Another Chess Mystery of Sherlock Holmes

Holmes had been very despondent for some weeks, and was looking even more gaunt than usual. For lack of any other idea, I practically forced him out of bed and made him accompany me to the chess club. Holmes was rather fond of peculiar chess puzzles, and I hoped that we might run across a game of interest to him. You may have read about some of these puzzles in the book “The Chess Mysteries of Sherlock Holmes,” by Raymond Smullyan. Holmes had no interest in actually playing the game, but he took pleasure in trying to deduce all the logical possibilities regarding both past and future moves related to unusual chess positions.

When we arrived at the club, there were several tables laid out with chess pieces, but one caught our attention immediately, because a game seemed to be in progress, yet the table was abandoned. We sat down as follows:

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Holmes
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[white pieces: pawns on b6, c2, and h2; rook on a1, knight on a2, bishop on b1, queen on c1, king on h1; black pieces: pawns on b7, c6, d5, c3, h3; rooks on f8, g7; bishops on d4, d7; knight on e2; king on a8.]  
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I studied the position carefully, but Holmes appeared distracted and seemed to pay no attention. Soon I exclaimed, “Holmes, I declare that I can checkmate you in three moves.” Holmes sighed and peered at the board for a minute. “Watson, are you entirely sure of your claim?” “Why, yes,” I said. “Let me show you.” Playing White, and with Holmes playing Black, the game proceeded as follows:

1. Q-a3  1...K-b8
2. Q-a7ch. 2...K-c8
3. Q-a8 mate
“You see, Holmes,” I said triumphantly. “Surely there is nothing you could have done to prevent this checkmate.” “Well, yes, I suppose you are right, Watson,” he replied, “at least from a certain philosophical point of view. But yet I am not entirely convinced.”

Before I could ask Holmes to explain this strange remark, two gentlemen joined us at the table, our old friends Colonel Marston and Sir Reginald Owen. “Ah, Holmes, I haven’t seen you at the club for some time,” said the Colonel in greeting, “and your friend Watson as well. What do you think of our game? I do believe I have the upper hand, would you not admit, Sir Reginald, despite the disadvantage of playing with the black pieces? “Yes, I confess that you are right,” he replied, “but I have only been playing seriously for last few moves, and once having fallen behind, there was little I could do to salvage the situation.”

At this point, Holmes interjected: “Marston, I see that you and Sir Reginald have been playing one of your highly unorthodox games.”

“Why, yes, I do admit that we have not been playing in a conventional manner. Perhaps you deduced that from the unusual placement of Sir Reginald’s bishop. Sometimes we play just to explore the many possibilities inherent in the game, without regard to who wins or loses, but we always adhere strictly to the rules.”

Holmes continued: “But I quite agree with you, Marston, that you are in control of the position. As it is clear to see, you can force a checkmate in only three moves.”

“But Holmes,” I whispered hurriedly, “did you not hear that Marston is Black? It is White that can checkmate in three, not Black. It appeared to me that Holmes’ depression must be deeper than ever to cause him uncharacteristically to pay so little attention to detail. Unfortunately, the Colonel overheard my rebuke, for he joined in by saying, “I do believe that Watson is correct, for even though it is my turn to move, I can see no way that I can force checkmate is less than 5 moves.” And, indeed, neither could Sir Reginald nor I.

To my chagrin, Holmes did not concede his error. “You may not see the solution, said Holmes, but it is plain to see right there in front of you.” “In that case, replied Colonel Marston, surely you would accept a wager. If you convince us you are right, I will pay you one pound, but if you are wrong, you must come to the club tomorrow and take us all out to dinner.” As he finished speaking, he gave a slight wink in my direction. Surely the Colonel had recognized my friend’s deteriorated condition and was trying to help me draw him back into a more normal manner of living.

Taking no notice, Holmes responded to the challenge. “Very well, I accept, but I do not wish to take unfair advantage of you. First we must be very clear about the terms of our wager. I win if I can demonstrate that Black can play in such a way that the game will end before he needs to play four moves. It might end in three moves, and it might end sooner. In any case, the result will be checkmate.”

“Nothing could be clearer,” agreed the Colonel. You may take the black pieces, then, and I will take White, and let us see if you can finish the game in less than four moves. “It is your move.” “But wait, you agree not to resign before the four moves are up?”

“Most certainly,” said Holmes, “both players must continue to play for four moves, as long as it is possible.”

Holmes moved his hand towards the rook on his bottom rank, hesitated for a moment, and then placed it back in his lap. “There is a slight problem,” he said. “I am not certain how to play.”
“You may have all the time you wish,” said the Colonel, “as I have nothing else to do this after-
noon.” “No, you do not understand,” said Holmes. “It is not a question of time. It is just that I do not know for certain how I should play. “In that case,” said Sir Reginald “surely you must con-
cede defeat. If you do not know how to play, and if time will not help you, then there is no alter-
native.”

“I most assuredly do not concede defeat,” said Holmes. “I can prove that Black can play and force mate in three moves, and yet I cannot yet be certain exactly how it can be done.”

At this point, I could not restrain myself from exclaiming, “But Holmes, surely that is impos-
sible. I have heard that in the realm of higher mathematics, it can sometimes be possible to prove that a solution to an equation is possible even though one can not solve for the exact result. But surely nothing of the sort could occur in a simple game. Why, there are only a finite number of possibilities, and surely we could check out all of them.”

“That is true, Watson,” said Holmes, “but nonetheless your conclusion does not follow. Perhaps it would help explain the situation if I let you finish your game, and we could continue this dis-
cussion later.”

All three of us stared at Holmes without knowing what to say. I will admit that he looked more like his old self. He was sitting up straighter and looked at each of us in turn with a twinkle in his eye, and yet his proposal was so unreasonable that I wondered if this time he was really going insane. Did he really hope that by observing our play we would somehow discover a solution that he had missed, and none of us could see? Or was he simply stalling for time?

Nonetheless, without even sitting down, the Colonel and Sir Reginald proceeded to finish their game, as follows:

1.... NxQ
2. NxN check 2.... K-b8
3. R-a8 check 3.... KxR
4. N-e2 4.... R-f1 check
5. N-g1 5.... Rxg1 mate

“I am sorry, Holmes,” said the Colonel generously, “but in truth I did my best and I don’t see how I could have made the game shorter, given White’s various threats.”

“Not at all, Marston,” replied Holmes, with assurance. “For now I know how to play.” Replac-
ing the pieces to their previous position, Holmes began with:

1.... R-b8

“It is your turn, Marston.”

“But Holmes, how could you possibly hope to cause a checkmate by retreating your rook into the corner,” I cried. And before I had quite finished, Marston added, “Really, Holmes, in fairness I must allow you to retract that move, because by hemming in your own king, you are allowing me to win immediately just by moving my knight. Your king will have nowhere to escape the di-
scovered check from my rook!”

After a moment, Sir Reginald began to laugh. I was embarrassed beyond words. How could he triumph so at Holmes’ humiliation? Maybe he was not aware of Holmes’ unusual condition, and smarted from previous occasions when Holmes had proven us all wrong.
“But don’t you see, Marston,” said Sir Reginald, you really can’t move your knight! That is, if you did move your knight, you would checkmate Black, and the game would not last for 4 moves. You might win the game, but you would lose your bet!”

“And why would I lose the bet?” demanded Marston. "Holmes said he would checkmate me in under four moves, not let me checkmate him!"

“No, you are wrong,” said Sir Reginald. “Holmes gave you fair warning. He said the game would end in checkmate. He didn’t say anything about who would checkmate the other! If you reflect upon his words, you will realize that he spoke them very carefully and very precisely.”

“In any case, I have not lost the bet, because I have not played yet, and nobody has yet checkmated the other,” said Marston, somewhat louder than necessary. And after thinking for a while, he played, and continued, as follow:

2. Q-e1    2.... R-g1 check
3. QxR    3.... BxQ

“Now you must admit defeat, Holmes,” I said, “because you have played your third move, and you have not even placed White in check.

“Not so,” said Sir Reginald. “For it is still White’s move. And White is still able to checkmate Black. And, more to the point, White must checkmate Black. For the only piece White has left that can move is his knight,...”

4. N-anywhere, mate

“... and he must make a move, because Marston himself insisted that the game could not be cut short by resignation! So Holmes wins the bet.”

And so, after some more discussion, Marston and I had to agree with Sir Reginald.

Holmes had for some time been quiet. But finally he spoke up, and to our astonishment, said: “Really, gentlemen, you are conceding defeat all too easily. For I have not yet proved my case. I have merely demonstrated one half of the necessary argument. Do you not wonder why I waited until you finished your game to play out one variation?”

“Why, yes, I do wonder why you created such an air of mystery,” said Colonel Marston, “and pretended not to know how to play, when all the time you knew full well how to proceed.”

“I will admit,” said Holmes, that I did see foresee that line of play, “but until I saw how you finished your game, I could not be sure that it was the right way to play.” “You see, coming upon your game in the middle placed me at a disadvantage, for I did not know what had proceeded before. In particular,...

“But Holmes,” I interrupted, “I don’t see how the preceding play could possibly be relevant. And even if it were relevant, what possible help would it be to observe a future line of play?”

At this point Marston interposed thoughtfully, “Well, I do see how the preceding line of play could sometimes be relevant. For example, in some positions it is not clear whether a player has the ability to castle or not, because we can’t tell whether the player’s king or rook has been previously moved. But I don’t see anything of this sort in our particular game. Let us hear what Holmes has to say.”
“I really did not intend to be so circuitous,” said Holmes. “But the future often tells us something about the past, just as the past tells us something about the future. Perhaps I should have asked directly about the past moves. For example, I could have asked you, Marston, whether you indeed made a capture with your pawn on h3 as your last move before the position we found when we arrived.”

“Yes, as a matter of fact, I did make such a capture,” said Marston.

Losing patience once again, I burst in and said, “But Holmes, how could you possibly know that? And what does that have to do with the checkmate? You are piling mysteries upon mysteries!”

“One thing at a time, Watson,” Holmes said patiently. “Marston has accused me of prolonging the mystery, but each question of yours diverts me further from the explanation.” Chastened, I resolved not to say anything more until Holmes finished.

“As for the pawn capture, Watson, that is really quite elementary. Sir Reginald assured us that in his last moves he was sincerely trying to play well. But White was the last to play in the position we found. What was White’s last move? Why did Sir Reginald leave his queen where it could be captured by the knight? Indeed, why didn’t he begin the checkmate sequence you found so quickly when we arrived? The only reasonable explanation was that a more important piece was at stake, his king. His king must have been in check, and he must have moved his king out of check. Where could it have come from? Not from g1, because the square g1 is in check from three pieces, the knight on e2, the bishop on d4, and the rook on g7. That is impossible: there is no way black could have moved so as to simultaneously place white in check in three different ways. So, white must have moved his king from g2 to h1. Now, what was black’s move prior to that?”

Sir Reginald was first to reply: “Yes, when white’s king was on g2, it was in double check, from the pawn on h3 and the rook on g7, and the only way this could have happened was for the pawn to have made a capture, moving from its previous position on g4 and exposing the king to check from the rook, as well as administering a check of its own. By why, then, did you need to ask?

“Because that is not the only other possibility,” said Holmes. There is one more. Or to be precise, there are several more, all of a similar nature.”

“Really, Holmes, this is too much,” I protested. There can be no other possibility. Black can’t have moved two pieces at once. His pawn can’t have captured starting from a different square. White’s king can’t have moved from h2, because he can’t share the square with his own pawn!”

“But yet, there is another possibility,” said Holmes. “Perhaps you are forgetting how little we knew when we entered the room. We did not know what moves had gone before. We learned who the players were, who played White, and who played Black,” (and as he said this, if I am not mistaken, he fixed me with his gaze for a moment longer than usual), “but we did not even learn where they were sitting as they played.”

“What could one possibly learn from that?” I cried out.

“Yes, Watson, what could one learn from that? Where do you normally sit when you play chess?”

“I sit on the white side, if I am playing white, and on the black side, if I am playing black.”
“Quite so.”

“For example, I am now sitting on the white side, and you are sitting on the black side.”

“Yes, we know that now. But we didn’t know that before.”

“How can you say that. Isn’t it obvious merely from the position that the white pieces are on my side, and the black pieces are on your side?

“Not necessarily,” said Colonel Marston. “After all, some of the black pieces: the knight and bishop, and two pawns are on your side now, and one of the white pawns is on Holmes’ side. It is evident that the pieces are able to move about. How do you know where the pieces were at the beginning of the game? As Holmes said, you didn’t see the beginning of the game.”

“But my side must be the white side,” I insisted. “Are you trying to tell me that the white pieces could have begun on your side of the board, Holmes, and marched down to my side, while my pieces marched over to your side, and they all politely stayed out of each others’ way, except for a few, mostly pawns, that were captured, and that Black has three pawns almost ready to be queened, but has considerably refrained from doing so?

“Exactly, Watson. That is just what I have been thinking. It may seem wildly improbable, but it not impossible. For example, the pawns could have side-stepped one another by capturing and changing files.”

“But it is truly impossible, Holmes, because the final position reached by Marston and Sir Reginald would not be a checkmate if the white pieces began at your side of the board. In that case, the white pawn on h2 could move backwards, so to speak, to capture the rook on g1!”

“Capital, Watson!” cried Holmes. “That is exactly how I reasoned. That is why I did not know how to play until I saw the completion of the game. Only then did I know for certain in which the direction the pawns were allowed to move.”

Once this mystery was cleared up, we were able to resolve all of the remaining problems. Marston was first to discover the other half of Holmes’ proof. Assuming that white moves from top to bottom in the diagram, there is only one way black can checkmate in 3, as follows:

1. N-g3 check (the white pawn now can’t capture the knight!)
2. K-g2 (the king is not under attack from black’s pawn on a3, which is moving up the board!)
3. K-g1
4. R-d2, discovered check and mate.

Sir Reginald pointed out why Holmes’ first line of play would have failed if White had been moving down the board. After 1...R-b8, and 2...R-g1 check, White could have defended with 3. PxR (promotes to queen), and when Black captures the new queen, White still has the old queen to play (and also the pawn on c7, which could queen on c8) and therefore is not forced to checkmate Black.

And I then chimed in: “Now I understand your other point, Holmes! As you explained, White’s previous move, starting from the position we first saw, was to move the king from g2 to h1, but when the white king was on g2, it might have been in check only from the rook on f8, because the black pawn on a3 might have been moving up the board, not downward as I presumed. In
that case, black’s previous move could have been a move with its rook, ending on g7, to deliver the check.”

“But Holmes,” added Marston, “why are you so sure that White’s previous move was a king move? Perhaps instead white moved the white queen to c1, a possibility you did not cover in your explanation. That move might not have been such a blunder as it seems, because perhaps white captured a black piece that was previously on c1, maybe even Black’s queen. And Black’s queen might have itself just captured a white piece on c1, perhaps a rook.

“Indeed, you are quite right. That is also possible,” conceded Holmes, “but in that case, would not Sir Reginald have captured the black queen on c1 with his knight on a2, thereby safeguarding his own queen and administering a check on your king? Sir Reginald has assured us he was playing his best at the end, and with this move, he could have turned the tables on you, so to speak.”

“And now gentleman, I hope you will all join me for dinner tomorrow night, for I can think of nothing better on which to spend the pound I have just won from Marston. I know that you all have been concerned about my health, but I assure you that nothing is wrong with me, and our afternoon together has quite lifted my spirits.”

We were all so pleased that we quite forgot to be annoyed with Holmes for making such a mystery out of the simple but far-fetched notion that the original position did not reveal the direction in which the pawns could move. I myself have recounted this story as I have merely due to my desire to convey the events as faithfully and truthfully as possible.

It was only later that I remember the moment we first arrived at the club, when I had displayed my checkmate in three moves for White. What had Holmes meant when he said I was right only in a philosophical sense? Upon reflection, I realized that if White had been moving from top to bottom, then my move 2. Q-a7 check would fail, as Holmes could have responded with 2…KxQ! Or, if I had tried instead, 2. Q-d6 check, followed by 3. Q-c7 check, it would have still met 3…KxQ, since in both cases the pawn on b6, moving down the board, no longer guards the Queen.

But there is indeed another sense in which I was right. Sitting as I was on the “South” side of the board, as long as it was my move, I could indeed win by checkmate in 3 moves. There are two possible cases. In the first case, the white pieces are moving upwards on the board, as I had presumed, and I was playing white, and my checkmate was valid. In the second case, the black pieces are moving upwards on the board, and sitting South, I should have played the black pieces instead of white! But then I still had a forced checkmate in 3, using the moves demonstrated by Marston. But I would have faced the same dilemma as Holmes. Even though I could have truthfully asserted (and, unwittingly, did assert) that I could checkmate Holmes in 3 moves, I would not have known how to do so, because I would not have known whether to play White or Black!