When is the next Thankgivukkah?

Doron Levy[1]

In the October 8, 2013 issue, the Boston Globe stated that the next time the first day of the Jewish holiday of Hanukkah will coincide with the American observance of Thanksgiving is "79,043 years from now, by one calculation" [1]. In contrast, on November 22, 2013, National Geographic stated that we will have to wait "until the year 79,811" for the next occurrence of Thanksgivukkah [2]. How come two respectable news outlets managed to provide similar yet different answers to the same calculation?

In principle, calculating the next occurrence of Thanksgivukkah should be a rather straightforward task. For any given year, one has to (i) calculate the date of the American thanksgiving, which is defined as the fourth (and not the last) Thursday in November; (ii) calculate the date of the first day of Hannukkah; and (iii) check if these two dates match. Instead of resolving the discrepancy in this calculation, I prefer to leave it as a challenge. Accordingly, this note will discuss some issues that one may wish to take into account in carrying out this task.

The holiday of Hannukkah is observed 8 days starting on the 25th day of the month of Kislev according to the Hebrew calendar. It is important to note that the Jewish day does not begin on midnight, but on the sunset before it. This means that every Jewish day overlaps with two "standard" days. While the first day of Hannukkah of the Jewish year 5774 coincided with Thanksgiving 2013, the first candle was actually lit on Wednesday night - as this is the beginning of the day of Thursday according to the Jewish tradition.

A Hebrew year consists of 12 months in a common year and 13 in a leap year. In the Hebrew calendar, leap years occur in years 3, 6, 8, 11, 14, 17, and 19 of a 19-year cycle. In a leap year, the month of Adar is renamed as Adar II and has 29 days instead of 30 days. An additional 30-days month (Adar I) is added before Adar II. Another confusing aspect of the Hebrew calendar is that there are two months (Heshvan and Kislev) that may have 29 or 30 days, depending on various factors. In some rare cases, the leap year can be extended by an extra day by moving the New Year holiday of Rosh Hashana one day forward. A comprehensive discussion can be found in [8, Chapter 7].

Gauss derived a formula for the date of the holiday of Passover, which falls on the first day of the month of Nisan (see, e.g., [10]). Given Gauss formula for the date of Passover it is a reasonable adjustment to compute the first day of Hannukkah, as all that is required is to figure out whether it is a leap year, and what is the exact number of days in the month of Kislev. These factors determine the number of days that separate Hannukkah from Passover.

How do we know when is Thanksgiving? Clearly, for any given year, if we know the day of the week of November 1, we can easily find the date of the fourth Thursday in November. Many algorithms were devised to find the day of the week for a given day. A rather comprehensive list can be found at [4]. Here I would like to mention three examples of interest to the attendees of the G4G meetings:

1. Corinda on page 71 of "13 steps to mentalism" presents a method for "A day for any date". This method is described as a mentalism effect in which "The performer invites members of his audience to call out any date they like; upon hearing the date, the performer gives the exact day of the week that that date falls on and delivers his reply within seconds. Everything is achieved by a quick calculating mental system."

   The algorithm that Corinda provides is one of the easier algorithms to memorize and mentally compute, which makes it suitable for stage demonstrations. It can be summarized as follows: (i) Add a quarter to the value of the last two digits of the year. (ii) Add the code value for the month (Table 1). (iii) Add the day of the month. (iv) Divide the total by seven. (v) The remainder tells you the day according to Table 1. This algorithm gives the correct date for years 1801-1900. When the date is in the twentieth century you must deduct two from the final remainder and when the date occurs in the eighteenth century, two should be added to the final remainder.

[1]Department of Mathematics and Center for Scientific Computation and Mathematical Modeling (CSCAMM), University of Maryland, College Park, MD 20742.

June	0
September, December	1
January (leap year), April, July	2
January, October	3
May	4
February (leap year), August	5
February, March, November	6

Sunday	1
Monday	2
Tuesday	3
Wednesday	4
Thursday	5
Friday	6
Saturday	0

Table 1: Months and Days for Corinda's algorithm

2. Lewis Carroll published an algorithm "to find the day of the week for any given date" [6]. This algorithm was discussed by Martin Gardner in [9]. A copy of Carroll's paper is shown in Figure 1.

3. The Doomsday Rule. The doomsday is the day of the week of the last day of February. The Doomsday Rule is a method devised by John Conway for computing the day of the week of any given date. A nice explanation of this algorithm can be found in the notes by Graham [3].

All algorithms for calendar calculations must account for leap years. The Julian calendar, imposed by Julius Caesar in 45 B.C., following the advice of Sosigenes, the Alexandrian astronomer, was based on a 4-year leap year cycle – one day being added every four years. The current calendar, the Gregorian calendar, was instituted by the Pope Gregory XIII in 1582. According to the Gregorian calendar, a leap year is a year that is divisible by 4, unless it is divisible by 100, unless it is divisible by 400. Leap years were established in order to provide occasional corrections to the length of the year, so that the calendar follows the "astronomical" year.

The astronomical Almanac online [5] defines a tropical year as the period of time for the ecliptic longitude of the Sun to increase 360 degrees. Since the Sun's ecliptic longitude is measure with respect to the equinox, the tropical year comprises a complete cycle of seasons, and its length is approximated in the long term by the civil (Gregorian) calendar. The mean tropical year is approximately 365 days, 5 hours, 48 minutes, 45 seconds. This estimate corresponds to approximately 365.24219 days. This value is actually does not remain constant as it changes over time due to a variety of reasons.

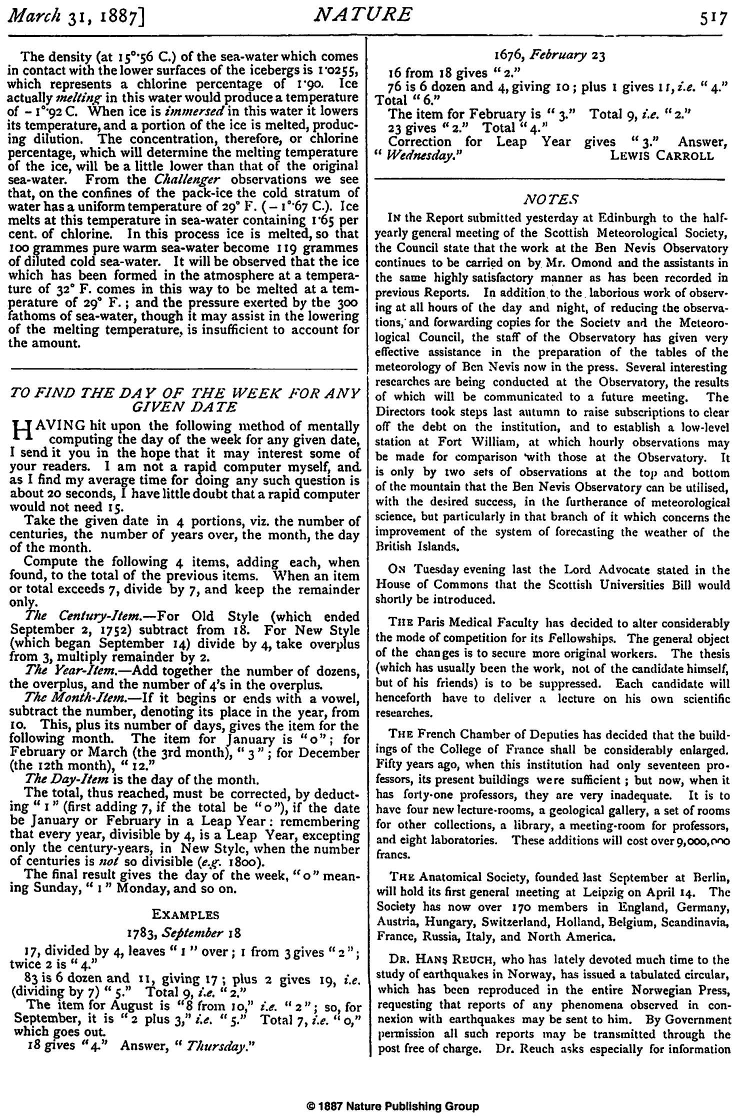
Leap years are nothing but a correction to the calendar in order to avoid too big of a drift between the calendar and the tropical year. In the Julian calendar, the average length of a year is 365.25. This is improved by the Gregorian calendar in which the average length of a year is 365 + 97/400 = 365.2425, a better approximation of 365.2422... The Julian or Gregorian calendars are not the only way to correct approximate the length of the year by adding days. One interesting alternative, involves approximating 365.24219 as a continued fraction:

$$365.24219 \approx 365 + \cfrac{1}{4 + \cfrac{1}{7 + \cfrac{1}{1}}} = 365\frac{8}{33} = 365.2424...$$

In practice, such an approximation can be implemented by defining 8 years in a 33-year cycle as leap years. The specific choice of which 8 years is of no importance. This is exactly the correction proposed by the Persian mathematician, philosopher and poet, Omar Khayyam: a 33-year cycle where the years 4,8,12,16,20,24,28, and 33 are leap years [11]. An even more accurate approximation can be obtained, e.g., by adding one term to the continued fraction (1/1 will be replaced by $1/(1 + 1/3)$). A calendar based on the resulting fraction (31/128) will correspond to an average year of 365.2421875 days.

Is there any real need for more accurate corrections? The error in the Gregorian calendar is approximately $|365.24219 - 365.2425| = 0.00031$ days per year. In comparison, the error in the Khayyam calendar is $|365.24219 - 365.2424242...| = 0.00023424...$ days per year. This means that the Gregorian calendar shifts by one day in approximately 3225 years, while the Khayyam calendar shift by one day in approximately 4269 years, rendering it a better approximation.

If we have to wait about 80,000 years for the next occurrence of Thanksgivukkah, multiple corrections will have to be made to the calendar (and in fact also to the Hebrew calendar). This means that any *correct* answer to the challenge, is guaranteed to be *wrong*...



March 31, 1887] NATURE 517

The density (at 15°·56 C.) of the sea-water which comes in contact with the lower surfaces of the icebergs is 1·0255, which represents a chlorine percentage of 1·90. Ice actually *melting* in this water would produce a temperature of – 1°·92 C. When ice is *immersed* in this water it lowers its temperature, and a portion of the ice is melted, producing dilution. The concentration, therefore, or chlorine percentage, which will determine the melting temperature of the ice, will be a little lower than that of the original sea-water. From the *Challenger* observations we see that, on the confines of the pack-ice the cold stratum of water has a uniform temperature of 29° F. (– 1°·67 C.). Ice melts at this temperature in sea-water containing 1·65 per cent. of chlorine. In this process ice is melted, so that 100 grammes pure warm sea-water become 119 grammes of diluted cold sea-water. It will be observed that the ice which has been formed in the atmosphere at a temperature of 32° F. comes in this way to be melted at a temperature of 29° F.; and the pressure exerted by the 300 fathoms of sea-water, though it may assist in the lowering of the melting temperature, is insufficient to account for the amount.

TO FIND THE DAY OF THE WEEK FOR ANY GIVEN DATE

HAVING hit upon the following method of mentally computing the day of the week for any given date, I send it you in the hope that it may interest some of your readers. I am not a rapid computer myself, and as I find my average time for doing any such question is about 20 seconds, I have little doubt that a rapid computer would not need 15.

Take the given date in 4 portions, viz. the number of centuries, the number of years over, the month, the day of the month.

Compute the following 4 items, adding each, when found, to the total of the previous items. When an item or total exceeds 7, divide by 7, and keep the remainder only.

The Century-Item.—For Old Style (which ended September 2, 1752) subtract from 18. For New Style (which began September 14) divide by 4, take overplus from 3, multiply remainder by 2.

The Year-Item.—Add together the number of dozens, the overplus, and the number of 4's in the overplus.

The Month-Item.—If it begins or ends with a vowel, subtract the number, denoting its place in the year, from 10. This, plus its number of days, gives the item for the following month. The item for January is "0"; for February or March (the 3rd month), "3"; for December (the 12th month), "12."

The Day-Item is the day of the month.

The total, thus reached, must be corrected, by deducting "1" (first adding 7, if the total be "0"), if the date be January or February in a Leap Year: remembering that every year, divisible by 4, is a Leap Year, excepting only the century-years, in New Style, when the number of centuries is *not* so divisible (*e.g.* 1800).

The final result gives the day of the week, "0" meaning Sunday, "1" Monday, and so on.

EXAMPLES

1783, *September* 18

17, divided by 4, leaves "1" over; 1 from 3 gives "2"; twice 2 is "4."

83 is 6 dozen and 11, giving 17; plus 2 gives 19, *i.e.* (dividing by 7) "5." Total 9, *i.e.* "2."

The item for August is "8 from 10," *i.e.* "2"; so, for September, it is "2 plus 3," *i.e.* "5." Total 7, *i.e.* "0," which goes out.

18 gives "4." Answer, "*Thursday.*"

1676, *February* 23

16 from 18 gives "2."

76 is 6 dozen and 4, giving 10; plus 1 gives 11, *i.e.* "4." Total "6."

The item for February is "3." Total 9, *i.e.* "2."

23 gives "2." Total "4."

Correction for Leap Year gives "3." Answer, "*Wednesday.*"

LEWIS CARROLL

NOTES

IN the Report submitted yesterday at Edinburgh to the half-yearly general meeting of the Scottish Meteorological Society, the Council state that the work at the Ben Nevis Observatory continues to be carried on by Mr. Omond and the assistants in the same highly satisfactory manner as has been recorded in previous Reports. In addition to the laborious work of observing at all hours of the day and night, of reducing the observations, and forwarding copies for the Societv and the Meteorological Council, the staff of the Observatory has given very effective assistance in the preparation of the tables of the meteorology of Ben Nevis now in the press. Several interesting researches are being conducted at the Observatory, the results of which will be communicated to a future meeting. The Directors took steps last autumn to raise subscriptions to clear off the debt on the institution, and to establish a low-level station at Fort William, at which hourly observations may be made for comparison with those at the Observatory. It is only by two sets of observations at the top and bottom of the mountain that the Ben Nevis Observatory can be utilised, with the desired success, in the furtherance of meteorological science, but particularly in that branch of it which concerns the improvement of the system of forecasting the weather of the British Islands.

ON Tuesday evening last the Lord Advocate stated in the House of Commons that the Scottish Universities Bill would shortly be introduced.

THE Paris Medical Faculty has decided to alter considerably the mode of competition for its Fellowships. The general object of the changes is to secure more original workers. The thesis (which has usually been the work, not of the candidate himself, but of his friends) is to be suppressed. Each candidate will henceforth have to deliver a lecture on his own scientific researches.

THE French Chamber of Deputies has decided that the buildings of the College of France shall be considerably enlarged. Fifty years ago, when this institution had only seventeen professors, its present buildings were sufficient; but now, when it has forty-one professors, they are very inadequate. It is to have four new lecture-rooms, a geological gallery, a set of rooms for other collections, a library, a meeting-room for professors, and eight laboratories. These additions will cost over 9,000,000 francs.

THE Anatomical Society, founded last September at Berlin, will hold its first general meeting at Leipzig on April 14. The Society has now over 170 members in England, Germany, Austria, Hungary, Switzerland, Holland, Belgium, Scandinavia, France, Russia, Italy, and North America.

DR. HANS REUCH, who has lately devoted much time to the study of earthquakes in Norway, has issued a tabulated circular, which has been reproduced in the entire Norwegian Press, requesting that reports of any phenomena observed in connexion with earthquakes may be sent to him. By Government permission all such reports may be transmitted through the post free of charge. Dr. Reuch asks especially for information

Figure 1: Lewis Carroll's Nature paper on finding the day of the week for any given date

References

[1] http://www.bostonglobe.com/news/nation/2013/10/07/convergence-hanukkah-thanksgiving-unleashes-creativity/MxbqB0YtKA2b8AtR7XGwsL/comments.html

[2] http://news.nationalgeographic.com/news/2013/11/131124-thanksgivukkah-thanksgiving-hanukkah-calendar-turkey-potatoes/

[3] http://people.cst.cmich.edu/graha1sw/Pub/Doomsday/Doomsday.html

[4] http://en.wikipedia.org/wiki/Determination_of_the_day_of_the_week

[5] http://asa.usno.navy.mil/SecM/Glossary.html

[6] Lewis Carroll, To Find the Day of the Week for Any Given Date, Nature, March 31, 1887.

[7] Tony Corinda, 13 Steps to Mentalism, Robbins & Co.

[8] Nachum Dershowitz and Edward Reingold, Calendrical Calculations, 3rd Edition, Cambridge University Press, 2008.

[9] Martin Gardner, The Universe in a Handkerchief: Lewis Carroll's Mathematical Recreations, Games, Puzzles, and Word Plays, pp.24–26, Springer-Verlag, 1996

[10] Zvi Har'El, Gauss Formula for the Julian Date of Passover, http://www.math.technion.ac.il/S/rl/docs/gauss.pdf

[11] Rickey, V.F., Mathematics of the Gregorian Calendar. The mathematical Intelligencer, **7** (1985), pp.53–56.