Four semi-chestnuts

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In collections of mathematical puzzles and when maths/puzzle people swap problems, there are quite a few problems that crop up again and again and these are of course chestnuts. The problem with the one hundred storey building and the glasses would be an example.

Some problems I first heard in the mid to late 80s seem to me as though they (to varying extents) ought to be more chestnutty than they are, and yet I encounter them much more rarely than I would expect to.

In the hope that they actually are relatively obscure and that G4G attendees agree with me that at least some of would deserve more of an airing, here they are. Note that they can all be done quite quickly (in a few seconds, indeed) without writing anything down. I freely admit that the first one is more silly than mathematical, and may be the best known of the four.

I was told in 2010 that (4) is the general version of a problem from the 1976 International (or possibly UK) Mathematical Olympiad. I would be interested to know how old these problems are.

1) What is the only integer whose name in English has its letters in alphabetical order?
   (For example, "two" is not the answer because t, w, o are not in alphabetical order.)

2) Please give an arithmetic progression of three integers whose product is prime.
   (For example, "10, 15, 20" is not the answer because 10*15*20 is not prime.)

3) I define a positive integer *n* to be "semi-one" if exactly 50% of the numbers from 1 to *n* (inclusive) "have a one in them", i.e. if they contain at least one digit "1" when written in base 10. For example, 2 is semi-one since 1 has a one in it, 2 does not, and 1 is 50% of 2. 16 is semi-1 because the eight numbers 1,10,11,12,13,14,15,16 have a one in them, the other eight do not, and eight is half of 16. Note again that a number with more than one "1" in it just counts as "having a one in it" the same as a number with just a single "1".

   The question is: Are there finitely or infinitely many semi-one numbers? And why? And a rider for those who say "finitely many" can be: "How large is the biggest one?" though I don't expect this can be done in a few seconds without writing anything down.

4) I give you a pile of *n* tokens (*n* is a positive integer) and tell you that if you divide it into sub-piles in any way you see fit (whole numbers of tokens in each sub-pile, though, please), I will multiply the sizes of the sub-piles together and give you that many pounds (or dollars, euros, etc. depending on where we are when we are doing this). If you choose to leave all the tokens in one pile I will just pay you *n* currency units.

   How do you maximise the amount I have to pay you? (I don't want to know what the amount is.) For example, if I give you 17 tokens you could split them into piles of 9 and 8 and I would have to pay you 72 dollars, but if you split them as 5,5,5,2 I would have to pay you 5*5*5*2 = 250 dollars.