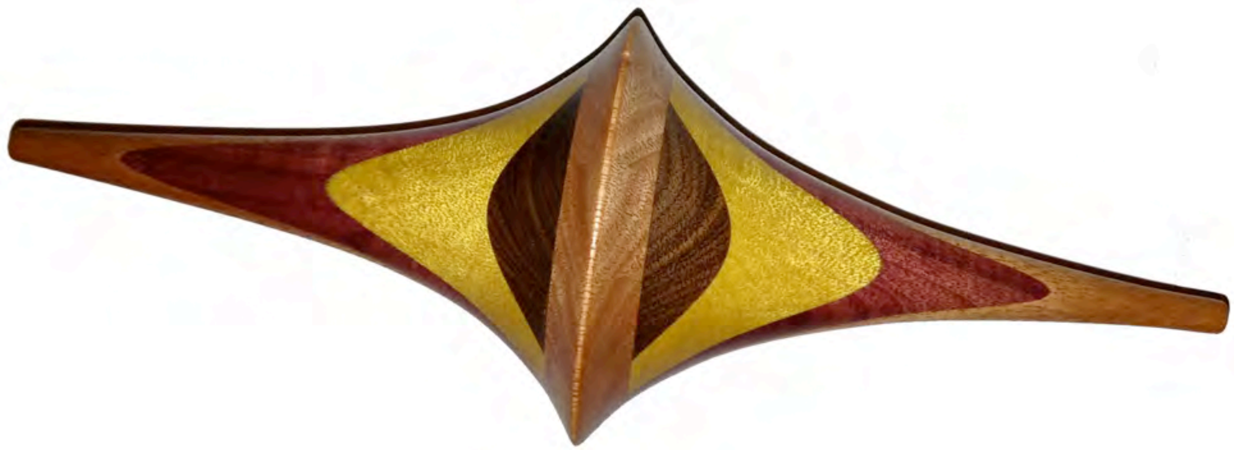
The Pseudosphere Uphill Roller



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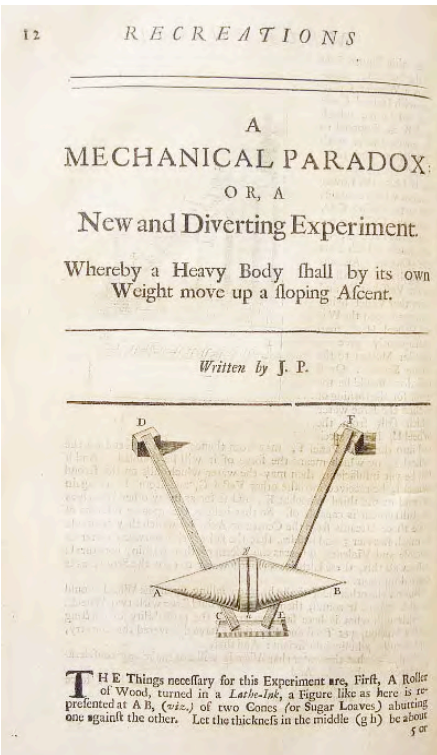
The Uphill Roller:

The "Uphill Roller" is a beautiful physics demonstration first reported by English mathematician William Leybourn in 1694. In the original version, a double cone placed on two divergent inclined ramps appears to roll "uphill", apparently violating the laws of physics.



12 RECREATIONS

A
MECHANICAL PARADOX:
OR, A
New and Diverting Experiment.

Whereby a Heavy Body ſhall by its own Weight move up a ſloping Aſcent.

Written by J. P.

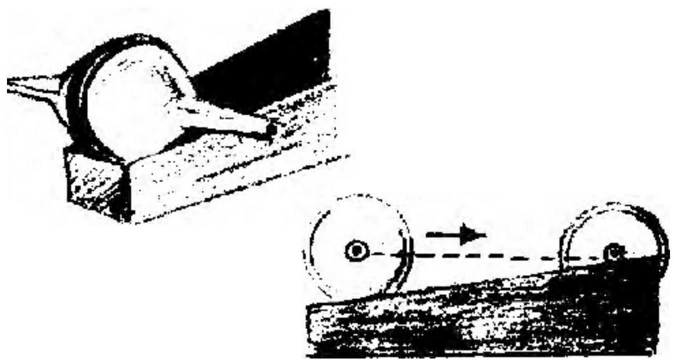
THE Things neceſſary for this Experiment are, Firſt, A Roller of Wood, turned in a *Lathe-Ink*, a Figure like as here is repreſented at A B, (*viz.*) of two Cones (or Sugar Loaves) abutting one againſt the other. Let the thickneſs in the middle (g h) be about

5 or

Left: 1694 description of "A Mechanical Paradox" employing a double cone; **Center**: 18th century wood and brass version in the Museo Galileo in Florence, Italy; **Right**: Portrait of William Leybourn.

Martin Gardner Cone Version

ANTI-GRAVITY CONE



This curious little toy seems to defy gravity. When it is placed at the bottom of a sloping track, it appears to run uphill!

The toy is a double cone, easily made from two plastic funnels. Use rubber cement to stick their rims together. The sloping track is cut from cardboard. You will have to experiment to get the slope just right, since the gradient will depend on the size of the funnels.

Arrange the track so the two sides are about an inch apart at the lower end, with a width at the other end equal to the length of the double cone. When the cone is placed at the bottom of this track, it slowly rolls to the top. Observe the cone carefully from the side and you will see what really happens. As the cone moves 'up,' the increasing width of the track lowers the cone so that its centre of gravity actually moves down.

(From Science Puzzles by Martin Gardner, illustrated by Anthony Ravielli.)

Martin Gardner Ball Version

Physics Trick of the Month

The Ball that Rolls Uphill

For this amusing demonstration you need two new wooden pencils, a deck of cards, and a table-tennis ball.

Divide the deck in half, placing the halves side by side as shown in Fig. 1. Place the pencils on the half-decks, their eraser ends touching and the other ends on the table about 1¼ inches apart. Put the ball at the center of the V to show how it rolls downhill and off the pencils to the table.

Move the half-decks so they are 1¼ inches apart as shown in Fig. 2. Rearrange the pencils so their eraser ends touch on the table, and their other ends are 1¼ inches apart, each resting on the inside edge of a half-deck. Place a weight (the edge of a book will do) on the eraser ends to hold them firmly in place.

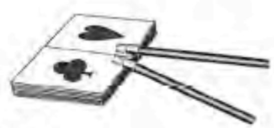


Fig. 1.

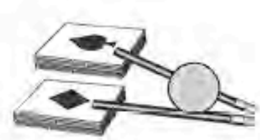


Fig. 2.

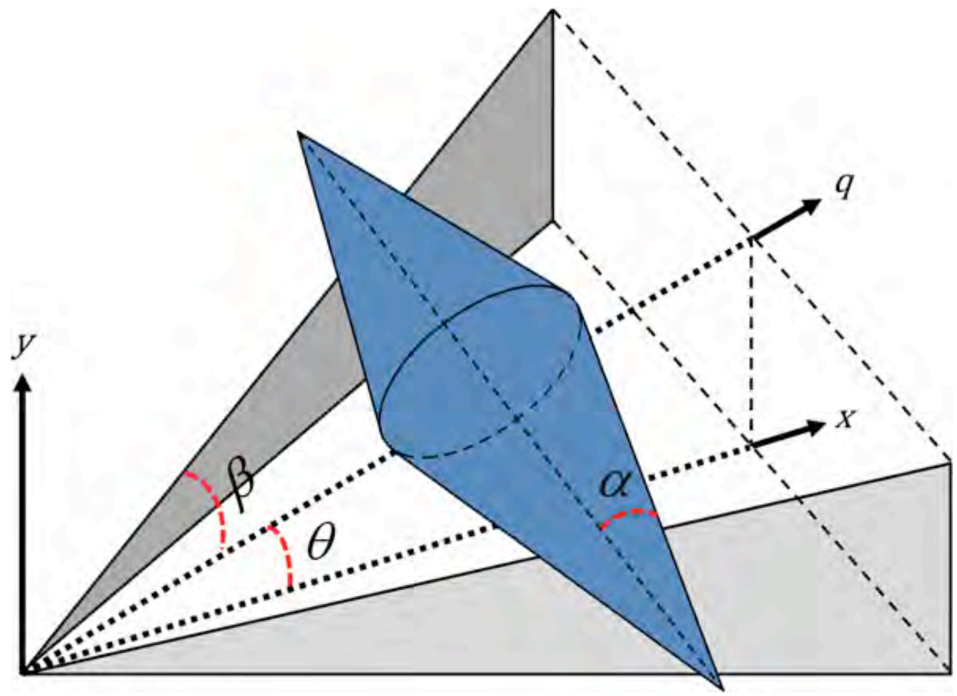
Place the ball in the middle of the pencils as shown. Which way will it roll? Because the cards are raising the ends of the pencils a fourth of an inch, students are likely to guess that the ball, as before, will roll toward the pencils' lower ends.

Surprisingly, it doesn't. It seems to roll *uphill* toward the pencils' higher ends! Actually, of course, the ball's center of gravity *lowers* as it moves toward the open end of the V, so it is really rolling downhill.

Martin Gardner, *Hendersonville, NC 28792*

(*From*: The Physics Teacher, **34**, 461, 1996.)

Uphill Rolling Condition



Design of a double cone and rails. Motion is *mainly* determined by the angles: α, β, and θ. The condition for the double cone to roll uphill is:

$$\tan \theta < \tan \alpha \tan \beta.$$

(From the paper: "Mechanical Paradox", European Journal of Physics, **32**, 1559, 2011 by Emilio Cortes and D. Cortes-Poza. However, a more detailed treatment shows that the actual spatial parameters also appear in the solution.)

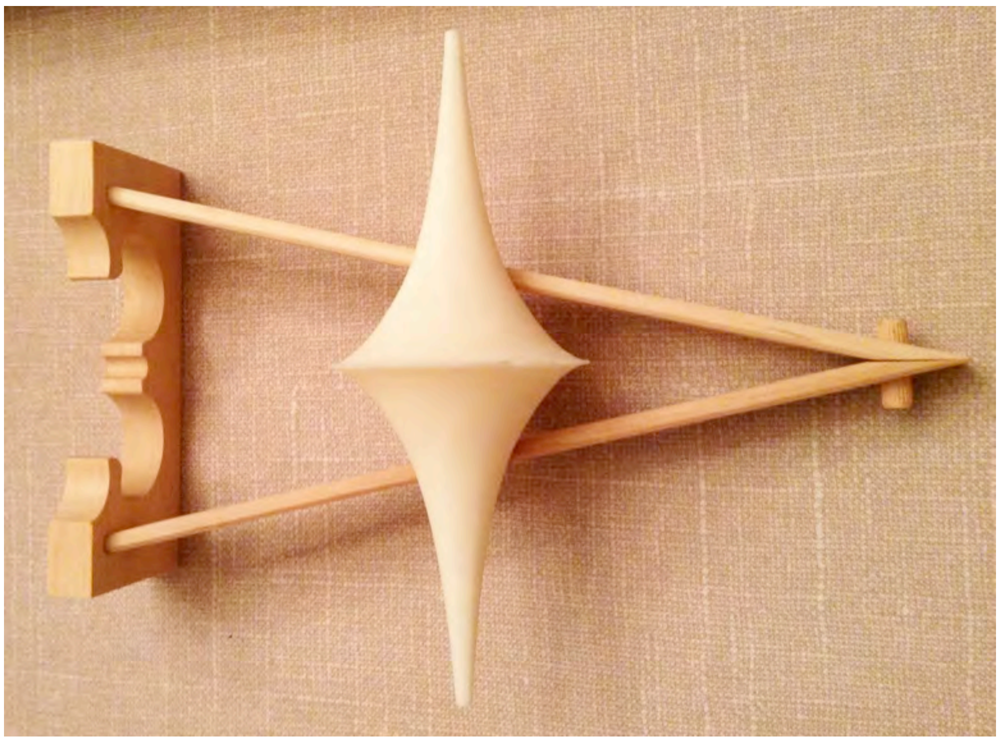
Classic Uphill Roller Demonstrations

The uphill roller is an example of the center of mass of an object *descending* under the influence of gravity. Over the past three centuries, this demonstration has remained essentially the same. It has been incorporated into art and science museum exhibits. It is also utilized in many – if not most – introductory physics courses.

Set of uphill rollers and ramps made for KB by wood artist/craftsman Randy Rhine.

The Pseudosphere Uphill Roller: A New Scientific Demonstration

By replacing the double cone with a pseudosphere, something *really* counter-intuitive can occur. Depending on the angles of inclination and divergence of the ramps, the pseudosphere can ascend *or* oscillate back and forth, reaching an equilibrium position near the center of the ramp.



Summary

A new “uphill roller” demonstration that utilizes a pseudosphere instead of a double cone has been devised (by KB). The detailed theory of the motion is left as an exercise for oscillatory individuals.

Acknowledgments

My thanks to former B.U. undergraduate Geshan Weerasinghe who programmed the rapid prototype pseudosphere file, to B.U. engineer David Campbell for fabricating our plastic version, and to Randy Rhine for designing and making the elegant wood pseudosphere and related objects shown in the presentation. The .stl files for the models will be posted on the web site for “Project LITE: Light Inquiry Through Experiments”:

http://lite.bu.edu

Project LITE was supported in part by NSF Grant # DUE - 0715975.