Chocolate Chip Pi

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Abstract

Have an urge for chocolate candies? Why not use the tasty morsels to approximate the value of pi? This paper begins finding the area of a partially eaten Hershey bar and then develops an integration technique with milk and white chocolate chips. The chocolatey techniques are used to estimate the value of pi. The methods easily motivate such standard methods of numerical integration as the rectangle method and the Trapezoidal rule.

Let's learn some Calculus with chocolate. We will construct chocolate mosaics with white and milk chocolate chips and use them for numerical integration.

1 Math at the bar

To begin, help yourself to 3 of the 12 smaller rectangles that comprise a Hershey bar. You can see what I chose to eat in Figure 1. The area of a Hershey's chocolate bar, ignoring depth, is 12.375 square inches since its dimensions are 2.25 by 5.5 inches. What is the area of the chocolate bar in Figure 1? To answer this, we could find the area of one of the small rectangles and multiply this figure by 9. Another approach is to note that only 9 of the 12 rectangles remain. So, the desired area is $(9/12)(12.375) = 9.28125$ square inches. This approach is the basis of our chocolatey Calculus.

Figure 1: Math on a Hershey bar

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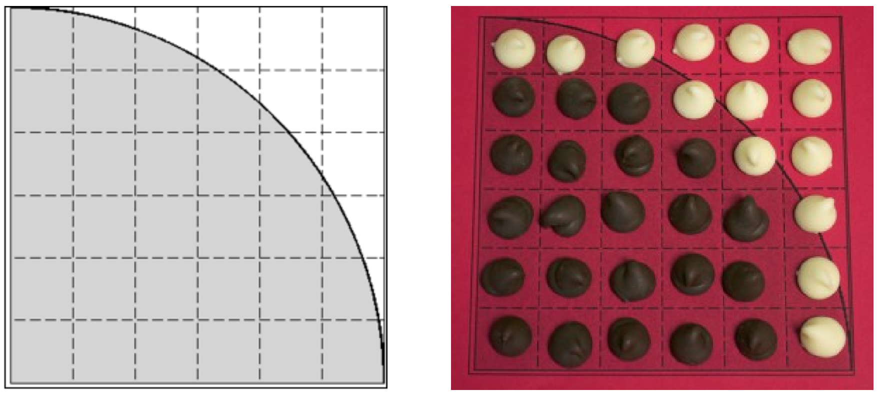
2 It only takes a quarter

Now, rather than a chocolate bar, let's use a 1 by 1 square containing a quarter circle as seen in Figure 2 (a). We just found the area of the uneaten portion of a chocolate bar. We'll adapt the same method to approximate the area of a quarter circle of unit radius, which equals $\pi/4$.



(a) (b)

Figure 2: Estimating π on an 8 by 8 grid

To begin, we overlay the square with a grid of smaller squares. For example, in Figure 2 (a), we have used a grid of 36 squares. We place a milk chocolate chip on any square contained entirely in the circle. All other squares contain a white chocolate chip. This produces the chocolatey mosaic in Figure 2 (b). We are now ready to approximate π.

The quarter circle has unit radius and is contained in a square with an area equal to 1. For the mosaic in Figure 2 (b), 22 of the 36 chocolate chips are milk chocolate. We estimate the area of the quarter circle as 22/36ths of the total area of the unit square. So, our approximation to $\pi/4$ is 22/36 also yielding $\pi \approx 4 \cdot (22/36) = 2.44$.

The squares containing the milk chocolate chips, from a certain perspective, can be viewed as columns; our approximation is the sum of the area of these chocolatey stacks. This is reminiscent of Riemann sums, which is a method from Calculus that approximates the total area underneath a curve on a graph, otherwise known as an integral. Riemann sums and our algorithm with chocolate mosaics both accumulate the area of rectangles, the methods differ in how they compute the height of the rectangles.

We can improve our estimate with a more refined grid and more chocolate chips. For example, the 11 by 11 grid in Figure 3 (a), results in placing 83 milk chocolate chips of the 121 total. This yields an estimate of 2.7438 for π.

Our estimates are improving but still have yet to reach 3. Suppose we use a 54 by 54 grid; 2916 chocolate chips were placed on such a grid with the help of public school teachers enrolled in the Charlotte Teachers Institute as seen in Figure 3 (b). The mosaics contained 2232 milk chocolate chips yielding the estimate $\pi \approx 3.06$. That's a lot of chocolate chips!

(a) (b)

Figure 3: Estimating π on 11 by 11 (a) and 54 by 54 (b) grids.

3 More with less

Let's improve our estimate with some clever manipulation. We fill the grid boxes using the same procedure but change how we compute π from the resulting mosaic. Now, we take the number of milk chocolate chips to equal the number of such chips in the mosaic and half the number of squares containing a white chocolate chip in which the underlying square in only partially contained in the circle. Looking back at Figure 2 (b), we see 22 milk chocolate chips and 11 white chocolate chips in which the associated square is contained partially in the circle. Our estimate becomes $\pi \approx 4(22+\frac{1}{2}(11))/36 = 3.06$. Notice, we have produced the same estimate to 2 decimal places using 36 chocolate chips versus 2916 as used in Figure 3 (b).

Using an 11 by 11 grid, the resulting mosaic seen in Figure 3 (a) contains 83 milk chocolate chips and 21 white chocolate chips on squares that contain only a portion of the underlying circle. Therefore, $\pi \approx 3.09$.

4 Be square and integrate

Not a chocoholic? Skittles, Starbursts, Cheerios or, for the calorie conscious, Sticky Notes could replace chocolate chips and be used to approximate π. In fact, you could even estimate this irrational number as you tile your bathroom floor or decorate a sheet cake!

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