Harmonic Magic Square

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About the magic square of the integer and inverse number.

Sum of inverse does not become constant, if composed with consecutive integers.
Because there is a prime consisting of $n^2/2 < p < n^2$
But, sum of inverse can be constant, if composed with distinct integers.
I created it, and was named "Harmonic magic square"

Example:

12x12 Harmonic magic square MinNb＝165 MaxNb＝8736 s1=25740 s-1＝1/56

5280	2184	165	720	780	572	2520	1848	2002	8736	660	273
231	336	616	840	2912	264	5460	495	1716	2340	4290	6240
1365	5040	1440	3696	7920	2860	504	182	390	1001	286	1056
4004	260	364	5005	1120	468	3080	1287	288	3960	5544	360
2080	2464	560	1456	252	8190	176	5720	990	2574	585	693
1638	224	2730	195	2772	1040	1386	520	7392	528	6435	880
924	420	8008	4620	1155	416	3465	1248	312	180	3432	1560
1872	3276	2016	6552	315	4680	308	4576	220	715	440	770
6930	6006	1092	728	858	4368	330	1680	1980	1320	240	208
429	630	6864	1232	5148	385	3744	280	1170	210	2288	3360
168	3640	455	462	2310	2145	672	624	3120	3168	396	8580
819	1260	1430	234	198	352	4095	7280	6160	1008	1144	1760

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5280+2184+165+720+780+572+2520+1848+2002+8736+660+273 = 25740
1/5280+1/2184+1/165+1/720+1/780+1/572+1/2520+1/1848+1/2002+1/8736+1/660+1/273 = 1/56

Trick: Product of the two position of the line symmetric is constant (Example: 1441440).
Since the same as the mirror of original to 1441440/n all the integers, even if you become inverse magic square.

Combine the horizontal line of the two Harmonic magic square of coprime, can be created the additional-multiplicative-harmonic magic square!

List of Harmonic magic square (and 8x8 Hermonic semi-magic square)

Size	MinNb	MaxNb	s1	s-1	p
semi 8x8	42	3960	6732	17/420	
10x10	88	3780	9504	1/35	
10x10	48	4620	9240	1/24	
12x12	24	13860	20790	1/16	
12x12	165	8736	25740	1/56	
12x12	11	25200	30800	1/9	
14x14	18	40040	51480	1/14	
64x64	3741	26771144400	34718611200	317936/917132337	100150851200400^{32}
64x64	49140	2938615240	6539097600	2544/56179409	144403552893600^{32}
64x64	105248	9294405750	21383208000	1/45747	9782176163760000^{32}
100x100	2499	5021276832000	5567067792000	1/2254	12548170803168000^{50}
100x100	205200	25139826712	60799939200	1/84847	5158692441302400^{50}
100x100	230230	22406690880	64645268688	1/79800	5158692441302400^{50}
100x100	12420	84193309200	128147169150	1/8160	1045680900264000^{50}

8x8 Harmonic semi-magic square MinNb＝42 MaxNb＝3960 s1=6732 s-1＝17/420

60	105	165	198	840	1008	1584	2772
990	112	315	54	3080	528	1485	168
88	432	2640	1080	154	63	385	1890
1232	77	1386	110	1512	120	2160	135
330	264	42	210	792	3960	630	504
231	2970	756	1680	99	220	56	720
336	2310	240	2520	66	693	72	495
3465	462	1188	880	189	140	360	48

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10x10 Harmonic magic square MinNb＝88 MaxNb＝3780 s1=9504 s-1＝1/35

1080	1386	2772	924	2160	154	360	120	240	308
189	112	864	528	1890	176	630	385	2970	1760
1485	2310	1440	140	560	594	2376	231	144	224
352	165	1056	440	99	3360	756	315	2016	945
330	160	135	1540	1320	252	216	2464	2079	1008
3696	1980	420	432	616	540	770	792	168	90
110	720	378	2640	504	660	126	880	462	3024
1512	1120	96	1188	990	336	280	3465	297	220
270	1155	495	88	1260	264	3780	672	288	1232
480	396	1848	1584	105	3168	210	180	840	693

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12x12 Harmonic magic square MinNb=11 MaxNb=25200 s1=30800 s-1=1/9

990	1155	924	550	23100	2640	105	12	504	300	240	280
90	18	176	75	50	700	396	5544	3696	1575	15400	3080
45	1100	660	200	35	12600	22	7920	1386	420	252	6160
1008	1232	60	13860	3600	5775	48	77	20	4620	225	275
330	616	315	720	525	11	25200	528	385	880	450	840
28	24	3960	264	99	560	495	2800	1050	70	11550	9900
80	693	84	13200	40	112	2475	6930	21	3300	400	3465
1260	6600	360	15	1680	308	900	165	18480	770	42	220
17325	5040	4400	1400	825	350	792	336	198	63	55	16
8400	132	25	126	66	2310	120	4200	2200	11088	2100	33
44	9240	36	210	150	3850	72	1848	1320	7700	30	6300
1200	4950	19800	180	630	1584	175	440	1540	14	56	231

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14x14 Harmonic magic square MinNb＝18 MaxNb＝40040 s1=51480 s-1＝1/14

36036	4004	264	1320	385	1155	364	1980	624	1872	546	2730	180	20
1456	1144	9240	3696	6006	6435	21840	33	112	120	195	78	630	495
1092	5148	3432	7920	440	35	72	10010	20592	1638	91	210	140	660
18	273	936	780	910	819	858	840	880	792	924	770	2640	40040
616	504	30	176	208	156	10920	66	4620	3465	4095	24024	1430	1170
1560	330	39	126	154	7280	70	10296	99	4680	5720	18480	2184	462
280	132	5544	143	65	8008	56	12870	90	11088	5040	130	5460	2574
315	2310	1848	12012	13104	16380	2002	360	44	55	60	390	312	2288
1287	240	4290	6160	15015	9009	63	11440	80	48	117	168	3003	560
1584	1260	3120	22	6552	468	286	2520	1540	110	32760	231	572	455
429	21	5005	585	990	260	3080	234	2772	728	1232	144	34320	1680
2145	30030	9360	1820	693	720	3640	198	1001	1040	396	77	24	336
3276	4368	8190	2860	6930	40	6864	105	18018	104	252	88	165	220
1386	1716	182	13860	28	715	1365	528	1008	25740	52	3960	420	520

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Enumeration of Polyominoes considering the symmetry

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Information on efficient method for counting the polyominoes.

Counting polyominoes on the entire time is required $O(4^n)$.

But, fixed polyominoes has been calculated up to 56 because known to have an efficient method of counting, and polyominoes with symmetry can be computed in time $O(n2^n)$ at most.

If counting for each symmetry and acquire appropriate heaviness and add all, can calculate the number of free polyominoes.

The polyominoes has 16 kinds of symmetries, and if use 9 kinds, can count up free polyominoes.

I was able to enumerate up to 45-omino by the calculation of approximately two months.

List of 16 symmetries

Fix	Asymmetry
FL	Line symmetry by center
CL	Line symmetry by corner
DL	Diagonal symmetry
FP	Point symmetry by center
EP	Point symmetry by edge
CP	Point symmetry by corner
F2	Two-way line symmetry by center
E2	Two-way line symmetry by edge
C2	Two-way line symmetry by corner
FD2	Two-way diagonal symmetry by center
CD2	Two-way diagonal symmetry by corner
F90	90-degree rotational symmetry by center
C90	90-degree rotational symmetry by corner
FA	Symmetry in all directions by center
CA	Symmetry in all directions by corner

	Fix*1	FL	CL*2	DL	FP	EP	CP	F90	C90
1side	1/4	0	0	0	1/4	2/4	1/4	2/4	2/4
2side	1/8	2/8	2/8	2/8	1/8	2/8	1/8	2/8	2/8
20		106004	36446	31822	7789	99783	39841	35	65
21		241203	0	120419	249283	0	0	57	0
22		409492	135268	120338	31348	386320	154642	0	0
23		928376	0	452420	964708	0	0	0	0
24		1587151	505861	457320	125802	1501661	602468	126	224
25		3586999	0	1709845	3748031	0	0	191	0
26		6169400	1903890	1745438	503948	5856748	2354592	0	0
27		13904736	0	6494848	14610484	0	0	0	0
28		24041597	7204874	6686929	2016677	22908230	9227412	461	790
29		54053950	0	24779026	57118440	0	0	658	0
30		93896826	27394666	25703792	8065830	89828072	36246300	0	0
31		210654990	0	94899470	223859532	0	0	0	0
32		367450477	104592937	99096382	32251819	353006402	142671941	1699	2851
33		822754494	0	364680344	879285686	0	0	2308	0
34		1440514144	400795844	383067646	128955260	1389925354	562600898	0	0
35		3219725534	0	1405619344	3460424846	0	0	0	0
36		5656283859	1540820542	1484352159	515657653	5482090062	2222092230	6315	10424
37		12622055937	0	5432421429	13642112667	0	0	8241	0
38		22242057564	5940738676	5764277096	2062335114	21655518914	8789217470	0	0
39		49559836758	0	21046198560	53865266960	0	0	0	0
40		87577573856	22964779660	22429257682	8250061654	85662816994	34809890792	23686	38496
41		194874338805	0	81716371069	212982833863	0	0	29853	0
42		345252222481	88983512783	87432657722	33011955188	339281765621	138027690188	0	0
43		767272498486	0	317917129256	843202561450	0	0	0	0
44		1362583793862	345532572678	341394729018	132132934138	1345314845818	547891969992	89432	143454
45		3024594142570	0	1239120776640	3342114787290	0	0	109268	0
46		5383141106732	1344372335524	1335080732960				0	0
47		11936178797526	0	4837744188806	13260804930438	0	0	0	0
48			20457802016011					339473	538667
49			0		52667141592300	0	0	403450	0

*1 Fixed:Use the results of the Iwan Jensen of A001168

*2 CL:Use A001168(n/2)

List of the number of Polyomino

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N	2-sided(A000105)	1-sided(A000988)
1	1	1
2	1	1
3	2	2
4	5	7
5	12	18
6	35	60
7	108	196
8	369	704
9	1285	2500
10	4655	9189
11	17073	33896
12	63600	126759
13	238591	476270
14	901971	1802312
15	3426576	6849777
16	13079255	26152418
17	50107909	100203194
18	192622052	385221143
19	742624232	1485200848
20	2870671950	5741256764
21	11123060678	22245940545
22	43191857688	86383382827
23	168047007728	336093325058
24	654999700403	1309998125640
25	2557227044764	5114451441106
26	9999088822075	19998172734786
27	39153010938487	78306011677182
28	153511100594603	307022182222506
29	602621953061978	1205243866707468
30	2368347037571252	4736694001644862
31	9317706529987950	18635412907198670
32	36695016991712879	73390033697855860
33	144648268175306702	289296535756895985
34	570694242129491412	1141388483146794007
35	2253491528465905342	4506983054619138245
36	8905339105809603405	17810678207278478530
37	35218318816847951974	70436637624668665265
38	139377733711832678648	278755467406691820628
39	551961891896743223274	1103923783758183428889
40	2187263896664830239467	4374527793263174673335
41	867273759121236342022 5	1734547518228643148 5513
42	344081766072795017795 92	6881635321429816936 2691
43	136585913609703198598627	273171827218863802383383
44	542473001706357882732070	108494600341169100991 6361
45	215560009110732422925 4415	431120018221251660104 9225
46		
47	340851055531238311581 80217	681702111062392753548 67268
48		
49		107983287733615453867 4417465