CYCLIC PATHS INSIDE PLATONIC SHELLS

Imagine a billiard ball bouncing around inside a cube. What is the path the ball must follow in order to hit each wall once and return to its starting point?

Quoting Martin Gardner: “The ball is assumed to be an idealized particle (or a light ray inside a solid with interior mirror surfaces), taking straight paths in zero gravity and bouncing off the sides in the usual manner: with equal angles of incidence and reflection on a plane perpendicular to the side against which it bounces.”

When a tetrahedron is centered on the origin, a symmetry exists in the locations where such a ball bounces off the sides of a tetrahedron. Given: Tetrahedron vertices; (1,0,b);(-1,0,b); (0,-1,-b):(0,1,-b) where b = Sqr(2)/2. The bounce points are: Left side (-a,0,-c); Front (0,-a,c); Right side (a,0,-c); and Back (0,a,c) where 0<a<1 and c = Sqr(2)*(0.5 - a). An infinite number of paths exist. Three possibilities are shown. When a = 0.4 the path is perpendicular to each side. The angle of incidence or reflection is 39.2 degrees.

When a cube is centered on the origin, symmetry also exists. Cube vertices are (-1,-1,-);(-1,1,1);(1,1,1);(1,-1,1):(-1,-1,-1);(-1,1,-1);(1,1,_1);(1,-1,-1). The coordinates of the bounce points are: Top (a,-a,1); Front (-a,-1,a); Left (-1,-a,-a); Bottom (-2,2,-1); Back (a,1,-a); Right (1,a,a). Where 0<a<1). Again an infinite number of paths exist. When a = 1/3 the path is perpendicular to each side. The angle of incidence or reflection is 54.8 degrees.

Data for the octahedron, dodecahedron and icosahedron were obtained from Mathew Hudelson. There are two angles of incidence/reflection for the octahedron: 27.5 and 45.6 degrees. For the dodecahedron: 26.0 and 37.4 degrees. And for the icosahedron: 28.3 and 45.1degrees.

References: Martin Gardner, “Bouncing Balls In Polygons and Polyhedrons”, Martin Gardner’s Sixth Book of Mathematical Games from Scientific American, 1971,p.29
Clifford A. Pickover, “Platonic Billiards”, The Math Book, 2009, p.414. Referenced Lewis Carroll, Hugo Steinhaus, John Conway, Roger Hayward, and Mathew Hudelson.
Mathew Hudelson,”Periodic Omnihedral Billiards in Regular Polyhdra and Polytopes”1996. Unpublished.