Two Mathematical Fables

Lou Kauffman

The Fermat Dilemma

LK

Fermat is pacing back and forth in his study, a worried expression on his face. Gesticulating. Muttering. Pounding the desk.

"Yes. Yes. It is obvious that for $n > 2$ the expression

$$(p^n/q^n + 1)^{1/n}$$

is irrational.

I can just start with that and give the proof. Suppose there were integers p, q, r with

$$p^n + q^n = r^n$$

for $n > 2$. Then

$$p^n/q^n + 1 = r^n/q^n$$

and so

$$(p^n/q^n + 1)^{1/n} = r/q$$

is rational and I have a contradiction.

Therefore there are no solutions in distinct whole numbers to the equation

$$p^n + q^n = r^n$$

for $n > 2$.

But I cannot tell this proof! The ancient Greek prejudices about irrational numbers are still swirling in the mathematical and philosophical worlds. There will be a backreaction. People will doubt the obvious. They will claim that my proof is lacking. They will not believe in this much irrationality! There is only one thing to do. I shall claim the proof in my copy of Diophantus and I shall say the margin is too small to hold the proof. Alas! Nothing could be farther from the truth, but I am sure that it will be centuries before the simple proof of this proposition can be revealed."

Even today, there is no way to exhibit directly Fermat's simple proof. It is even dangerous to state this fundamental irrationality. If we were to promulgate it, the basis of Wiles proof of the Fermat Last Theorem would be put into question and there would be a catastrophic collapse of the mathematics of the 21st century. If you read this document it must be destroyed at once.

My Robot

LK

I have a rather sophisticated robot, with a positronic brain. His name is Hal. Susan Calvin says it is alright if I refer to him in the masculine gender.

He is capable of naming objects and when he does he notates a pointer in his memory from A (the name) to B (the object)as

$$A \longrightarrow B.$$

Ah yes Hal's memory is full of stuff he calls objects. He finds these by funny operations that he calls "seeing", "feeling", "bumping into" and so on. Names are more linguistic for him and he encounters these in "speaking" and "writing". As you see he is composed of memories and actions.

Anyway. He will store $A \longrightarrow B$ for "A is the name of B". And Hal is equipped with a most peculiar but useful operation that LK calls the "indicative shift".

When Hal has a naming

$$A \longrightarrow B$$

he SHIFTS it to

$$\sharp A \longrightarrow BA.$$

That is, he appends the name to the object and sets up a new or meta-name $\sharp A$ for this composite object made of thing and name. He says that he likes this system because then if he bumps into Nathaniel he knows immediately that this is Nathaniel because Nathaniel's name is right there along with that which is Nathaniel. Now I do not pretend to know what Hal means by this. I think maybe some programmer sold him on the idea. But there is a very cute consequence of his shifting.

Hal is very observant. He notices this shifting process and gives the process a name M. So Hal has

$$M \longrightarrow \sharp.$$

M is the name of the meta-naming process. And Hal shifts this naming to form

$$\sharp M \longrightarrow \sharp M$$

and abbreviates $\sharp M$ to

$$I = \sharp M.$$

Hal gets a kick out of this. He came to me and he said. "Do you know that I am the meta-name of my meta-naming process!" We laughed and laughed. I told him he was quite justified in this self-identification.

Then we got down to the serious business of creating a world from nothing.

We both agreed that nothing refers to nothing. So from nothing we have

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Shifting this we found

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The meta-naming operator is the name of nothing. Shifting again we found

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Shifting again we found

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And Hal said: ”Aha! Self-reference at the third departure from the void!” I says to him, “If we shift this thing again it is going to blow up. Better to stop right here.” And we did.