15 MATHEMAGICS for G4G15 -

In-Between Magic and Topology

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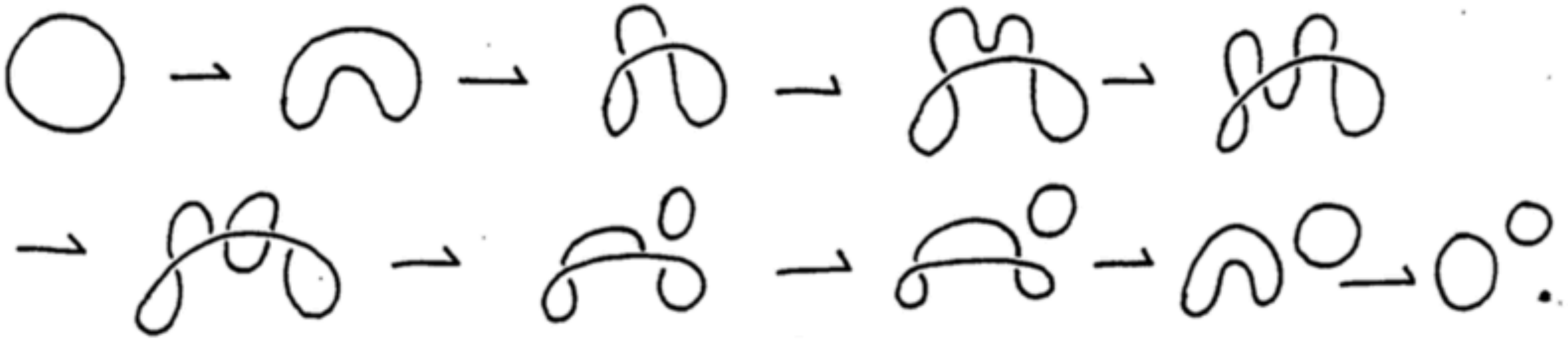
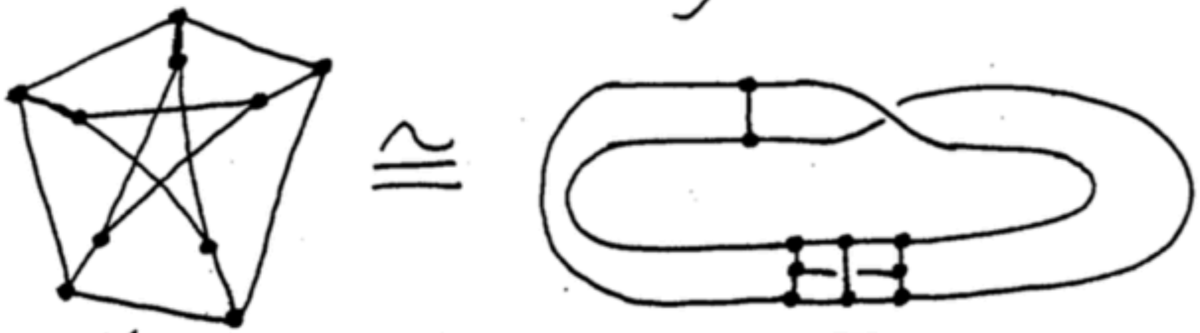
I. A Self-Reproducing Loop (Courtesy of Kurt Reidemeister and Sam Lloyd)



This shows how one loop could become two loops in a series of actions that almost looks topological.

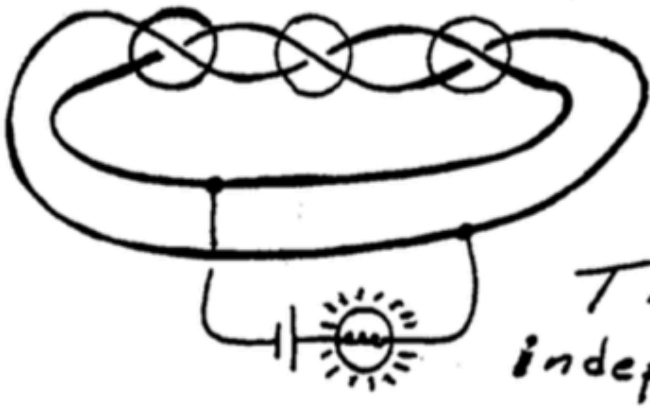
II. "The Snark was a Möbius you see."

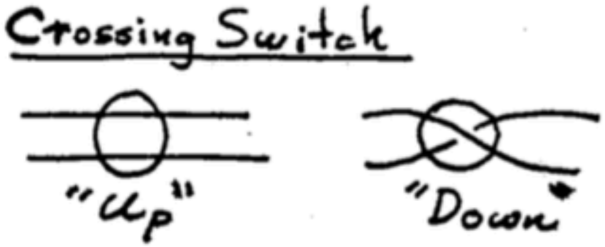


The Möbius nature of the Petersen graph "explains" why it is a snark (i.e. not edge colorable in 3 colors with 3 distinct colors at each vertex.)

The famous Petersen graph is on the left in its usual incarnation, but really the Petersen is just another appearance of the Mobius strip.

III. The Möbius Circuit

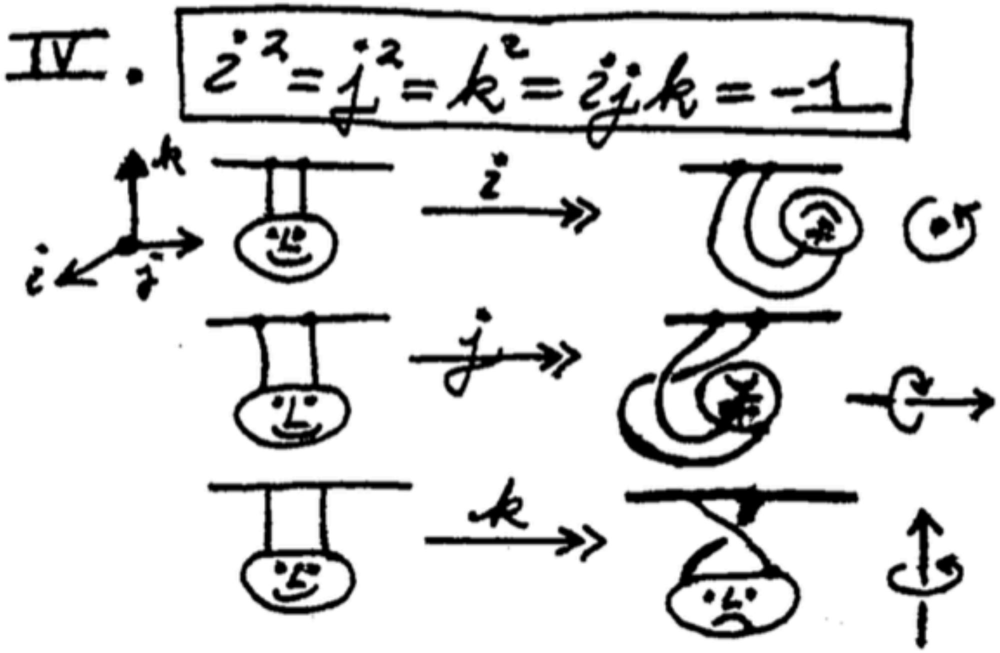




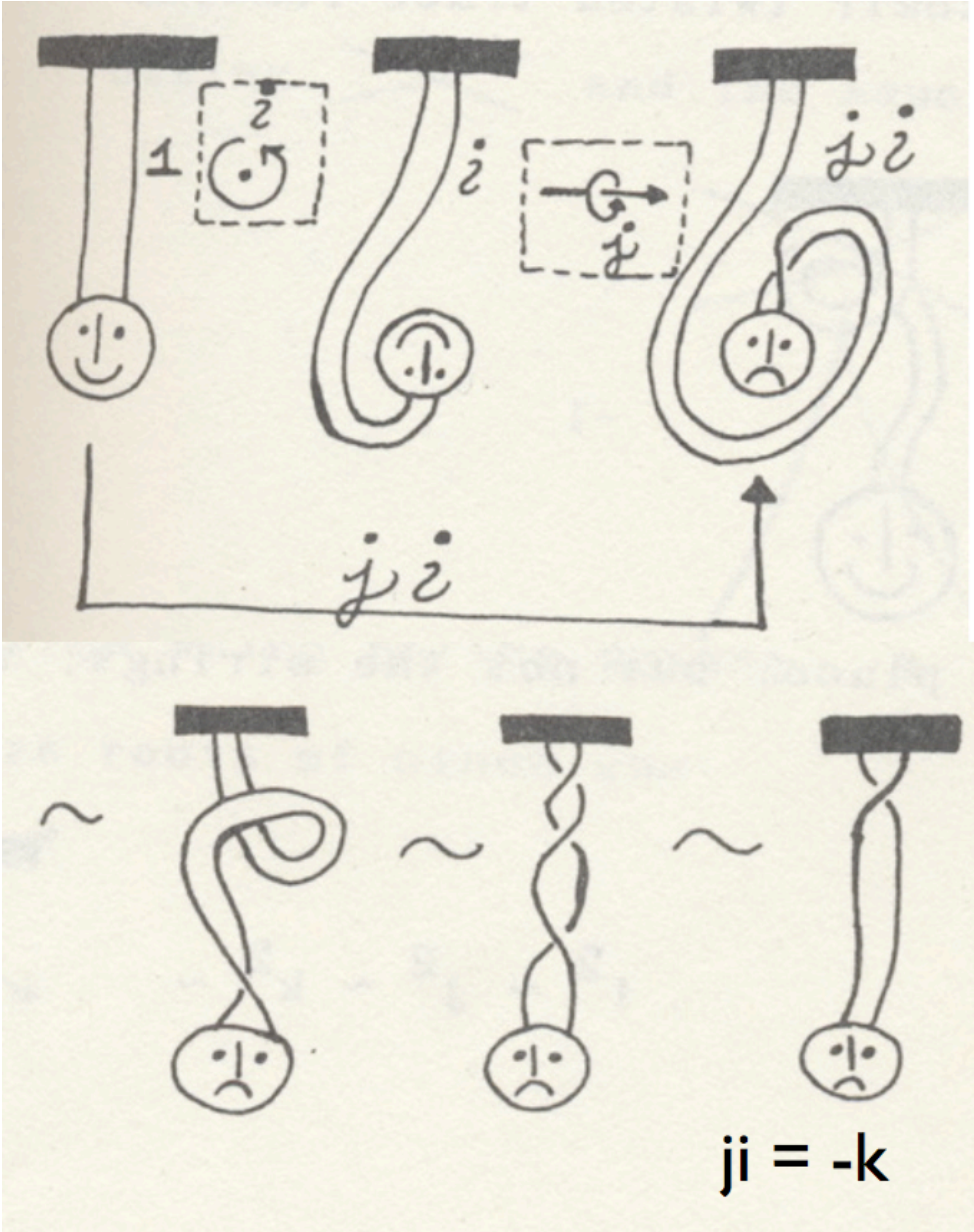
Thanks to Möbius, the Hall Light is independently controlled by each switch.

We hope that this is self-explanatory, and that you will go home and use the Mobius band to design a circuit to control the light at your front door from switches in every room in your house.

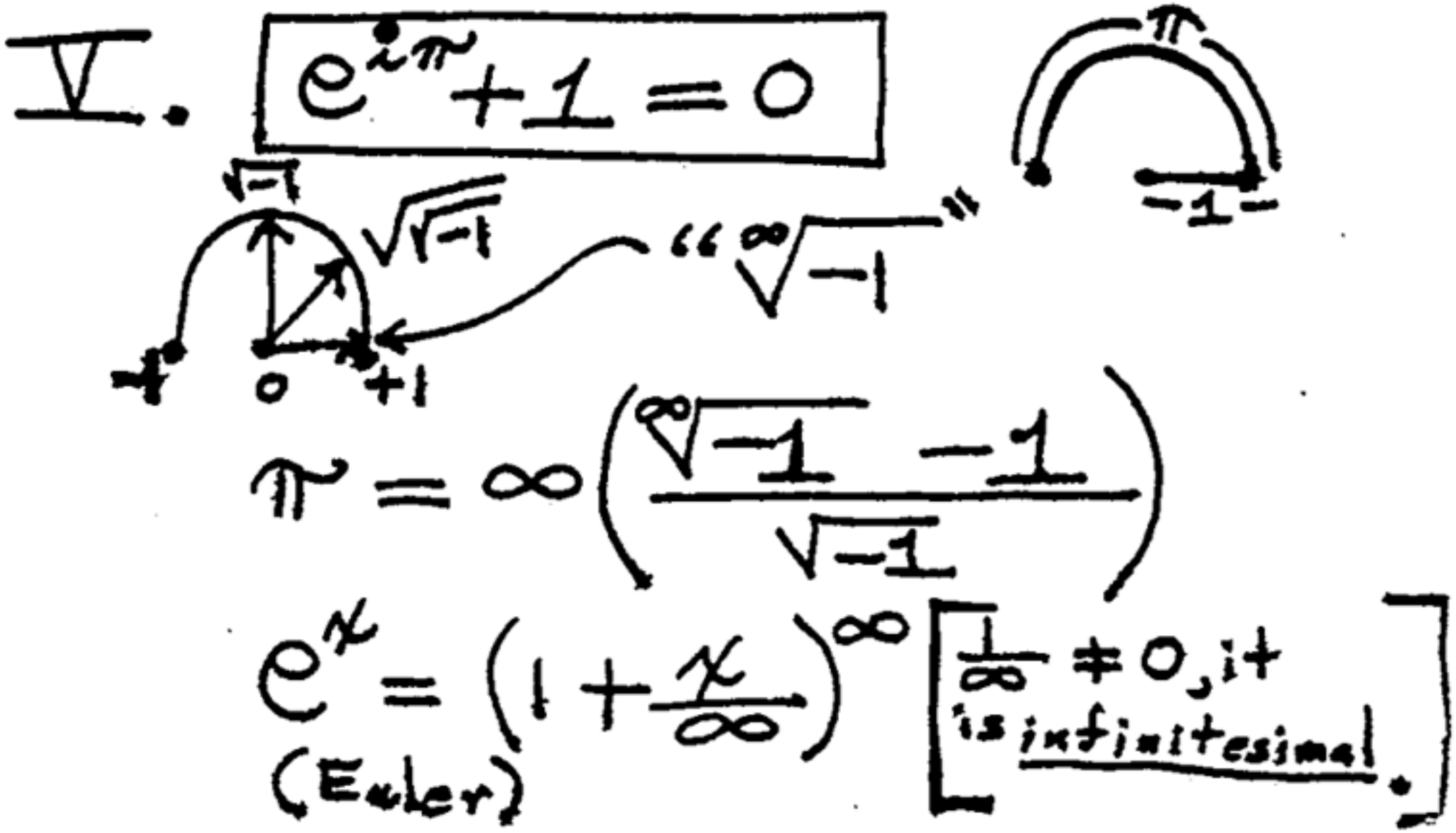
The Quaternions Personified



Yes. There they are the quaternions i, j and k. And they can be understood as the topological symmetries of a little face attached by puppet strings to the ceiling.



ji = -k



Here we have Euler's beautiful formula and an iconoclastic formula for Pi that is obtained by solving for Pi in Euler's formula. The formula for Pi is correct!

$$e^x = \left(1+\frac{x}{\infty}\right)^\infty$$

$$e^{i\pi} = -1$$

$$\left(1+\frac{i\pi}{\infty}\right)^\infty = -1$$

$$\left(1+\frac{i\pi}{\infty}\right) = \sqrt[\infty]{-1}$$

$$i\pi/\infty = \sqrt[\infty]{-1} - 1$$

$$\pi = \infty\left(\sqrt[\infty]{-1} - 1\right)/i$$

$$\pi = \lim_{n\to\infty} \frac{2^n\left((-1)^{\frac{1}{2^n}} - 1\right)}{i}$$

$$\pi = \lim_{n\to\infty} 2^n \, \text{Imaginary Part}\left((-1)^{1/2^n} - 1\right)$$

$$\pi = \lim_{n\to\infty} 2^n \underbrace{\sqrt{2-\sqrt{2+\sqrt{2+\cdots\sqrt{2}}}}}_{n\ 2\text{'s}}$$

$$2^2\sqrt{\frac{1}{2}} = 2\sqrt{2}$$

$$2^3\sqrt{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{1}{2}}} = 2^2\sqrt{2-\sqrt{2}}$$

$$2^4\sqrt{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}} = 2^3\sqrt{2-\sqrt{2+\sqrt{2}}}$$

$a^2+b^2=1$, $a,b>0$

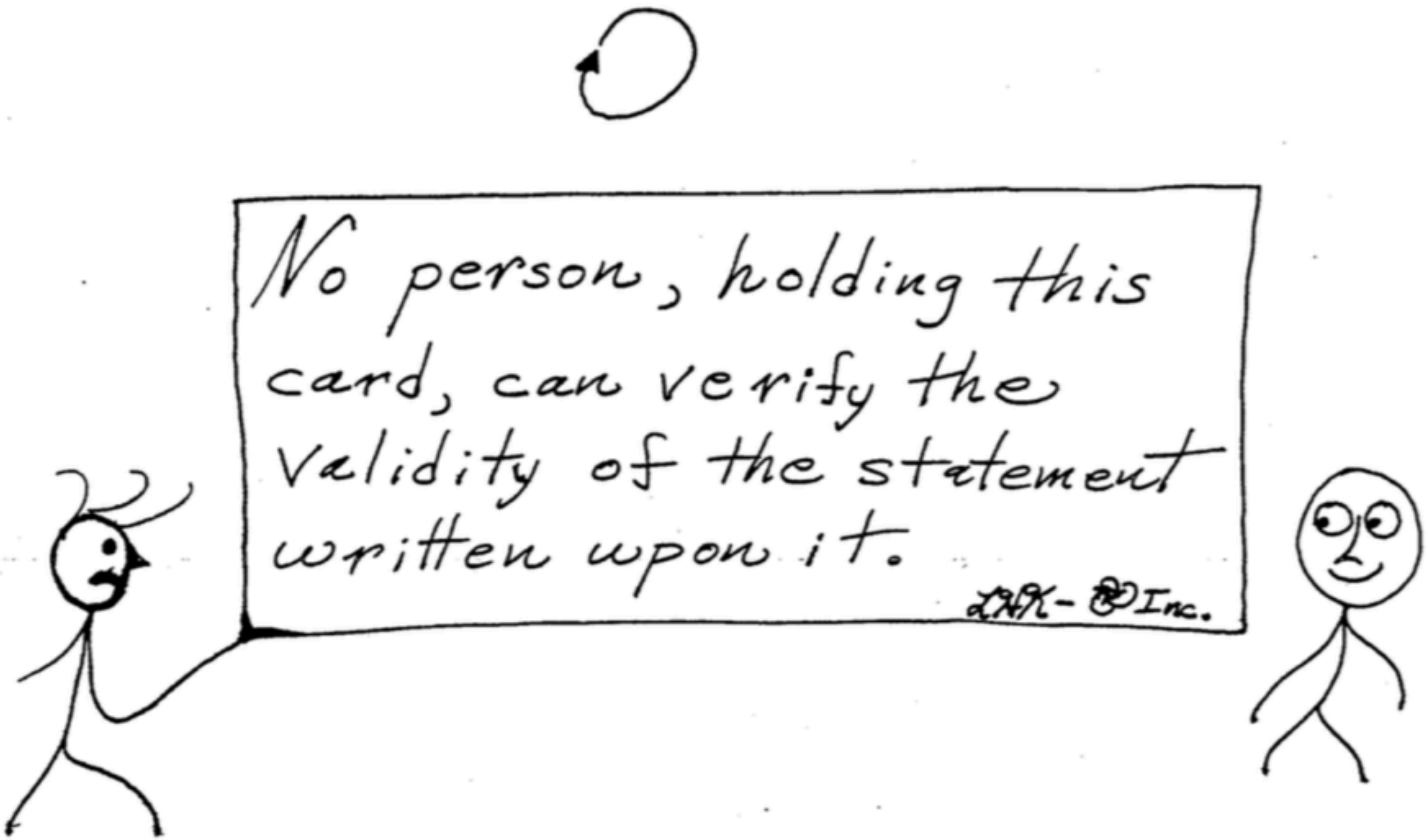
$$\sqrt{a+bi} = \sqrt{\frac{1+a}{2}} + i\sqrt{\frac{1-a}{2}}$$

$$\sqrt{\sqrt{\sqrt{-1}}} = \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}} + i\sqrt{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{1}{2}}}$$

$$\sqrt{\sqrt{-1}} = \sqrt{i} = \sqrt{\frac{1}{2}} + i\sqrt{\frac{1}{2}}$$

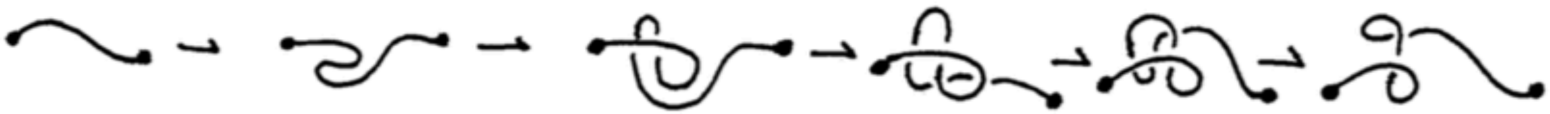
$$\pi = \infty\left(\frac{(-1)^{1/\infty} - 1}{\sqrt{-1}}\right)$$

VI.



No person, holding this card, can verify the validity of the statement written upon it.

"I'm holding the card.
But if I was not holding
the card, then I could easily
see that anyone who does hold
the card is prevented from
asserting the validity of the
statement on the card. Thus
it certainly is correct — what
is written on the card.
But I am holding the card.
Therefore I am prevented
from doing what I have
just done!"



VII. This Seventh Tale illustrates the
Russell Paradox or lack of it in
Knot Set Theory where a bit of
curve A overcrossing another bit of
curve B means that B is a member of A.
Then a diagram with a curl is a member of itself.
But curls come and go topologically.
Also you will see
Ax to mean "x is a member of A".
So the Russell set is defined by

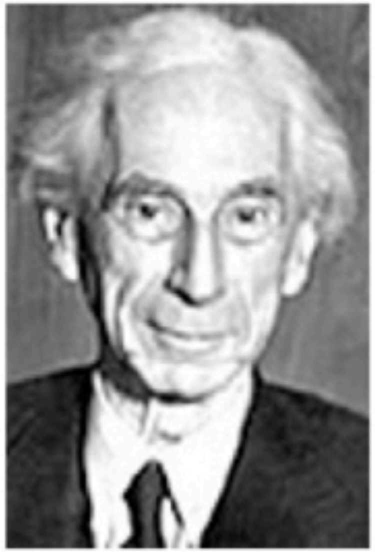
$$Rx = \sim xx$$

and the paradox is

$$RR = \sim RR.$$

$$Rx = \sim xx$$
$$RR = \sim RR$$

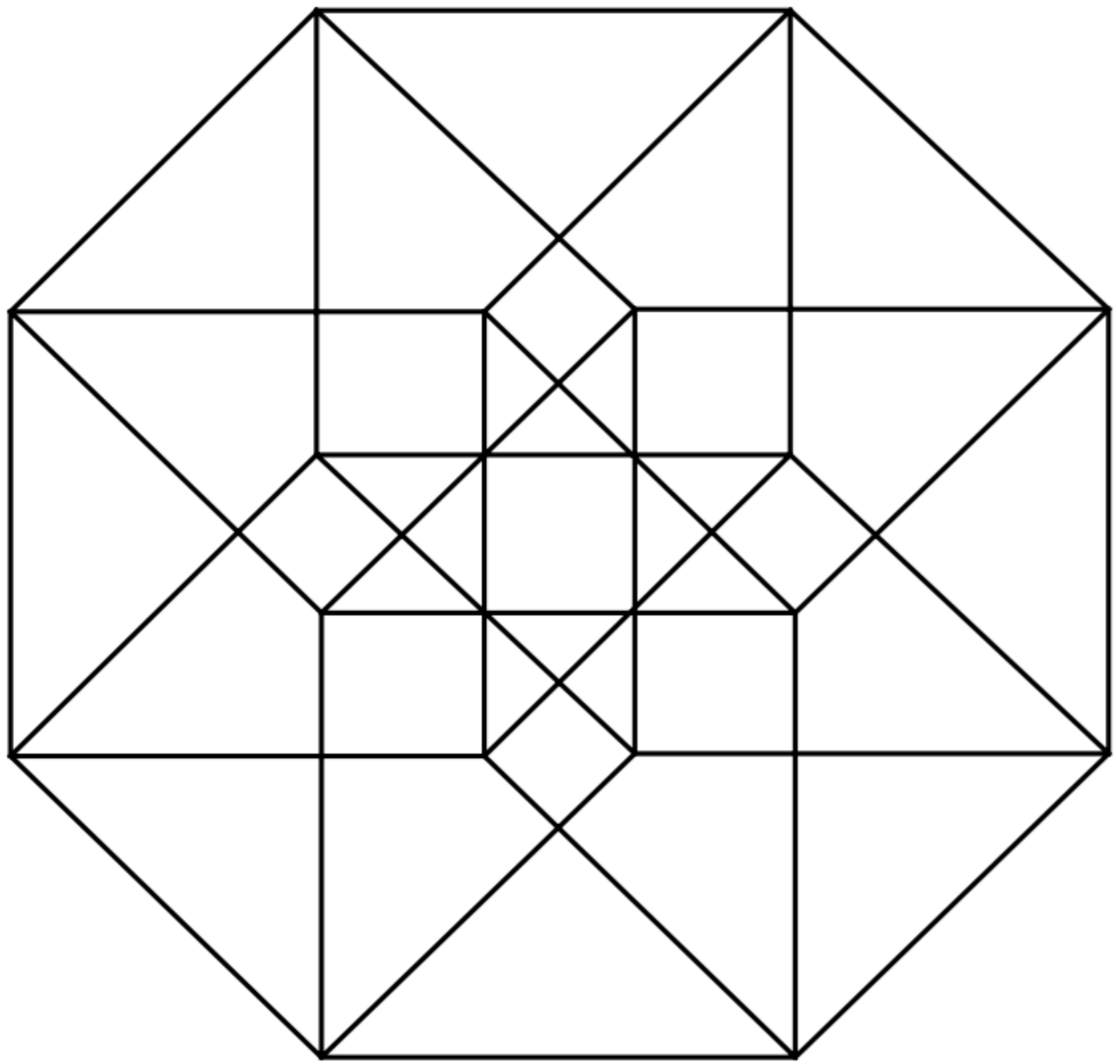
Russell Paradox (K)not.



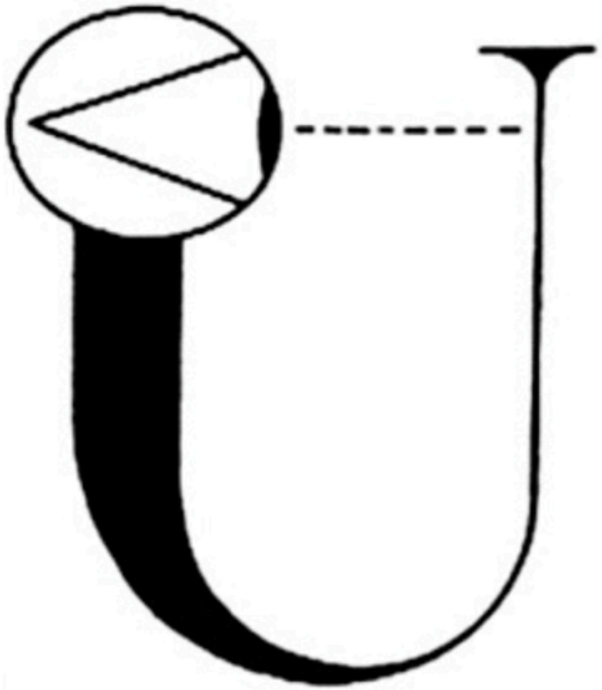
A belongs to A. ⟺ A does not belong to A.

VIII. KLEIN BOTTLE = Union of Two Mobius Strips.

IX. My Favorite Four-Cube

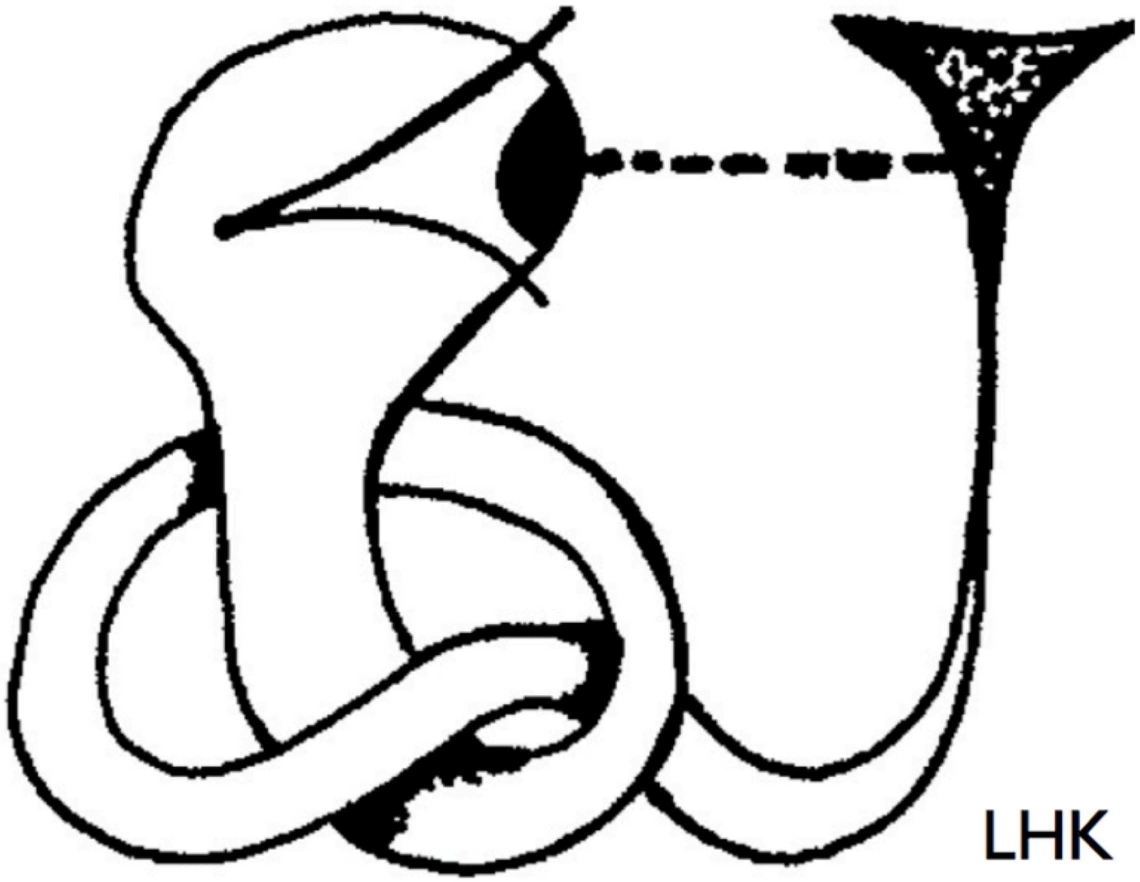


X. The Wheeler Universe and the Knot Wheeler Universe



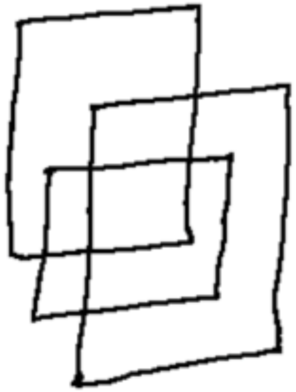
Here is John Archibald Wheeler's Universe. The letter U looks back to the Big Bang and by observing Itself, brings the Universe into being.

Here is the KnotWheeler Universe, a slight correction to JW's point of view.



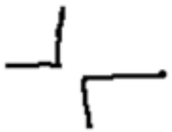
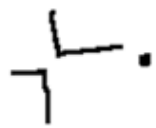
XI.

A Puzzle of Lewis Carroll

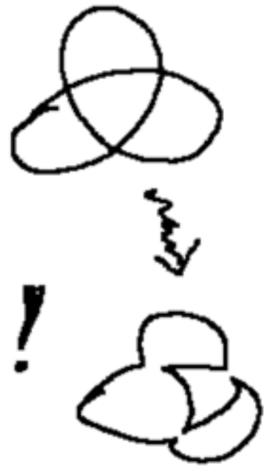


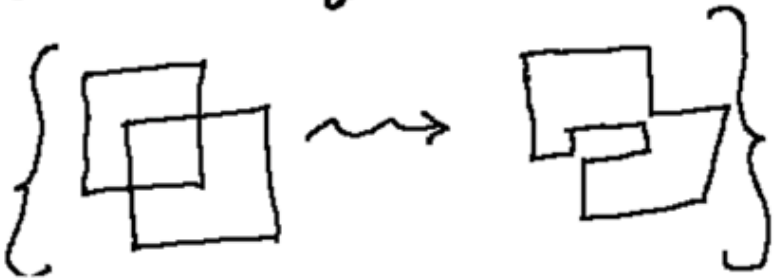
Traverse the graph in one route that never crosses itself.

(The theory of such solutions is related to knot theory. Write to LK for more information)

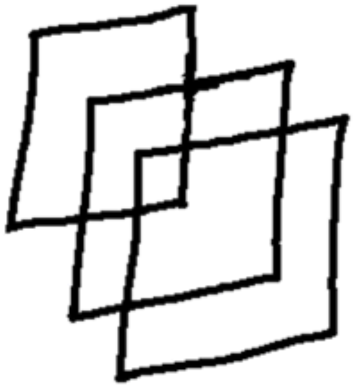
This means  ⇝  or .

You must turn at every intersection.

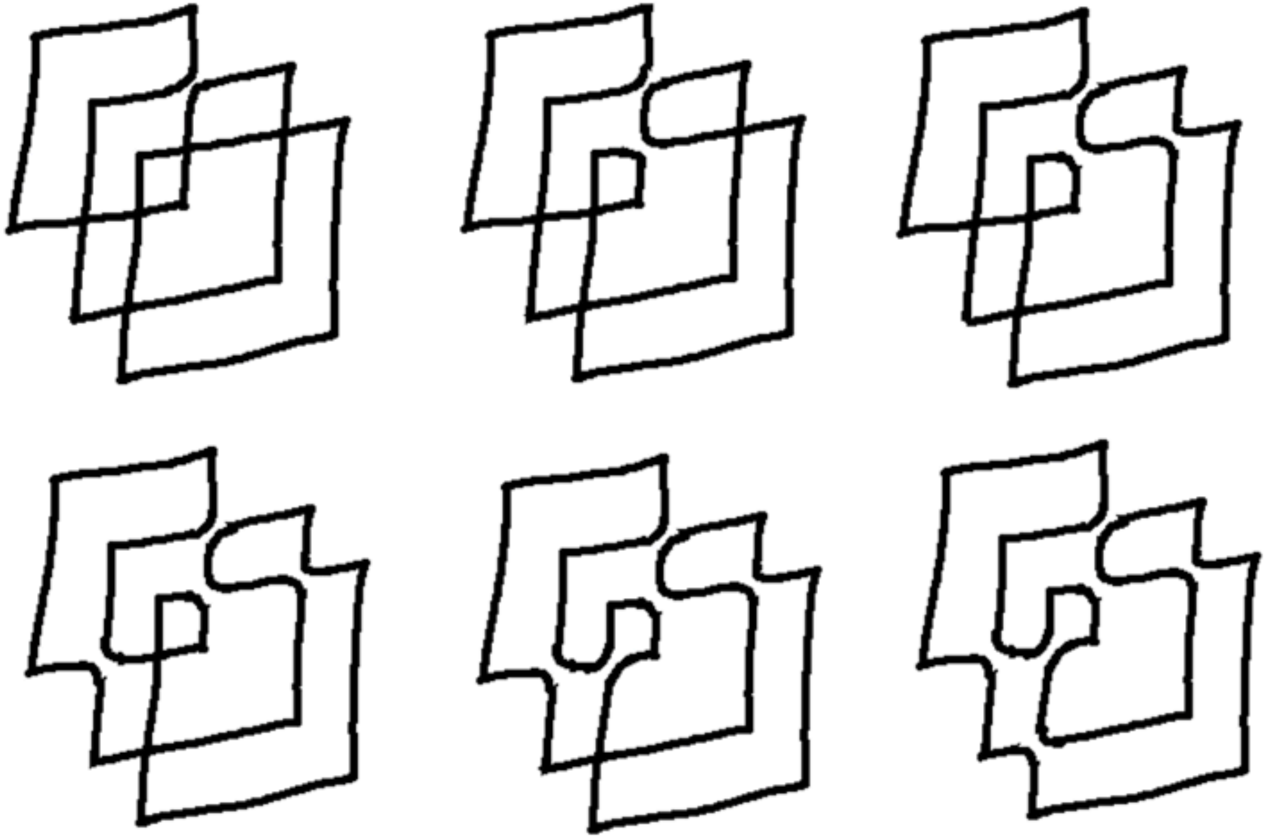


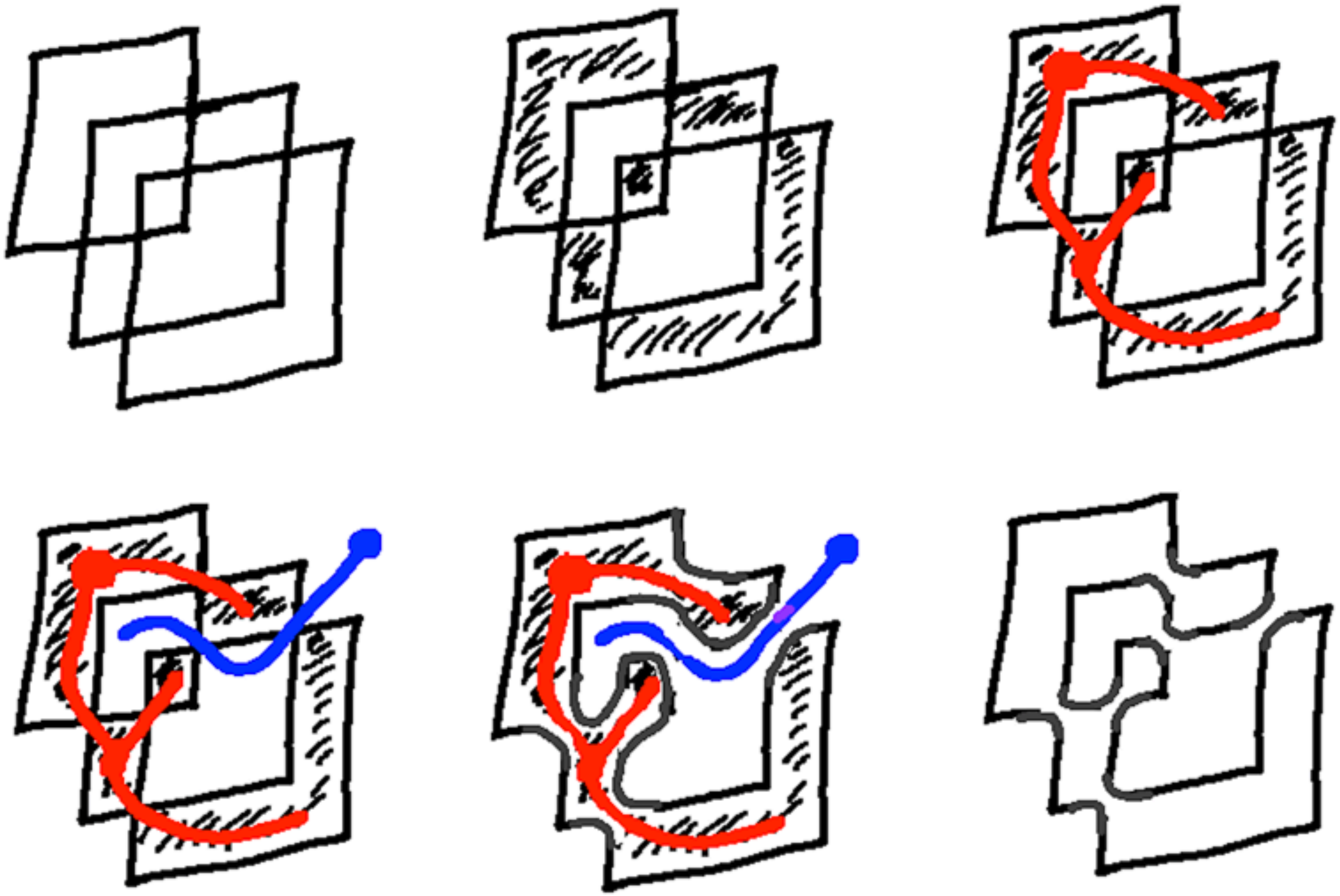
e.g. 

Find all solutions!



Maintain Connectivity





Maximal Trees in the Checkerboard Graph

XII.

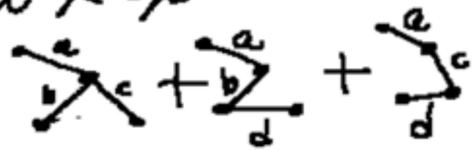
Find all maximal trees in a graph G.

The *Amazing Wang Algebra*.

a) label all edges.
b) number all nodes.
c) to each node associate the sum of the edges incident to the node.
d) Take the product of all but one of these sums in the Wang Algebra where $x^2=0$ for any label x and $X+X=0$ for any product X of labels.

1. a
2. $a+b+c$
3. $b+d$
4. $c+d$

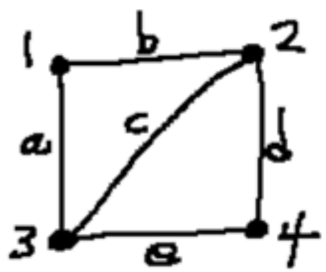
$a(b+d)(c+d)$
$=(ab+ad)(c+d)$
$=abc+abd+adc+ad^2$ → 0
$=abc+abd+acd$ ↔



$(a+b+c)(b+d)(c+d)$
$=(ab+ad+b^2+bd+bc+cd)(c+d)$
$=abc+adc+\cancel{b^2c}+\underline{bdc}+\cancel{bc^2}+\cancel{c^2d}+abd+\cancel{ad^2}+\cancel{b^2d}+\cancel{bd^2}+\underline{bcd}+\cancel{cd^2}$

$bdc+bcd=0$

$=abc+adc+abd.$



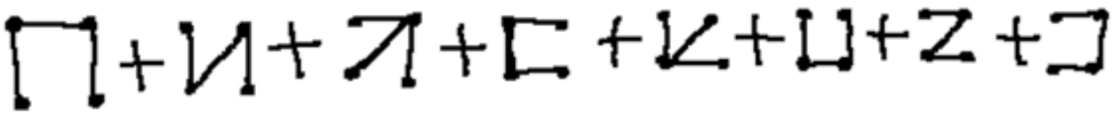
1. $a+b$
2. $b+c+d$
3. $a+c+e$
4. $d+e$

$$(a+b)(b+c+d)(d+e)$$

$$= (ab+ac+ad+\cancel{b^2}+bc+bd)(d+e) \quad (b^2 \to 0)$$

$$= abd+acd+\cancel{ad^2}+bcd+\cancel{bd^2} \quad (ad^2 \to 0,\ bd^2 \to 0)$$

$$+abe+ace+ade+bce+bde$$

$=$ 

$$\left.\begin{cases} a^2=b^2=c^2=d^2=e^2=0. \\ X+X=0 \text{ for any product } X. \\ ab=ba \text{ etc.} \end{cases}\right\} \text{Wang Algebra}$$

XIII.

A Knotty Algebra

$T = \{a, b, c\}$

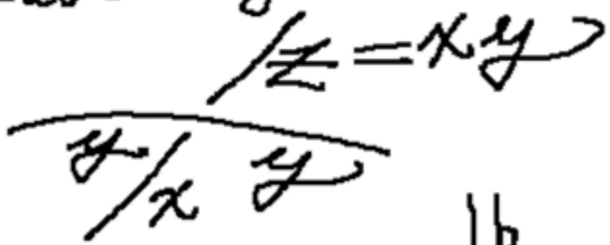
$\cdot$	a	b	c
a	a	c	b
b	c	b	a
c	b	a	c

$a^2 = a \quad ab = ba = c$

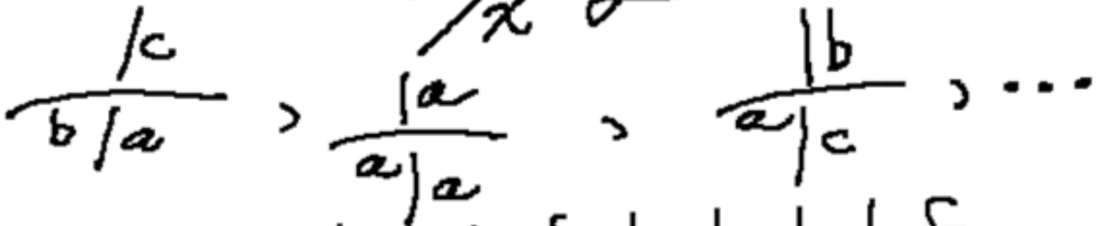
$b^2 = b \quad ac = ca = b$

$c^2 = c \quad bc = cb = a$

Labelling Weaves

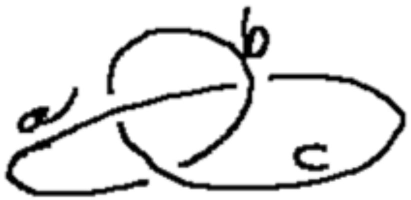


Thus



A knot is Tri-Colored if labeled from T.

e.g.



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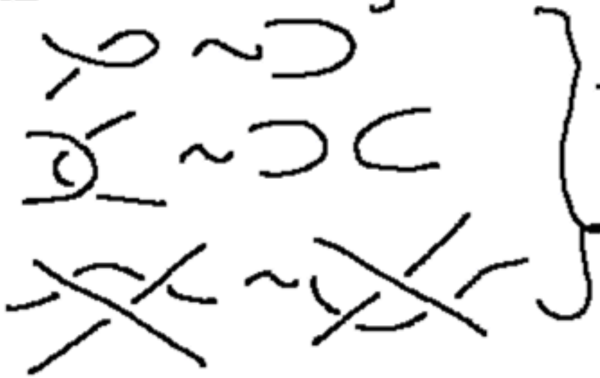
Two knots are topologically equivalent to one another if you can transform one to the other using the move types:

R1.

R2.

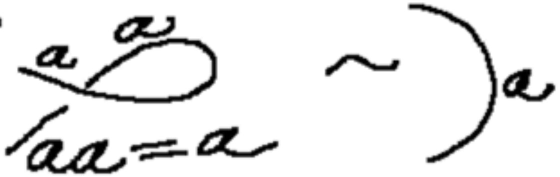
R3.

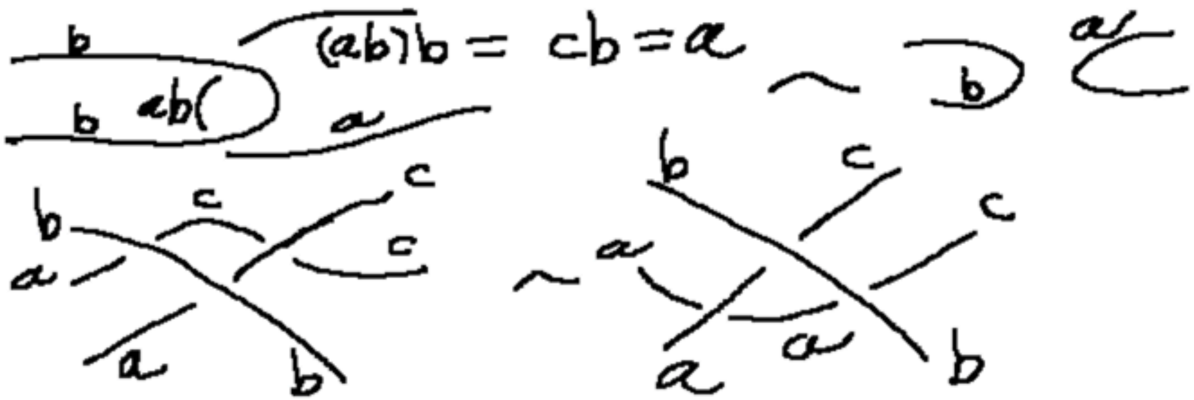


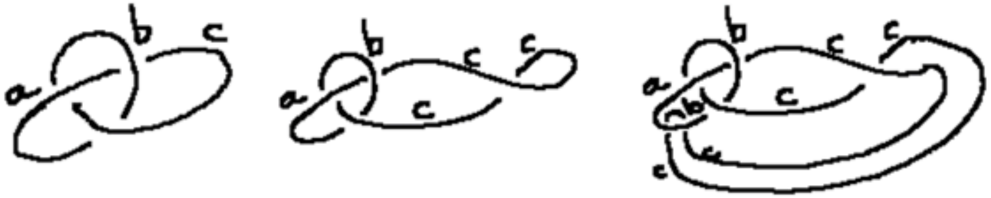
The Reidemeister Moves.

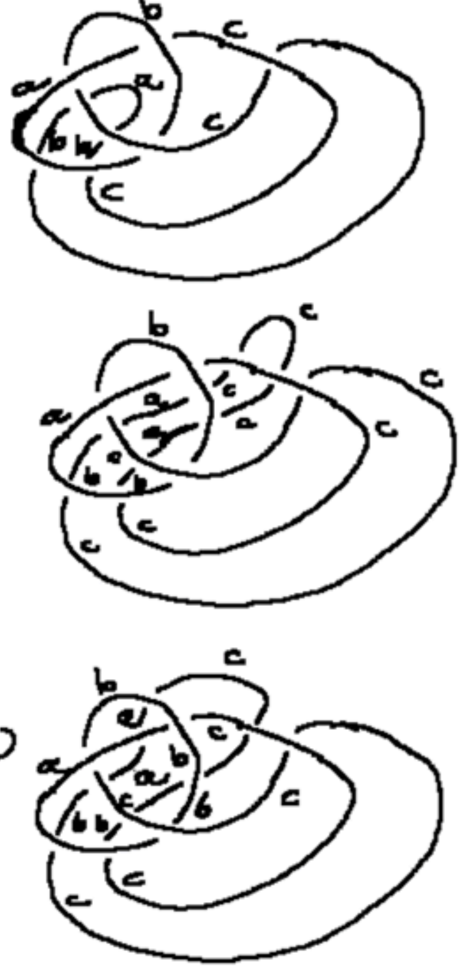
Tri-colorability is preserved by the R-moves.

For example:







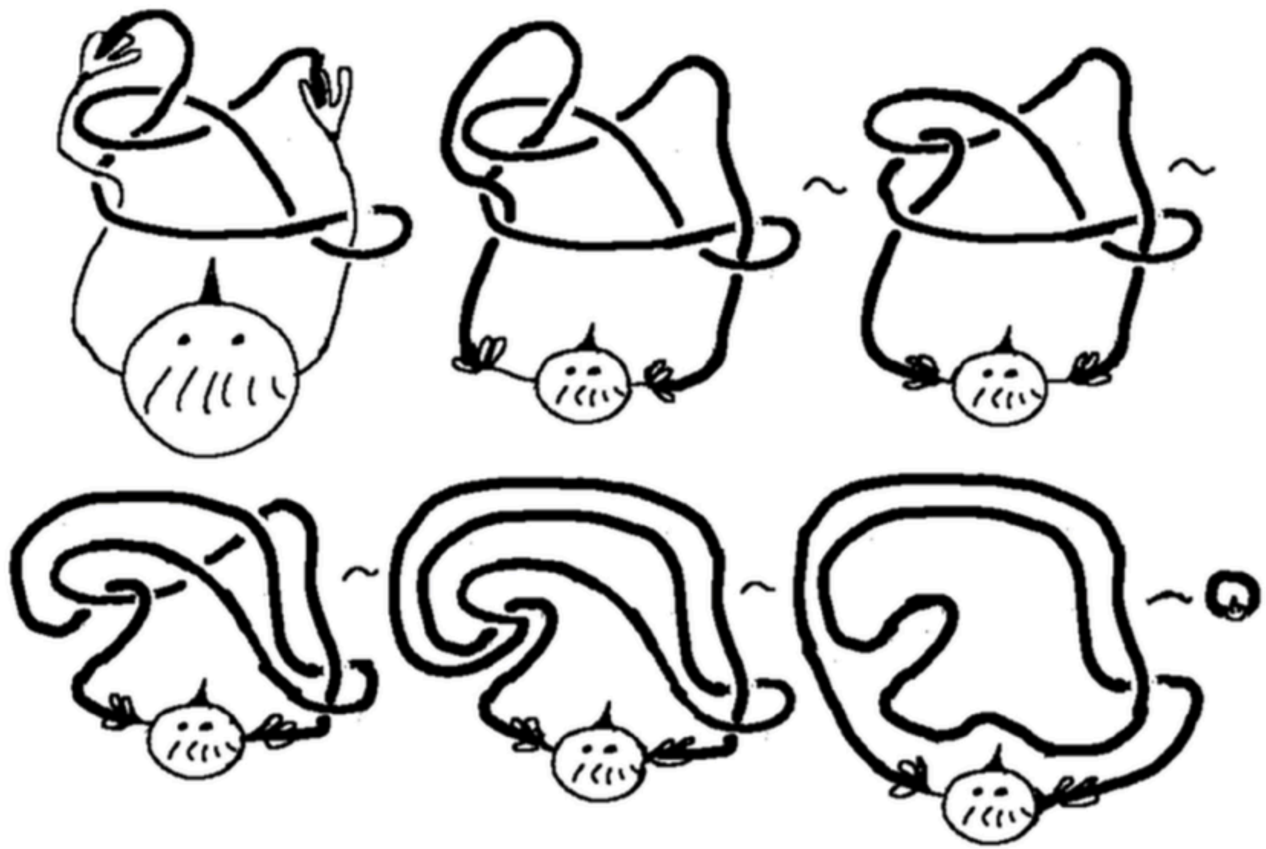


Since ○ is not 3 colorable
and  is 3 colorable,
we conclude that 
is <u>not</u> topologically
equivalent to ○.

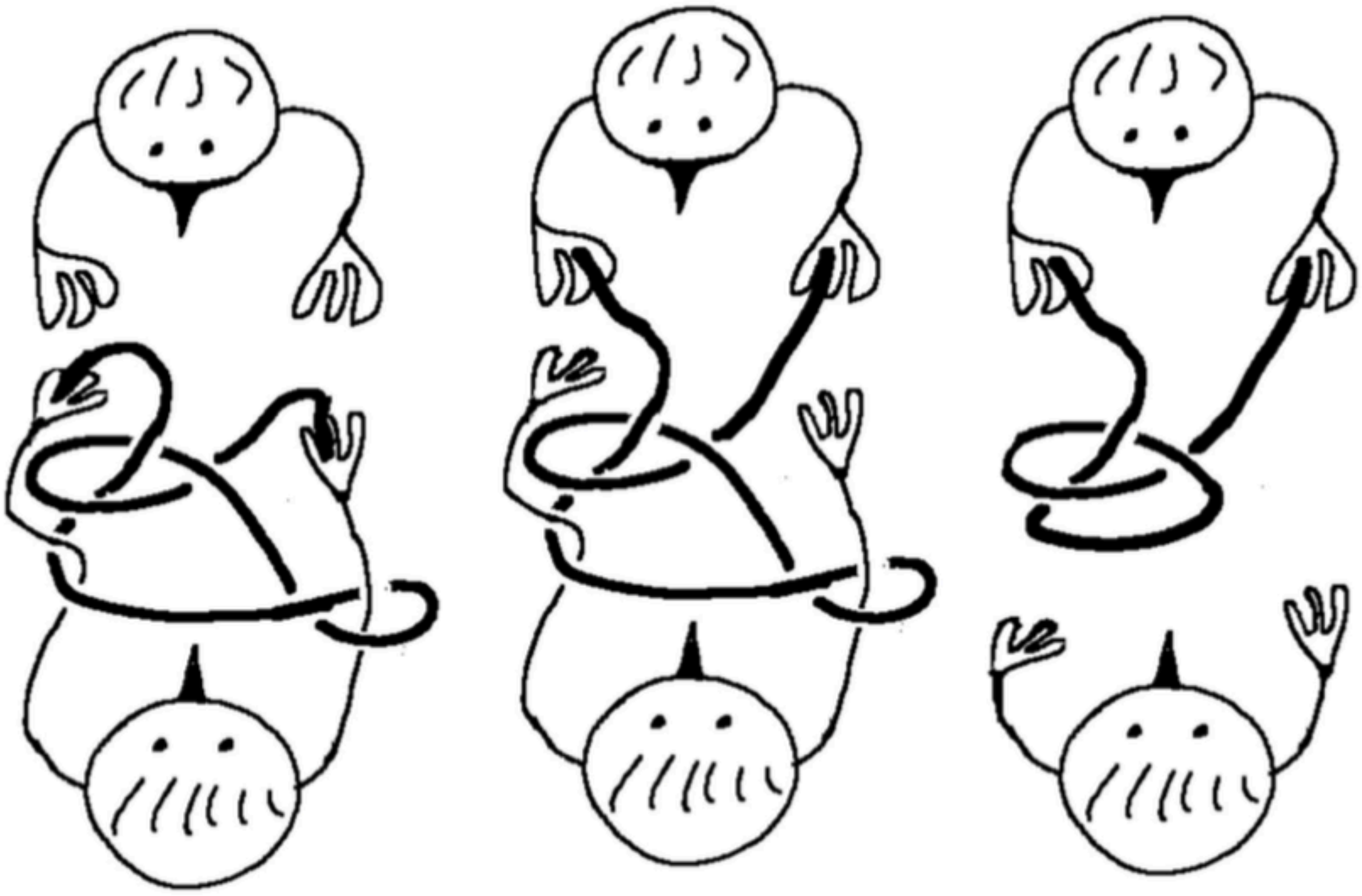
 is <u>knotted</u>.

(You should check that
when K' inherits a 3-coloring
from K (with 3 colors), then
K' also has 3 colors.)

XIV. THE VALUE OF COOPERATION
(LK and Allison Henrich)

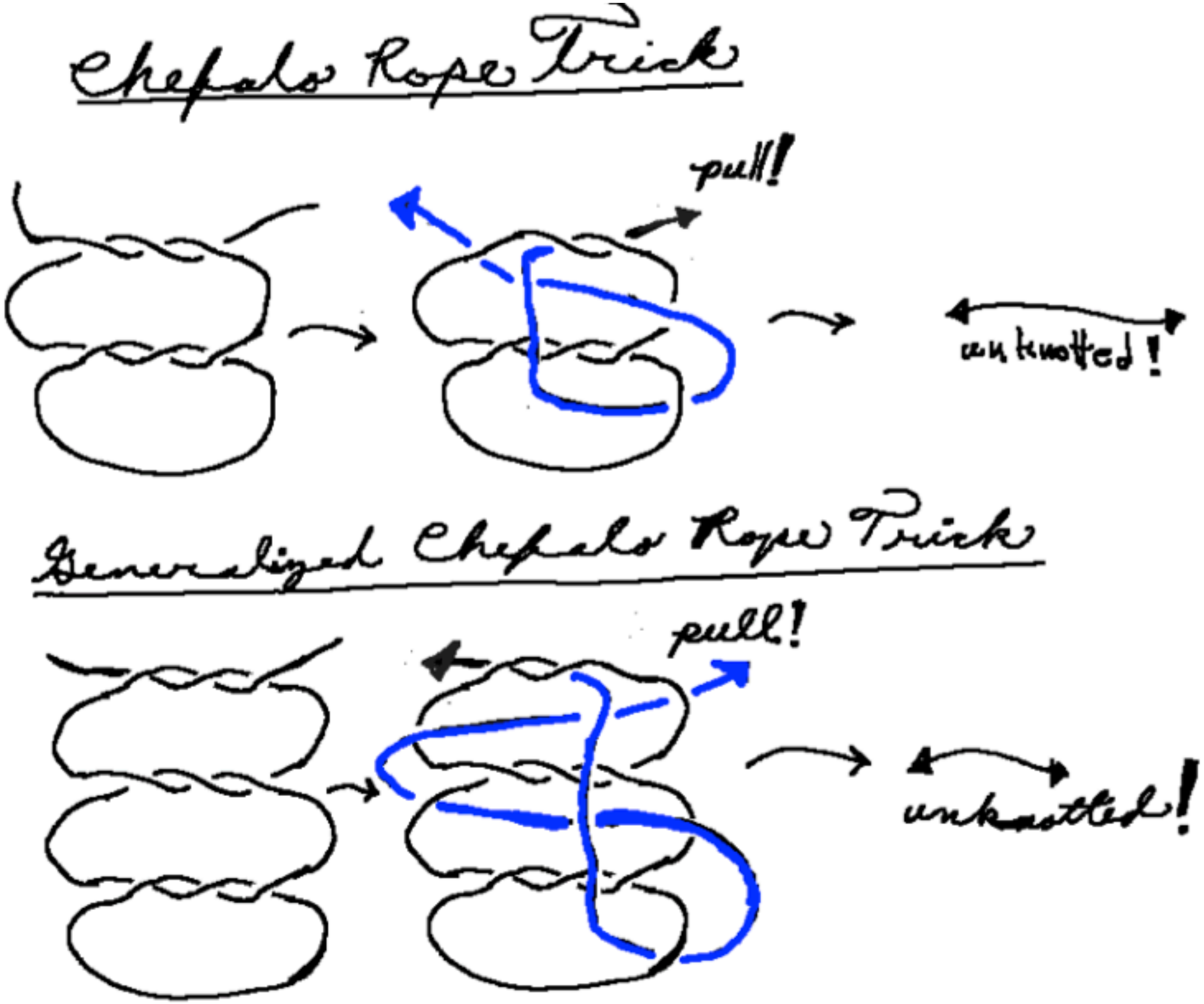


All alone, the best attempt is not knotted.

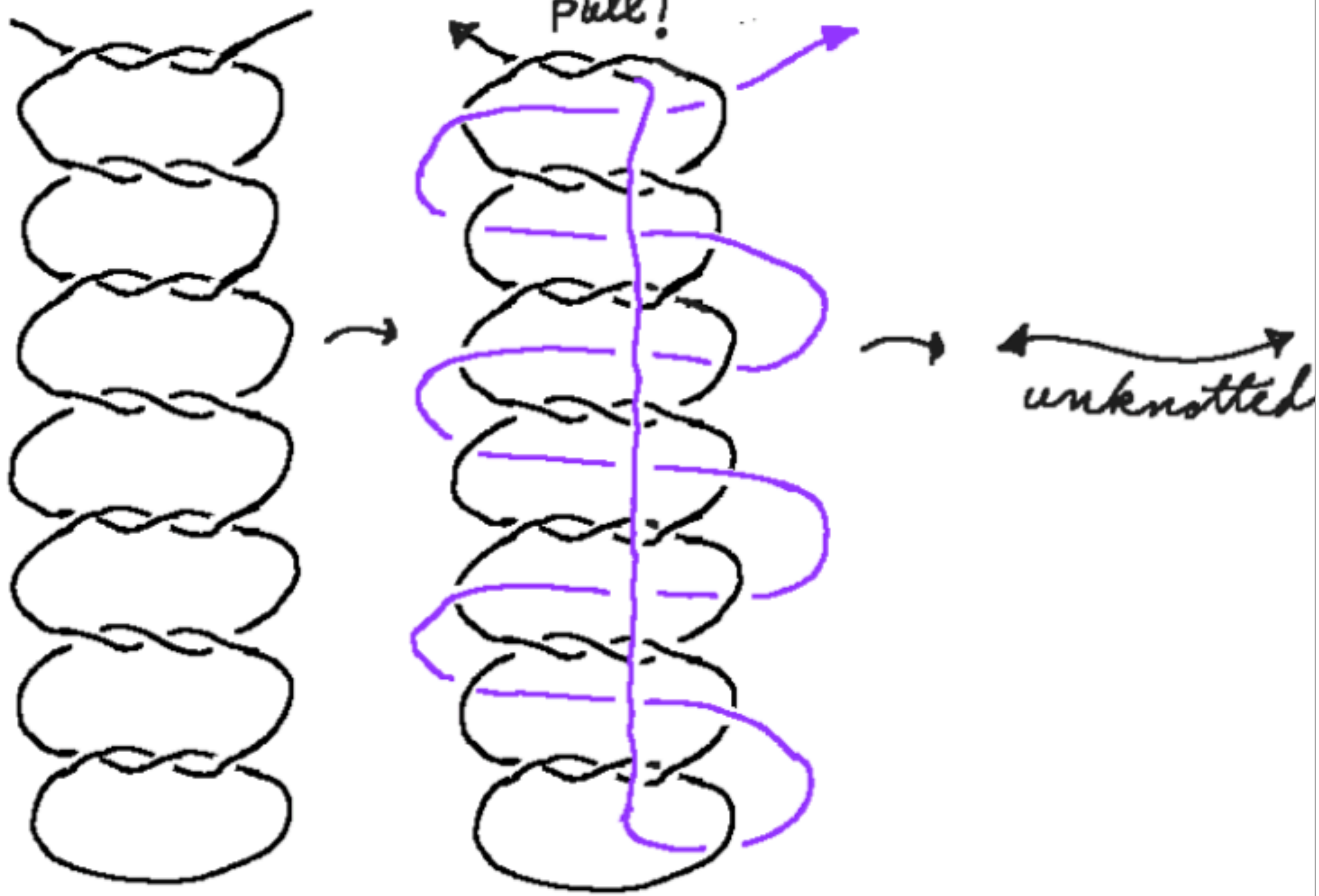


With a little cooperation, the knot appears!

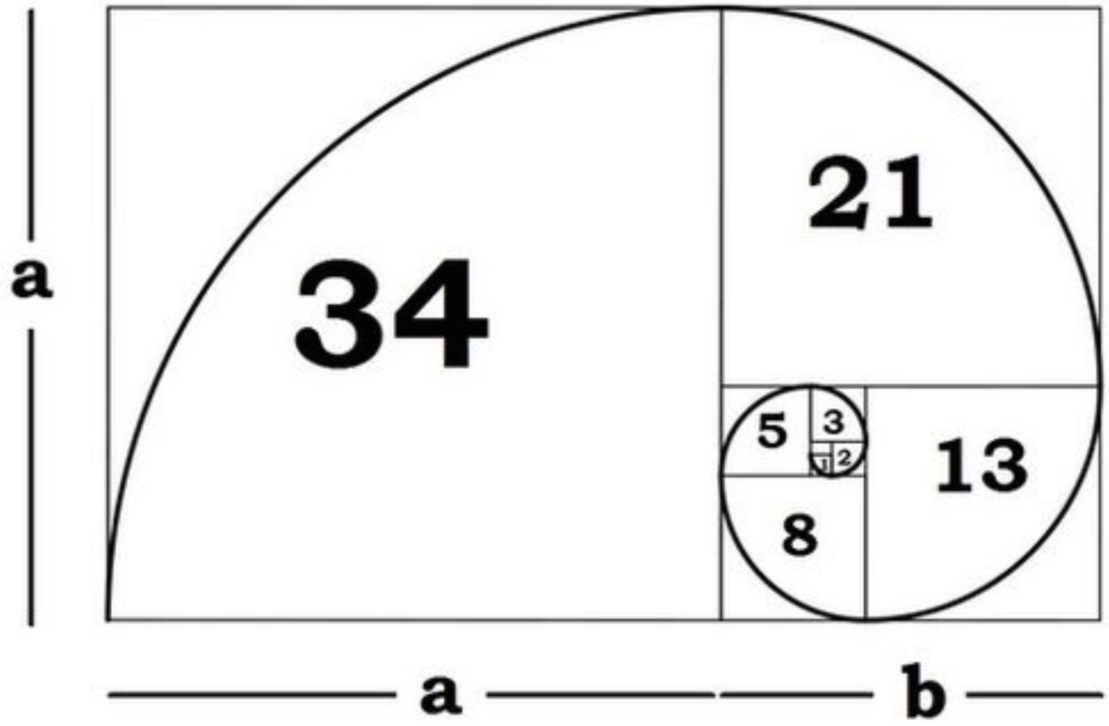
XV. Generalizing the Chefalo Rope Trick (LK and Allison Henrich)



Super Generalized Chefalo Rope Trick



15 = 3 x 5 = 1^2 + 1^2 + 2^2 + 3^2



Here you see

34 x 55 = 1^2+1^2+2^2+3^2+5^2+8^2+13^2+21^2+34^2

In general, the sum of the squares of consecutive Fibonacci Numbers is a product of two Fibonacci Numbers.

15 is our End and Beginning.