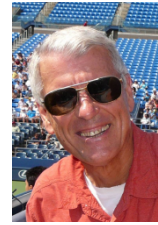


**ELMO'S  
EXCELLENT  
EXCURSION**  
ENGAGING EXERCISES FOR  
EAGER ENTHUSIASTS  
BY DICK HESS



**PUZZLES AND  
RECREATIONAL  
MATH**

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**Introduction:** These 26 puzzles are prepared for the Gathering for Gardner 11 held in Atlanta, from 19 March to 23 March 2014. Elmo is an eager math enthusiast who works through each of the 26 problems to create a final enlightened ending. Find solutions **A** to **Z** to duplicate Elmo's feat.

Enjoy Elmo's excellent excursion.

**Happy puzzling from Dick Hess.**

**A**

Elmo participates in a race with 7 others on a straight track. He overtakes the runner in second place.

**A** = the place number he is now in.

**B**

**B** = the largest known integer such that its square differs from a factorial by one.

**C**

**C** = the number of different integer pairs,  $(m,n)$ , with  $m < n$  such that  $m+n+mn = 2012$ . Negative integers are allowed for  $m$  and  $n$ .

**D**

Recently Elmo was married on the first day of the month; on the last day of the month his wife gave him a chess set for his birthday. He was married and received the chess set on the same day of the week he was born. **D** = Elmo's age when he got the chess set.

## E

There are 9 slips of paper with "7" on five of them and "18" on the other four. Five of the slips go on the hats of logicians A, B, C, D and Elmo (who is blindfolded) in some order. The other four slips remain hidden. Logicians A to D can see the numbers on the others' hats but not their own. The logicians are error-free in their reasoning and have all the information given so far. They are asked in turn to identify their number.  
A: "I don't know my number."; B: "I don't know mine."  
C: "I don't know mine."; D: "I don't know mine."  
**E** = Elmo's number.

## G

Elmo bought 7 toys for his nieces costing 750, 810, 900, 1230, 1740, 2610 and 2850 cents. Two nieces received toys totaling the same number of cents. **G** = that number.

## I

Two circular disks have radii of 333 and 372. The larger is held fixed while the smaller rolls around the outside of the larger disk without slipping. At the start, point P on the smaller disk coincides with point Q on the larger disk. **I** = the number of rotations of the smaller disk when points P and Q next coincide.

## K

Elmo loads a wagon with 574 kilograms of watermelons to take to market. At the start of the trip they are 99% water but they partly dry out through evaporation during travel so that by the end of the trip they are 98% water. **K** = the weight of the watermelons in kilograms at the end of the trip.

## F

Find the rule for combining numbers as shown below. In the diagram 34 and 16 combine to produce 18; 32 and 18 combine to produce 14 and so forth. Use the rule to determine **F**.

34	32	36	46	64	75	50	35	34	
16	18	14	22	<b>F</b>	40	35	15	20	12

## H

Elmo's uncle, who is younger than 70, notes that his age in years is of the form  $p^2q$ , where p and q are different prime numbers. A year ago his age was of the same form. **H** = his age.

## J

Each of logicians A, B and Elmo wears a hat with a positive integer on it. One hat has a number that is the sum of the numbers on the other two. They see the numbers on all hats except their own. They are error-free in their reasoning and have all the information given so far. A states "I don't know my number." B states "My number is 720." **J** = the number on Elmo's hat.

## L

Elmo has chosen four positive integers, a, b, c and d and written five of the six sums a+b, a+c, a+d, b+c, b+d and c+d. The written sums are 11, 28, 35, 49 and 56. **L** = the largest of Elmo's four original numbers.

## M

Elmo has 1400 red balls and 1400 blue balls. He places the largest number of balls possible in an urn so that if two are picked at random without replacement then the chances are exactly even that they will match in color.  $M$  = half the number of balls initially in the urn.

## N

Elmo and Andy wish to buy a computer game and each has a positive whole number of dollars. Elmo finds he is \$4 short; Andy finds he is \$64 short. If they pool all their money they still are unable to buy the computer game.  $N$  = the price of the computer game.

## O

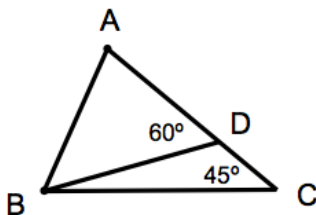
Elmo notices the clock's second hand is exactly on a second mark and exactly 18 second marks ahead of the hour hand.  $O$  = the number of minutes past the hour.

## P

Elmo gives the proof sheets of his new book to two different proof-readers. The first reader finds 86 typos; the second finds 140. Strangely though, only 43 typos are found by both.  $P$  = the expected number of typos not found by either proof-reader.

## Q

The triangle has  $AD = 2$  and  $CD = 1$ . Without using trigonometry Elmo has found  $Q$  = angle  $BAD$  in degrees.

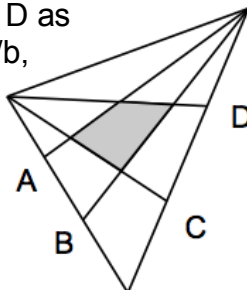


## R

Elmo, his young son, Elmo, Jr., and their dog, Ace, run a race of 1000 meters. Elmo beats Elmo, Jr. by 425 meters and Ace beats Elmo by 520 meters. If each runs at a constant speed then  $R$  = the number of meters Elmo, Jr. is beaten by Ace in the race.

## S

The triangle has unit area with sides trisected at points A, B, C and D as shown. The shaded area is  $a/b$ , reduced to lowest terms.  $S = 10 \cdot b + a$ .



## T

Elmo has found a number such that if its square and its cube are written down each digit from 0 to 9 appears exactly once.  $T = 300 +$  Elmo's number.

## U

Elmo takes a cruise of 4900.5 nautical miles from Panama to Honolulu. His boat starts from rest in Panama and proceeds at a constant 1 knot per hour to Honolulu.  $W$  = the number of hours the cruise takes.

## V

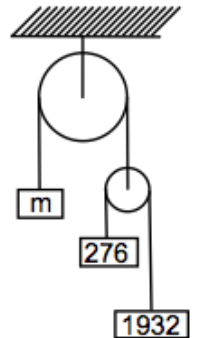
Elmo has a lampshade in the shape of a regular dodecahedron and notices a fly has settled on one vertex. It randomly chooses an edge at that vertex and walks along it to an adjacent vertex, arriving in 17 sec. It continues to repeat the process, taking 17 sec. for each step and possibly backtracking.  $V$  = the expected time in seconds to reach the vertex directly opposite the starting vertex.

## W

Elmo's fish pond has a horizontal bottom and vertical sides. To make a platform he places three identical, impermeable cubical blocks side by side resting on the bottom. Placing each block causes the pond's water level to rise. The second and third blocks each cause the level to rise 111 mm. The first block causes a smaller rise in water level.  $W$  = how many mm. the water level is above the top of the completed platform.

## X

An ideal pulley system has weightless and frictionless elements except for the weights shown. The weight,  $m$ , is chosen so that in the first few seconds after release from rest it will neither go up nor down.  $X = 1001m + 1$ .



## Y

Elmo takes his crop of pumpkins to the wholesale market. He sells the 42 lightest pumpkins to customer A and notes that they account for 25% of the total weight. He sells the 50 heaviest pumpkins to customer B and notes that they account for 30% of the total weight.  $Y$  = the number of pumpkins originally in the crop +  $J$  + 100.

## Z

Elmo is inspecting a right triangle and an isosceles triangle each having integer sides. The sides of the right triangle have no common divisor. Elmo notices that each of the two triangles has area  $a$  and each has perimeter  $p$ .  $Z = 30a + 13p + 44$ .

# SOLUTIONS

## SOLUTIONS

A = 2	I = 235	R = 724
B = 71	J = 360	S = 709
C = 8	K = 287	T = 369
D = 28	L = 47	U = 99
E = 18	M = 1352	V = 595
F = 28	N = 66	W = 74
G = 4590	O = 24	X = 966967
H = 45	P = 97	Y = 627
	Q = 75	Z = 724076

## ENLIGHTENED ENDING

Placing the solutions in order produces Euler's number, the base of natural logarithms,

$e=2.718281828459045235360287471$   
3526624977572470936999595749669  
67627724076...

Elmo hopes you are Ecstatic, Ebullient,  
Exhilarated and Enthralled