos, magic stores, magic clubs, magic conventions, online resources, etc., it looks as if everything is accessible. Or is it really? Facing an overwhelming number of resources for studying anything online, from quantum mechanisms to lock picking, the situation is no different with magic. Yet being able to access the information does not necessarily mean that the road to becoming the next Penn and Teller or David Copperfield is obvious.

So what should you do if you want to seriously engage in studying the art of magic? What follows is a collection of ideas that will hopefully help you achieve your goal.

#1: Doing a magic "trick" is not performing magic. Mastering the workings of a magic effect does not necessarily mean that you can *perform* magic. Clearly, being able to execute the technique is critical for a successful magic show, but that is not enough. Performing magic is not only doing an effect. Performing magic is how one effect connects to another effect; it is what you say, how you look, where you look, what you feel, what you project, and what emotions you create in your audience. Study the effects, but *think* about *performing* magic. Measure your success not only by how "clean" your routine is from a technical point of view, but by the overall impact your show has on your audience, that is, by the magical experience you create.

#2: The most important secret. This may sound silly, but there really is no way to perform magic without *performing* magic. Perform as much magic as you possibly can. Do it everywhere and do it at every opportunity. Performing magic helps you understand what to do when something goes wrong. Converting mistakes or mishaps into a magical moment

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is a skill that every magician must possess. You should think about pitfalls in your show and plan for them in advance, but the best practice comes with experience, and experience comes with performing magic.

#3: What is your target audience? It is likely that when you start thinking about magic you have no target audience. However, if you intend to consider magic seriously, it is best if you figure out sooner than later what kind of magic you would like to perform, or at least figure out your target audience. You should not spend your money on buying big illusions if you expect to perform in living rooms. Do not expect to be able to perform every kind of magic. The principles of magic can be studied using items you may not ever use in an actual show. For example, one of the classical items that is widely used for studying magic is sponge balls. You should probably not count on performing any sponge balls routine in an actual magic show (though some magicians still think it is a good idea). But studying how to vanish an object, how to hold the attention of the audience, misdirection, patter, creating a routine, all these can be done using simple objects (and a lot of practice). That will be a good way to start.

#4: Should I read a book or should I watch a video? Both. With so many magic DVDs out there, it is quite tempting to study by watching videos only. There are some great benefits to watching videos: it is wonderful to be able to feel as if you are learning directly from the great masters. The ability to pause, repeat, and watch in slow motion, are all very valuable. Books, on the other hand, are generally more difficult to process. The difference between videos and books is not just in the visual aspects. Think about the difference between watching a motion picture and reading a book. Good magic requires imaginations. It is important to learn from the masters, but it is equally important to spend time *thinking* about magic, and thinking about magic comes naturally when you read a magic book and are forced to think about what the effect will look like. An unclear technique might force you to come up with your own variation. Creativity is highly sought after, and I believe that reading books helps in that respect.

#5: What books should I read? Many books have been written on almost every aspect of magic. While most books cannot be bought in your neighborhood bookstore, many books are available from online magic dealers (and in magic stores). If I had to choose one book series to read, I would still go for the Tarbell Course in Magic. Even though the Tarbell series was written many decades ago, all eight volumes contain invaluable information. There are many contemporary books, but I am still a big believer that it is better to start with the classics. And when it comes to classics, books that should be on your reading list are "The Royal Road to Card Magic" by Jean Hugard and Frederick Braue and "The Five Points in Magic" by Juan Tamariz. Do not read books only for the "secrets". Many books focus on other aspects of performing magic, such as creating a routine, managing the audience, the experience of magic, and many other elements of a successful magic show.

#6: What videos should I watch? After making the attempt to convince you that watching videos should not necessarily be your #1 priority, I would still like to provide some specific recommendations. L&L Publishing has compiled a series of DVDs under the title of "World's Greatest Magic". These are relatively inexpensive DVDs, which can be purchased from many magic dealers, including from L&L Publishing directly (www.llpub.com). It is gradually becoming a collection of encyclopedic proportions. Specifically I recommend to take a look at the "professional rope routines", "Sponge balls", and "Cups & Balls" videos, but take a look at the other volumes as well. Check also the Michael Ammar and Jeff Mcbride videos. These videos are an excellent source for students of all levels.

#7: How much should I practice? This is probably one of the easiest questions to answer: as much as you need to. Remember that performing is also a form of practice, but you do not want to perform magic without sufficient practice. The more you practice the better you get. You should practice enough so that you will not have to think about the technique during the performance. Enough to be able to devote all your attention to creating a magical moment, so that you can pay enough attention to the audience, without having to think about what to do next. That part should be fully automated. If you plan on speaking during your show, write down your text and practice. If you plan to involve more people, make sure that everyone knows what to do and when to do it. If you use music, make sure you know the order of the songs, their timing, and how to control your audio system… Practice in front of a mirror, practice in front of a video camera (and watch yourself!), practice in front of people. Once you already have a show and you want to check new effects, it is best to try one new effect at a time. See if it works. See how it is received. Mixing new and old material in one show helps to smooth out any glitches that might happen with material you try out for the first time.

#8: A common mistake of beginners. Students of magic of all ages make many mistakes. It is good to make some mistakes. After all, this can be a great way to learn. However, some mistakes can and should be avoided. One of the most common mistakes you can make is buying every magical prop you see. It is likely that most of the stuff you buy, will never be used. Sometimes you may think that a prop is great, but you should always pause and think: is it going to fit my show? Isn't it too big (or too small)? Can I actually build a routine that makes use of the object? Do I really need to spend all this money on an effect that will hardly take three minutes to perform? A good example is gaffed playing cards. There is nothing wrong with gaffed cards. Amateurs and professionals use them alike. But there is so much wonderful card magic that can be done with plain playing cards. So do you really need to run and buy the expensive stuff? Does it add anything to your show? It might. All I recommend is to *think* about it in advance.

#9: **Study others but do not copy.** Your goal should be to perform magic that will represent who *you* are and what *you* can do. No one wants to see additional clones of generic magicians or clones of existing magicians. You should develop your own personality and find ways of reflecting your *self* in your magic show. This does not mean that you should not learn by watching other magicians. In some sense, watching other magicians is the best way of learning magic. Study others but do not copy their work.

#10: The magical experience. You want to be able to create a *magical moment* for your audience. This is the ultimate goal of every magician. It is not to "fool" the audience. It is not necessarily to make the audience laugh or to scare your audience. As a magician you want your audience to emerge from your show thinking: "WOW". How do you get there?

There really is only one way to do it: you should *think.* Think, think, and think again about your show. Not only from where you see it, but also from the point of view of the audience. Beyond that? Watch other magicians. Good magicians and bad magicians. Try to understand the difference and to transform the better points into your act.

Epilogue:

If you would like to consider magic at a level that is beyond a plain hobby start from the beginning. Build a proper foundation for your show. Plan everything and rehearse everything. Think about *all* the aspects of your show and analyze them. Most importantly – do not hesitate to ask questions. One of the biggest secrets of the profession is that most magicians actually like to teach and to give advice. All that they want to see is that the question is being asked by a *serious* student of magic. Good luck!

List of Authors G4G10 | VOLUME 1

List of Authors G4G10 (cont.)

G4G10 Exchange Book

Volume 1

Art, Games, & Magic

Atlanta, Georgia

MARCH 28 – APRIL 1, 2012

An aura of magic permeates a Gathering 4 Gardner. It's not just the presence of magicians, eager to display amazing feats of sleight of hand and sleight of mind. It's the pervasive spirit of ferocious creativity and antic playfulness among all the participants, whether magician, mathematician, artist, writer, inventor, engineer, scientist, toymaker, or puzzle master, that makes a Gathering such an enchanting and exhilarating experience.

Numbering in the hundreds, the members of this potent jumble are there to honor and remember Martin Gardner, whose many writings, particularly on recreational mathematics and magic, have had such a profound and lasting influence on their lives.

—excerpt from the Preface by Ivars Peterson

Published by:
Gathering 4 Gardner