

I met Nancy while helping out at one of these festivals. They're an exciting way to play with math. —SV

The Julia Robinson Mathematics Festival

by Nancy Blachman

It was March, 1972. My tenth-grade math teacher, Mr. Forthoffer, was handing out another problem set from Saint Mary's College. We didn't have to do them, so some kids just left them on their desks. I had so much fun with the ones we'd gotten earlier in the year that I couldn't wait to start working on these. The first few problems were usually pretty straightforward, and solving them would boost my confidence for tackling others. I never could solve all of them, but I enjoyed trying.

I liked experimenting—cutting a 10x10 square into two pieces to make a rectangle, plugging values into equations, learning more about the problems as I worked. Even after I solved a problem, I liked thinking about whether there was an easier way to solve it.

Students who scored high on that year's qualifying problems would be invited to the Saint Mary's Math Contest at the end of the school year. Schools all over the San Francisco Bay Area sent busloads of students. I didn't actually care much about going. For me, the fun was in the problems I was doing at home.

I went to the contest that year and the next two, but it was a disappointment. Sitting alone in that room all day, without being able to discuss my ideas with my father—it wasn't nearly as much fun as the problems I got from school. To this day I remember them with deep fondness.

When I teamed up with Josh Zucker and the Mathematical Sciences Research Institute (MSRI) in 2006 to create something new, I was determined to bring back the best of the now-defunct Saint Mary's Math Contest, and leave behind what didn't work. We decided to emphasize fun, creating a mathematics festival instead of another competition. The festival would have dozens of tables with mathematical problems,

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puzzles, games, and activities, each with a facilitator to help students stay connected with the math.

We wanted the festival to nurture students, so we let them work individually or in groups. We hoped to attract students at a wide range of abilities with math at all levels, so we chose problems and activities that would connect to one another. We started each set with simple problems everyone could work out, leading to progressively more difficult questions (we even included unsolved research problems). We hoped to have so many problems that not even a mathematical genius could solve all of them during the festival.

The festival needed a name to match its spirit. Julia Robinson was a great mathematician who, along with two other mathematicians, was renowned for solving Hilbert's tenth problem. She lived not far from us in the San Francisco Bay area, and was a distinguished mathematics professor at the University of California at Berkeley for many years, until her death in 1985. It felt perfect to honor her legacy with this festival.

In March 2007, the first year we ran the festival, we were concerned that we might not get many students to sign up, but within a few weeks the festival was oversubscribed. With more registrants than space, we asked our sponsor, Google, for a tent to accommodate more students. They came through, and the day of the festival started out sunny and chaotic.

It all fell into place, with hundreds of students eagerly approaching the problems we'd devised. There were thirty tables with activities, puzzles, games, and problems. When we announced that sandwiches were available for lunch, many of the kids would not stop working. We may not have managed to feed their bodies, but we surely fed their minds! The prizes from Google were icing on the cake.

The response was so enthusiastic that we've been able to make the festival an annual tradition. And we've grown, offering festivals in over a dozen locations—California (eight different locations), Connecticut, Washington D.C., Michigan, Texas, North Carolina, Arizona, Virginia, Washington (state), and Wyoming.

My goal was to inspire, delight, and challenge children, as the Saint Mary's Math Contest did for me, but with more collaboration and less competition. Thanks to its many sponsors and volunteers, the Julia Robinson Mathematics Festival is a success, and I've seen my dream come true.*

* You can find more information at: JuliaRobinsonMathFestival.org

PUZZLE

St. Mary's Math Contest Sampler

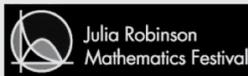
Adapted from problems by Brother Alfred Brousseau

1. A company named JULIA has an advertising display with just the five letters of its name, lit up in various colors. On a certain day the colors might be red, green, green, blue, red. The company wishes to have a different color scheme for each of the 365 days of the year. What is the minimum number of colors that can be used for this purpose?
2. How would you decide whether a number in base 7 is even, based on its digits?
3. Given the sequence 1, 2, 4, 5, 7, 8, 10, ... where every third integer is missing, find the sum of the first hundred terms in the sequence.
4. Find the sum of the cubes of the numbers from 1 to 13. Now find the sum of the cubes of the numbers from 1 to n .
5. Using exactly five 5's, and the operations $+$, $-$, \times , \div , and factorial (!), represent each of the numbers up to 30.

EXPLORATIONS

Candy Conundrum^{*}

by Joshua Zucker



Colors

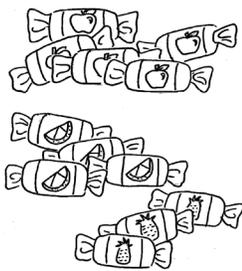
1. Some years ago, a candy company advertised the large number of flavors that could be made by mixing their candies in your mouth. How many are there really?
2. You have 5 red apple candies. How many different nonempty sets of candies could you put in your mouth?
3. You have 5 red apple and 4 green lime candies. How many different nonempty sets of candies could you put in your mouth?
4. You have 5 red apple, 4 green lime, and 3 yellow pineapple candies. How many different nonempty sets of candies could you put in your mouth?

Flavors

We'll consider two sets of candies to be the same flavor if the ratio of candies of each color in one set is the same as the ratio in the other. For instance, 2 green and 1 yellow is the same flavor as 4 green and 2 yellow; it's $\frac{2}{3}$ green, or a 2:1 green:yellow ratio. Similarly 3 red is the same flavor as 2 red: pure red!

5. You have 5 red candies. How many flavors could you make?
6. You have 5 red and 4 green candies. How many flavors could you make?
7. You have 5 red, 4 green, and 3 yellow candies. How many flavors could you make?

^{*} Find more problems at: JuliaRobinsonMathFestival.org



Geometry

There's a geometric interpretation of all of the above. For instance, with 5 red and 4 green candies, the possible combinations are ordered pairs. (2,1) and (4,2) are the same flavor, for example.

8. Expressed geometrically, what does it mean for two different sets of candies to be the same flavor? Assume for now that there are only two colors.
9. Describe a geometric way of understanding how many different flavors there are. Compare it to the numeric approach.
10. What does symmetry tell you about the number of flavors with k red candies and k green candies?

Generalizing

Now let's try for some bigger patterns.

11. If you have 1 candy of each of n colors, how many different flavors are possible? If you have k candies of 1 color, how many different flavors are possible? OK, sorry, that was too easy.
12. If you have 2 candies of each of n colors, how many different flavors are possible? If you have k candies of each of 2 colors, how many different flavors are possible?
13. Generalize as much as you can!
14. How do the previous answers change if the candies are large, with an upper limit to how many fit in your mouth at once?
15. What can you say about the relative probability of various flavors if you pick a random handful of size n out of a set of candies? Start by considering some easy cases, where n is small, and there aren't too many different flavors, and plenty of candies of each flavor (since n will limit you, it gets more complicated if you also have limits due to running out of candies).
16. What other questions can you think of about how to count combinations of candies?