Mixed Hands Blackjack
A New Blackjack Problem

Bill Liles
lilesw@gmail.com

June 25, 2016

1 Statement of the Problem

This problem follows standard Blackjack rules except for one exception. In standard Blackjack, a player may play more than one hand but the player is not allowed to move cards from any other there hands to any other of their hands.

In this version of the game, the player is allowed to pool all the cards in all of their hands and redistributed them into the same number of hands consisting of two cards each. For example if the player receives two hands with hand one containing an Ace and a three and hand two contains a ten and a seven, the player could rearrange the cards so that one hand now contains an Ace and a ten (Blackjack) and the other had contains a three and a seven for a total of ten. But once the hands have be rearranged, they cannot be rearranged again.

The problem is what is the best strategy for rearranging the cards to maximize winnings.

2 Variations

There are a number of variations to the above problem, some are which are listed here. For example, the player could rearrange the cards so that some of the new hands can have more than two cards. Or there could be multiple players and they could interchange cards between themselves, which include buying and selling cards. Or the dealer is also allowed to have multiple hands, in which case, which hand do the players play against? Or hands that have not busted are allowed to be rearranged multiple times. They are many such variants.

3 Relationship to casino games

In the problem formulated above, the player plays multiple hands and can rearrange the cards amongst those hands. There is a version of this played in casinos, called Blackjack Switch or Blackjack Swap. In this version of the game, the player plays two hands and is allowed to interchange the second card dealt to each hand.

In our version, the player can play more than two hands and all cards can be rearranged.

4 Terminology

In this paper, the following terminology will be used.

(1) A,2,3,4,5,6,7,8,9,X represents the numeric value of a card, where A represents Ace and X represents a 10 count (i.e. a ten, Jack, Queen or King).

(2) M, N, P, Q, R, S represent arbitrary cards

(3) H represents the number of hands

(4) C represents the number of cards in all hands $C = 2 * H$

(5) ! represents factorial $n! = n * (n - 1) * ... * 1$

(6) !! represents double factorial $n!! = n * (n - 2) * ... * 1$ for n odd

(7) ? represents summation of integers from 1 to n expressed as $n?$

(8) $R(n)$ represents the number of possible hands arrangements for n cards.

(9) $P(n)$ represents the number of distinct hands

5 Number of possible hands orderings

Assuming one could not rearrange the cards between hands but could order the hands will be played in, how many such orderings are there? For example assume the first hand dealt contained A2 and the second hand contained 34. By reordering the hands, the first hand will now contain 34 and the second hand contain A2.

This is the same question as how many ordering are there of n objects. In this case the number of objects is H so the number of orderings in $H!$.

In the actual play of a game, if the player is card counting, this could have a slight theoretical advantage.

6 Number of possible rearrangements

Given C cards and that each hand must have two cards in the hand, how many arrangements are possible. Examples will be given for 1,2 and 3 hands.

Remember that the order of the cards in a Blackjack hand are irrelevant.

In the following examples, the cards within a hand are listed vertically and the hands are spaced apart horizontally. A small horizontal gap means that the hands are one arrangement. A larger horizontal gap signifies a new arrangement.

Example for one hand using cards M and N.

M
N

For one hand there is only one arrangement. $R(2) = 1$

Example for two hands using cards M, N, P and Q.

M	P		M	N		M	N
N	Q		P	Q		Q	P

For two hands there are three arrangements. $R(4) = 3$
For three hands we can generalize from two hands using cards M,N,P,Q,R and S. Notice that card M is paired with cards N, P and Q in the different arrangements. So given the initial hands as being:

M	P	R
N	Q	S

Card M will be paired with cards N,P,Q,R and S. That is five different pairings. For each pairing, there are four other cards to be arrange. From the example of two hands, we know that there can be three hand arrangements for four cards. Thus the total number of arrangements for three hands is $5 * 3 = 15$ or $R(6) = 15$.

This can be express as $R(C) = (C-1) * R(C-2)$. This can be re-expressed in non-recursive terms as $R(C) = (C-1)!!$. Remember that C is the number of cards. This can be also expressed in terms of the number of hands, H, as $R(H) = (2*H-1)!!$.

The above was all done assuming that the order of the hands does not matter. If the order of the hands does matter than we need to include that into the possible arrangements. From Section 5 we know there are $H!$ orderings so the total number of rearrangements when ordering is important is $H! * R(H) = H! * (2*H-1)!!$.

7 Number of possible different distinct hands

This is slightly different than the prior section. The question is what is the number of possible distinct hands. If we refer again to the example of two hands we have the different rearrangements as:

M	P	M	N		N
N	Q	P	Q	Q	P

The distinct hands are MN, MP, MQ, NP, NQ, PQ. Thus there are six distinct possible hands. The formula for calculating this is straight forward. Pick any card, in this case M. It must be able to be paired with all possible other cards. If there is a total of C cards, then the number of possible pairings is $C-1$. Now pick any other card, in this case N, and it must be able to be paired with all other cards, but it has already been paired with the first card picked, so there are $C-2$ possible new pairings. Continue for the next card, which in this case is P. But it has been already paired with the prior two selections so there are only $C-3$ pairings possible.

So the total number of possible distinct hands, $P(H)$ is simply the summation of all of these numbers. This can be expressed as shown below.

$$P(H) = \sum_{i=1}^{2*H-1} i = (2*H-1)?$$

8 Minimum number of unique hands and arrangements

So far, all the calculations have been based upon all the cards in all the hands are distinct. That does not have to be the case.

For example, with two hands, all four cards could be the same such as a four. Is this case the number of distinct hands is one and the number of rearrangements is one.

For three hands from a single deck, there could be four cards of the same value, and the remaining two cards with the same value as each other, and different from the four cards of like value. In this case, the number of distinct hands is two and the number of arrangements is also two.

For four hands from a single deck, there could be four cards of the same value, and the remaining four cards having the same value as each other, and different from the first four. For example, there could be four fives and four sevens. In this case, the number of distinct hands is three and the number of rearrangements is three.

I am still working on the general formula for the minimum number of unique hands and arrangements when duplicate cards are allowed.

9 Conclusion

In this paper, a new version of Blackjack was presented that raises different mathematical problems than the traditional version. In future work, the general formula to determine the minimum number of unique hands and arrangements will be presented, and also a strategy for selecting an optimal arrangement of the cards.