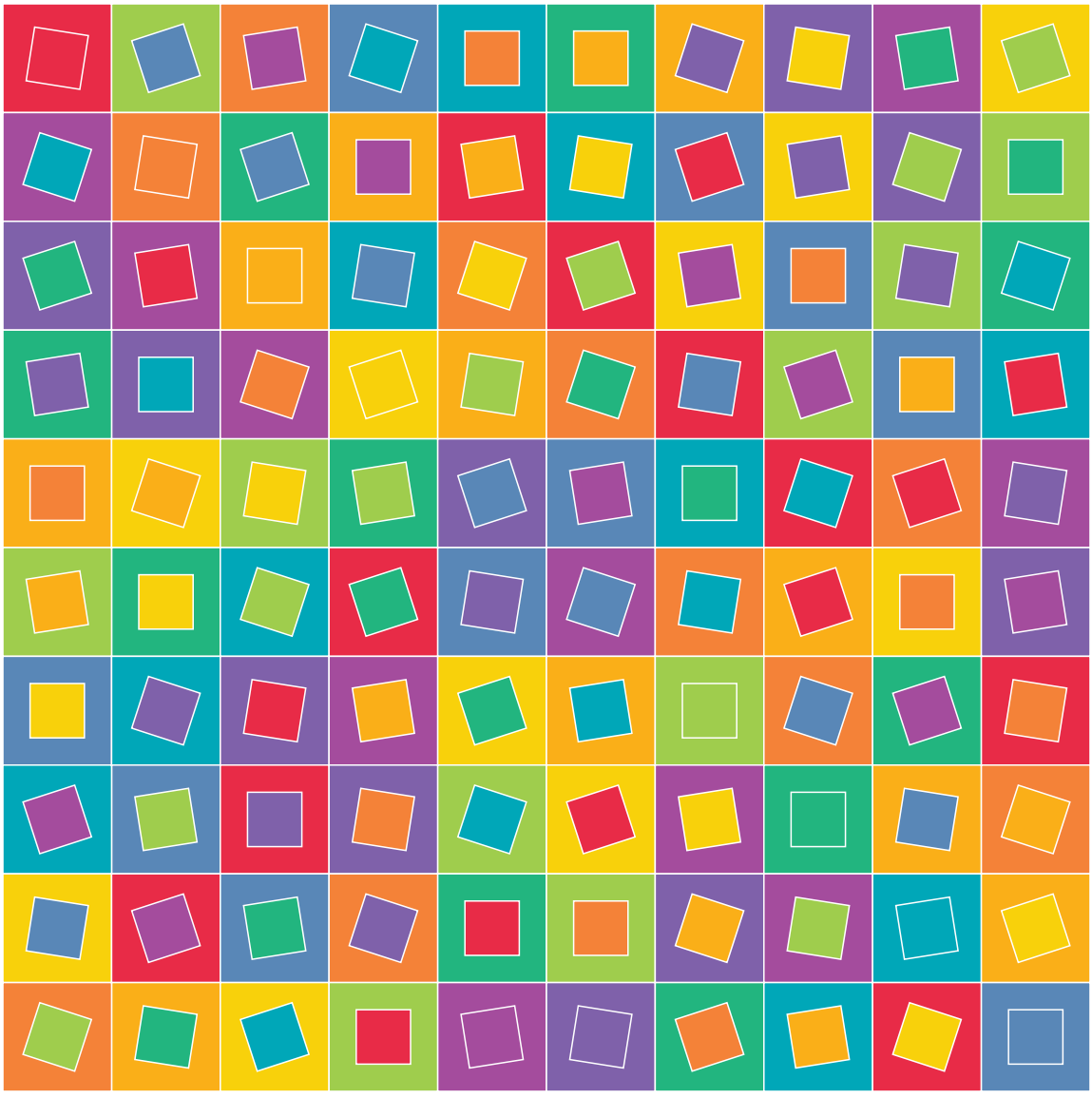
Quilt 100

Two Mutually Orthogonal Latin Squares of Order 10



"Quilt 100" is a design based on a pair of Mutually Orthogonal Latin Squares (MOLS). I was searching for subject matter for a 100-page artist's book, and realized that a pair of MOLS of order 10 could be used to generate a "quilt" pattern of 10 rows and 10 columns using 10 colors. This two-dimensional quilt could then be turned into an accordion-style book of 100 pages through appropriate scoring, cutting, and folding.

Pairs of MOLS of order 10 were first discovered in 1959 by Parker, Bose, and Shrikhande, thereby contradicting a long-standing conjecture by Leonhard Euler. He had suggested that it might not be possible to find MOLS for orders of the form (4k+2). In 1901, it was proven that the order-6 case (the so-called "36 Officers Problem") has no solution. This further strengthened the conviction among mathematicians that Euler's conjecture must be true. In 1959, however, MOLS of orders 10, 14, 18, etc. were found. Since these arrays disproved the conjecture, they were dubbed "Euler's Spoilers." Martin Gardner discussed this topic in Chapter 14 of his book New Mathematical Diversions, updating an earlier *Scientific American* column on the subject.

1	5	2	8	7	6	3	9	0	4
0	2	6	3	1	7	8	4	9	5
9	0	3	7	2	1	4	8	5	6
6	9	0	4	3	2	1	5	8	7
3	4	5	6	9	8	7	1	2	0
5	6	7	1	8	0	2	3	4	9
8	7	9	0	4	3	5	2	6	1
7	8	1	9	5	4	0	6	3	2
4	1	8	2	6	5	9	0	7	3
2	3	4	5	0	9	6	7	1	8

A

1	8	0	7	2	3	9	4	6	5
7	2	8	0	3	4	1	9	5	6
6	1	3	8	4	5	0	2	9	7
9	7	2	4	5	6	8	0	3	1
2	3	4	5	8	0	6	7	1	9
3	4	5	6	9	8	7	1	2	0
4	9	1	3	6	7	5	8	0	2
0	5	9	2	7	1	4	6	8	3
8	0	6	9	1	2	3	5	7	4
5	6	7	1	0	9	2	3	4	8

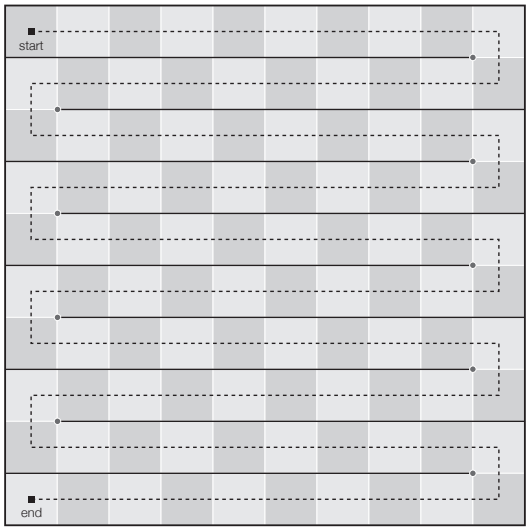
B

3	0	1	4	2	2	4	3	1	0
4	3	0	2	1	3	0	1	4	2
0	1	2	3	4	0	1	2	3	4
1	2	4	0	3	4	3	0	2	1
2	4	3	1	0	1	2	4	0	3
1	2	4	0	3	4	3	0	2	1
2	4	3	1	0	1	2	4	0	3
0	1	2	3	4	0	1	2	3	4
3	0	1	4	2	2	4	3	1	0
4	3	0	2	1	3	0	1	4	2

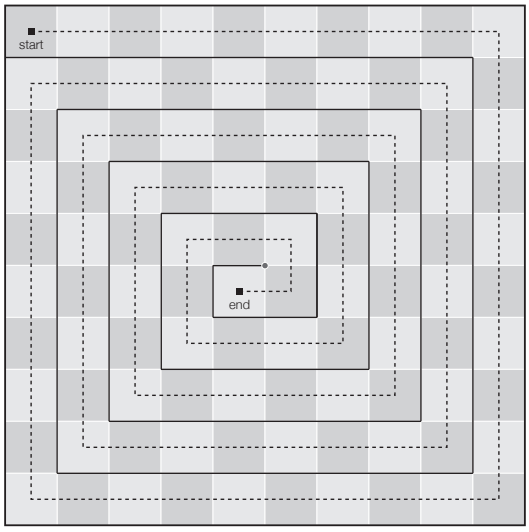
C

Arrays A and B above are MOLS of order 10. To create the "Quilt 100" design, their digits are translated into colors (red for 1, orange for 2, ..., magenta for 0). The elements of A determine the colors of the larger squares, and B controls the coloring of the smaller, inner squares. Due to orthogonality, all 100 possible color combinations occur. Note that eight of the double-squares (red-red, orange-orange, etc.) fall on the main diagonal, while two of them do not.

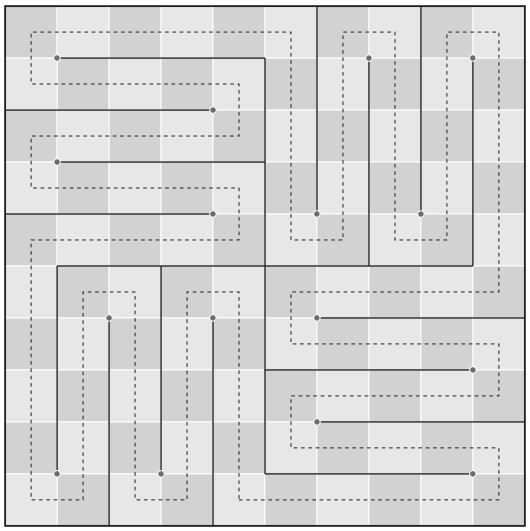
Array C is made up of four MOLS of order 5. The values of C are used to determine rotations of the quilt's smaller squares.



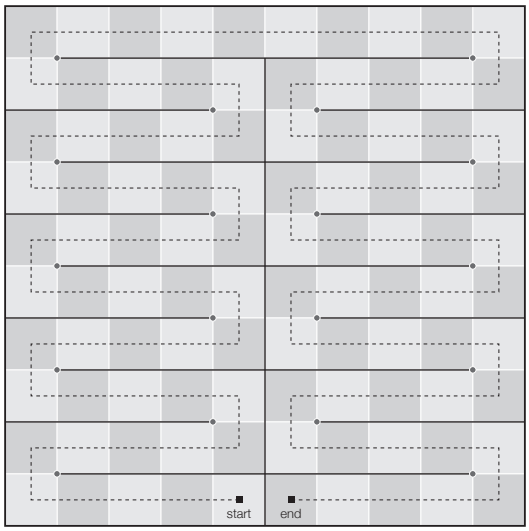
Long Snake



Spiral



Pinwheel

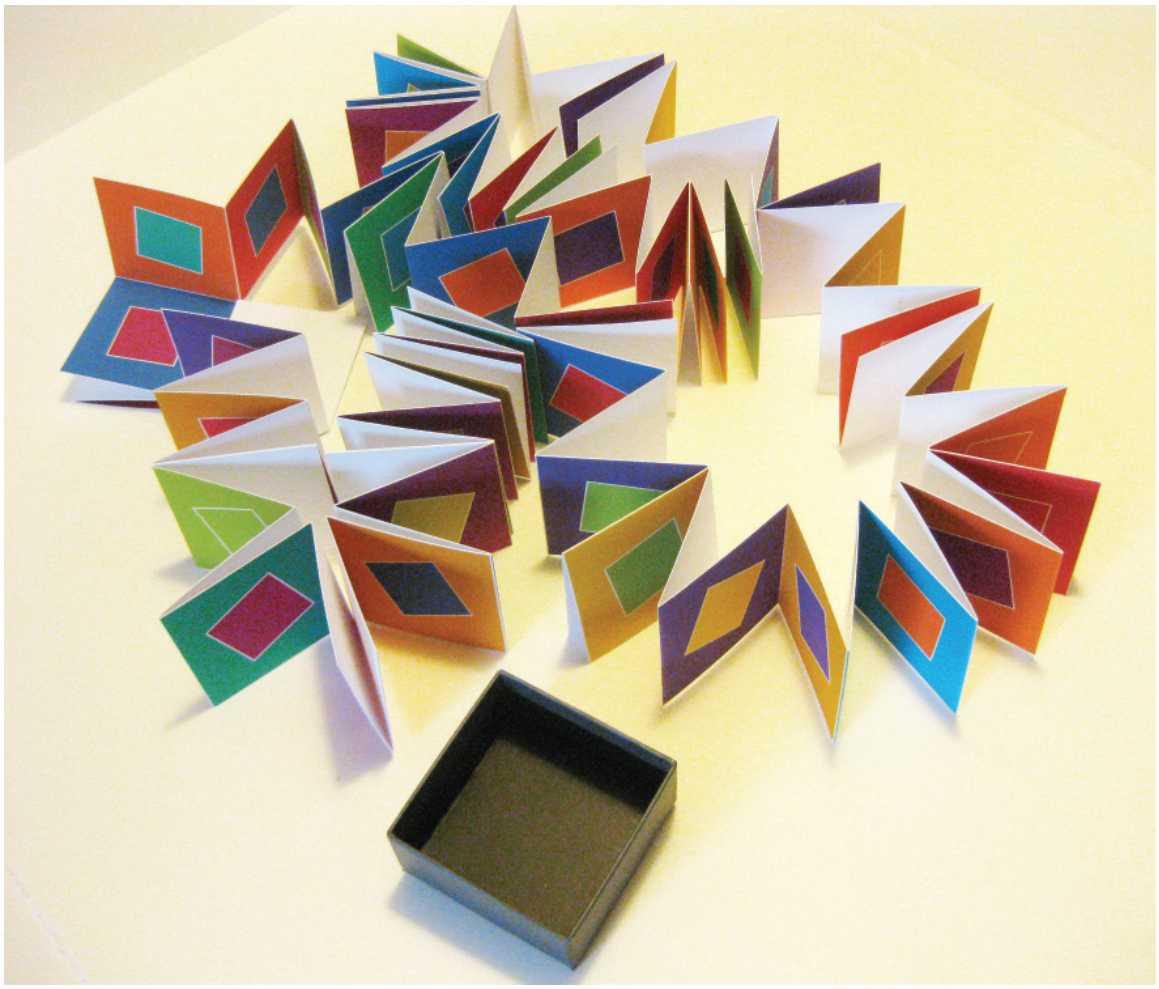


Double-Loaded Snake

There are many ways to convert the “Quilt 100” design into a folded book. Four possible schemes are shown above. Solid black lines represent cuts, and dotted lines indicate the folding path. The folds alternate in direction (mountain/valley) and occur along the borders of adjacent squares. Each of these methods begins with a single piece of paper and leads to a folded accordion-style book of 100 pages. The “start” and “end” squares are indicated, where relevant.

The **Long Snake** layout has 10 “runs” of 10 squares, and unfolding it allows row-wise verification of the quilt’s MOLS characteristics. The **Spiral** scheme is more interesting to unfold, and it has three full row/column runs before they start to shorten in length. The **Pinwheel** plan generates an “endless” strip, with consistent runs of 5 squares. Its structure has little relation to the quilt content, however. And, unlike the other methods shown, it cannot be folded up to a single-square footprint.

The **Double-Loaded Snake** scheme has a single run of 10 squares, with the rest having length 5. The “start” square is the front cover of the book, and the “end” square is the back cover. These correspond to the two double-squares of the quilt design that do not fall on the main diagonal.



The completed “Quilt 100” book is shown above. The starting point is a 24 x 24 digital print of the quilt design. The Double-Loaded Snake scheme is used to cut and fold the print into an accordion-style book. The folded book has a footprint of 2.4 x 2.4 inches — the size of a single large square. It is 1.6 inches thick and fits snugly into a small box. The entire folding path is 240 inches long, or 20 feet. It can be experienced as a folded strip of paper with various “runs” and “turns,” or it can be flattened out and pieced back together to recreate the original quilt design.

Due to the structure of the quilt and the folding scheme used, the front cover of the book is the magenta-magenta square and the back cover is the purple-purple one. This provides a conceptual link between the quilt and the folded book, since the two special double-squares in the quilt also have a special role in the book structure. Note that in the flat quilt they are adjacent, while in the folded book, they have maximum separation.

The book can be enjoyed without knowing the underlying math or the “Euler Spoilers” story. It can be seen as a color-swatch book containing all 100 combinations of 10 colors on 10 colors. The single long-run of 10 squares in the middle of the book corresponds to the top row of the quilt. In this row, it can be observed that each of the 10 colors appears exactly once in the large squares and once in the small ones. This might lead to further exploration to see if the other rows/columns share this characteristic. Naturally, they do, because of the underlying properties of MOLS.