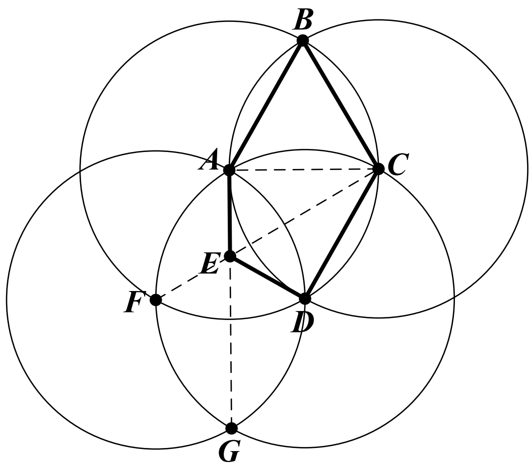
Marjorie Rice's pentagonal "Versatile"

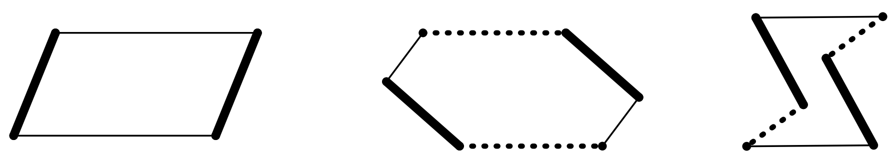
--Doris Schattschneider, Moravian College

Make your own--- with geometry software or with a compass and straightedge:



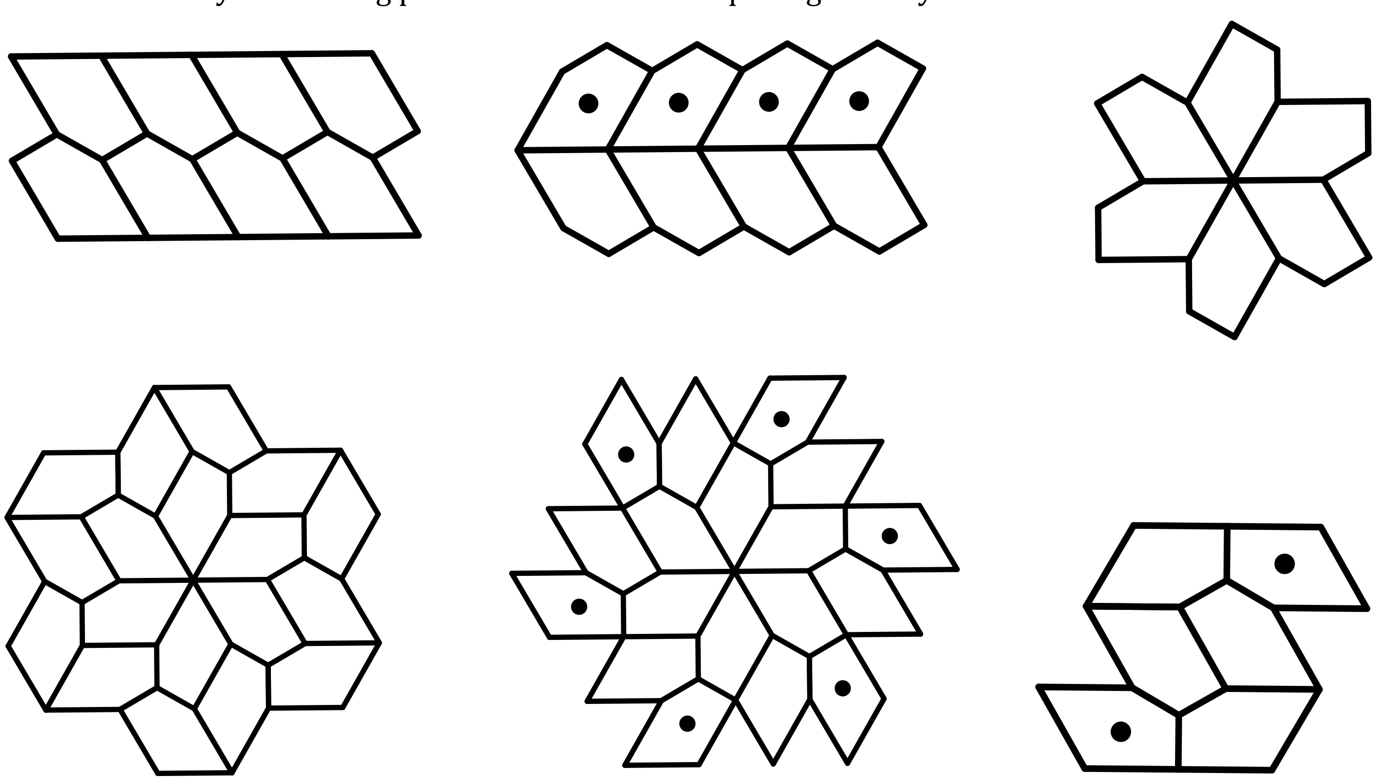
Construct two circles having radius AC, one with center A and the other with center C.
Label their points of intersection B and D as shown.
Draw AB, BC, and CD; then AB = BC = CD.
Construct a circle with center D having radius DC, and label F its intersection with the circle centered at A.
Construct a circle with center F and radius FD and label G its intersection with the circle centered at D.
Draw line segments AG and FC; let E be their intersection.
Draw AE and ED.
Since E is the center of equilateral triangle AFD, it follows that AE = ED, and $\angle E = 120°$. It is easily verified that $\angle B = 60°$, $\angle C = 120°$, $\angle D = 90°$, and $\angle A = 150°$.

Any patch of tiles that is a generalized "parallelogram" or "par-hexagon" can fill the plane using only translations. The boundary of such a patch can be partitioned into two pairs or three pairs of curves that have this property: the curves in each pair are congruent, and are translates of each other. Schematically, the boundary of such a patch looks like one of these (here "curves" are shown as straight edges; edges that match by translation have the same style of line):



Six ways that the pentagon versatile can tile the plane are shown below. Only a small patch of each tiling is shown—each patch is a generalized parallelogram or par-hexagon that can be translated to fill the plane with its copies. Tiles that have dots are reflected versions of the plain tiles.

How many other tiling patches of this versatile pentagon can you find?



Marjorie Rice's "Versatile." Copy and cut out the tiles; assemble them into tiling patches.