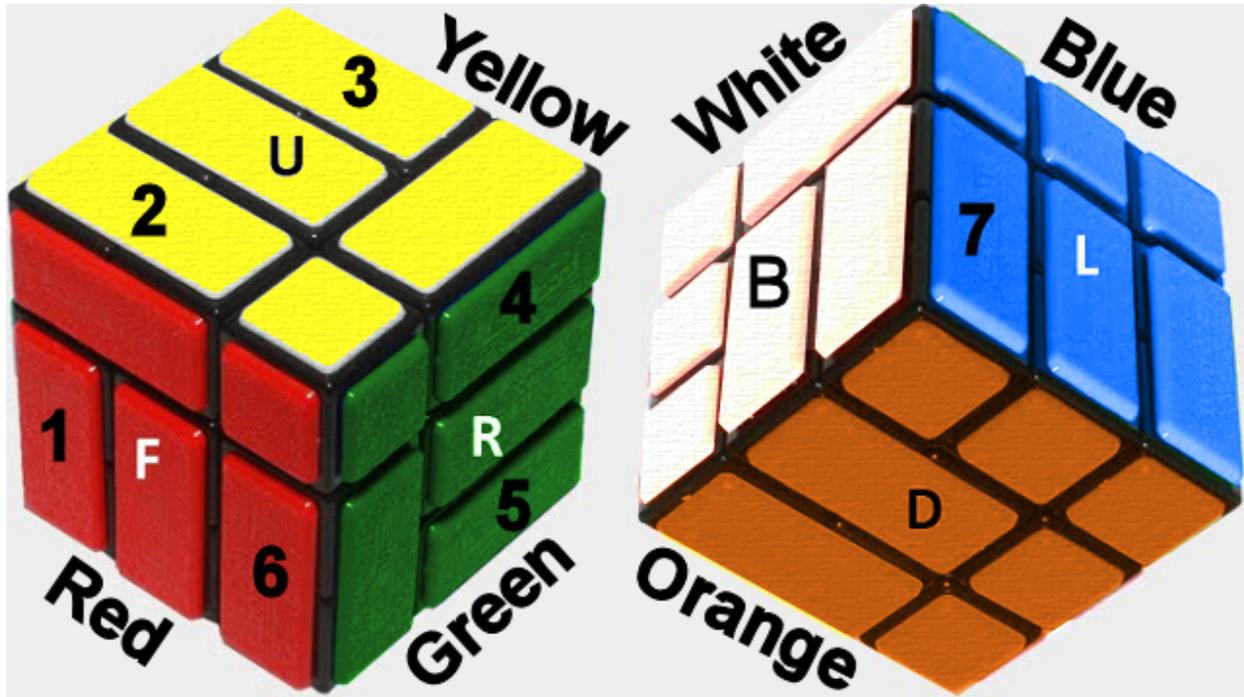


# The Bandaged Cube

An analysis and solution offered in tribute to  
Professor Ernő Rubik & Professor David Singmaster  
for the 13<sup>th</sup> Gathering for Martin Gardner.

By

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Preface: Since not all orientations of the Bandaged Cube are equal it makes sense to add a couple of conventions to simplify the discussion of the Bandaged Cube. Above is the coloring on the cube used for this discussion. If your cube is differently colored, you'll need to make an adjustment to cover that. I will also assume that if you're interested in the Bandaged Cube that you're fairly competent with the basic Rubik's Cube's movements and notation.

Analysis and conventions:

Colors = Y, G, R opposite O, B, W respectively.

There is only one single cubelet; YGB. Looking at the cube with this cubelet in the URF corner, gives you Yellow on Up, Green on Right, Red in Front, Blue on Left, etc. This is what I will call the Home position.

There are seven other movable couplets and these will be denoted by their two long edges. GY, YR, RG (group-1), GO, RB, YW (group-2) and finally the BW piece.

The WO centers are connected – leaving just the other 4 centers to turn (YRG and B). The YRG sides are rotationally symmetric. I looked for an operator that would leave the BW piece untouched. I found the following, really nice one:

**F R U F R- F2 U-**

Let's call that the "Move" and label it M.

Perform M in home position and you'll notice that RG piece stays in place.

I numbered the remaining 5 edges as follows (6 and 7 being the other two).

- 1. RB**
- 2. YR**
- 3. YW**
- 4. YG**
- 5. GO**
- 6. RG**
- 7. BW**

I think of it as just a clock-wise numbering by looking at the single-cubelet (with Yellow on top of course).

This is a nice 5-cycle 1->2->5->4->3

Doing this 5 times, of course restores the Cube.

The Move (M) can be done with Yellow, Green or Red on top (keeping the single cubelet (YGR) in the upper-front-right spot).

I knew that the 5-cycle wasn't going to be enough. At this point, I reasoned as follows. It's easy to get the centers aligned. It's easy to pop in BW (piece-7). It's easy to get one of the (Group-1) pieces in, which is then invariant under M from the right orientation (RG) (piece-6).

Using the 5-cycle, I can get one more in but that still left me with 11 unique positions to get to. There are 3 2-swaps and 4 3-cycles ( $2*4 + 3 = 11$ ). Doing the 5-cycle from different directions only let me find one of the 11 (what I call the 23 (which swaps 2-3 and 4-5 (the 1 cube is now invariant by using M repeatedly until it's in place). I needed one more interesting move and I found one. I call it The "Swap" Move and I use S to note it (because it has to do with Swapping a piece in and out of the BW spot).

Here it is...The "Swap" Move:

**(U2 L U- F- L- U- F) (R U) (U2 L U- F- L- U- F) (U-R-)**

1->2->5->3 and 4->6 A 4-cycle and a 2-cycle.

U2 L U- F- L- U- F (accomplishes a lot but I think of it as getting the Yellow Red to go to the White-Blue spot (or 2->7). The (RU) then let's you do the move again to put 7->2. The final (U-R-) gets the cube back in its symmetrical form so the M operator can work.

With this operator (we'll call it S) and Y, G, R for the M operator with that color on top. (Y on top is still the home orientation and the numbering). We have all we need to solve the cube from any position.

23 **G2R-**

24 **RGS**

25 **YSG2Y2**

2 **GYSY2**      4-5-3

3 **GSR-Y2**     4-5-2

4 **SG2Y-**      3-5-2

5 **Y2SG2**      2-3-4

So, that's it. Here's the 5-step process I use to solve the Bandaged Cube.

1. Get the cube in the F, U & R bandaged symmetry without worrying which of the 7 bandaged pieces are located.
2. Get the BW (7) in place by using the first part of the "Swap" Move.
3. Get RG (6) in place by using M from either R and G on top.
4. Get RB (1) by doing M with Y on top (or just Y).
5. Then look at 2, 3, 4, 5.

If one of them is in place use the corresponding formula (2, 3, 4, or 5) above.

(You may need to do this twice (or backwards) depending on the remaining 3-cycle).

If none of them is in place look at where the YR (2) exchanged with – either 3, 4, 5 and use that formula (23, 24, 25) from above.

## Bonus Cube Fun Fact:

Q: What's special about  $(R2L2F2B2U2D2)(RBLFR)$ ?

A: On each of the 6 faces 3 of the colors appear twice and 3 of the colors appear once.

I originally named any state that met these criteria as "The Most Random State" of the cube. Shortly after, I asked myself if I could create a cube that met this Most Random State criteria as well as having the property of being bilateral symmetric on all sides. I achieved that as well; but leave it as an exercise for you to enjoy if you're so inclined. I called that the Most Non-Random Random State but I never found a short algorithm to produce that. Swapping the order of the two operators above by putting the Checkerboard operator after RBLFR also results in "The Most Random State" criteria being met.

## About the Author:

I am Joe Cassavaugh. I am the CEO/Designer/Engineer of Puzzles By Joe.

I graduated in 1979 with a B.S. in Mathematics from Rensselaer Polytechnic Institute.

I discovered the Rubik's Cube at the end of 1980. Although slow by today's standards, I still solve the cube in 34-ish seconds on average using a modified version of my original solving method.

I have been programming since 1981.

I have been a software engineer since 1996.

I have been a game developer since 1991.

From 2004-2009, I created the Mah Jong Quest trilogy for iWin.

I have been an Indie-Game-Dev since 2010.

I have created a fairly successful series of games for the casual PC/Mac download market called Clutter and I'm about to release the 8<sup>th</sup> game in that series. It is its own sub-genre of the Hidden Object Games genre. Although the main game mechanic is targeted at the Hidden Object crowd, there are many unique minigames which would appeal to the Recreational Mathematics crowd. In addition to those games, I have several free games from 2000-2003 that may be of interest to those of us that enjoy more logic-based puzzles. Those include Recon, a logical Battleships game (based on the puzzle game that was popularized by Games Magazine), Rack'Em, a logical Pool game and GAPWAR, a matching-edge game.

My email address is [JoeCassavaugh@aol.com](mailto:JoeCassavaugh@aol.com)

If you send an email and mention the Gathering of Gardner and whether you prefer PC or Mac, I will send you a free link to the latest Clutter game.

You can also find the free logic-based windows (PC) games at:

[www.puzzlesbyjoe.com/other-games](http://www.puzzlesbyjoe.com/other-games)