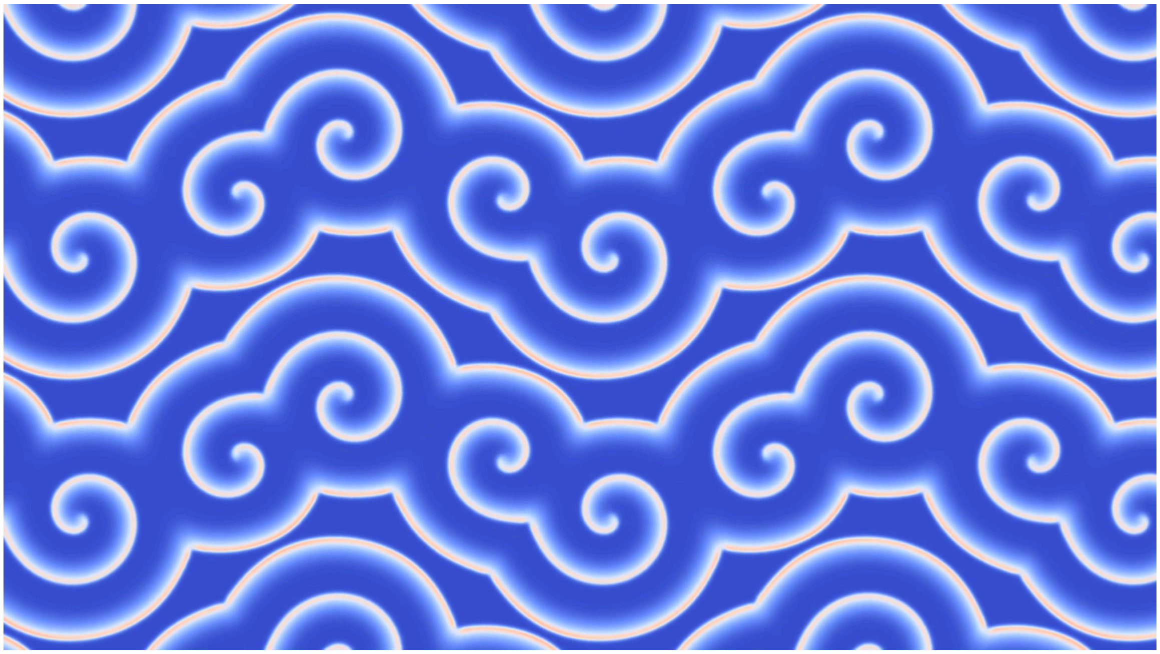

SymSim - creation of natural animated symmetric patterns using Symmetric Simulation

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Introduction

The motivation of this work is the development of algorithms to create animated seamless patterns with discrete symmetry in various geometries.

It is relatively easy to create a seamless pattern with symmetry generated by pure reflections. The basic kaleidoscope is constructed in such a way. Reflections are continuous functions and the resulting patterns are continuous. Animation of such a kaleidoscope keeps the pattern continuous. However discontinuity of derivatives along reflection lines generates obvious visible artifacts.

In case of more general symmetries it is rather difficult to make the pattern even continuous. A way to make symmetric patterns with arbitrary symmetry is the rubber stamp approach used by M.C.Escher in his tessellation work. This requires very difficult manual fitting of the tiles. No general way to animate such patterns exists.

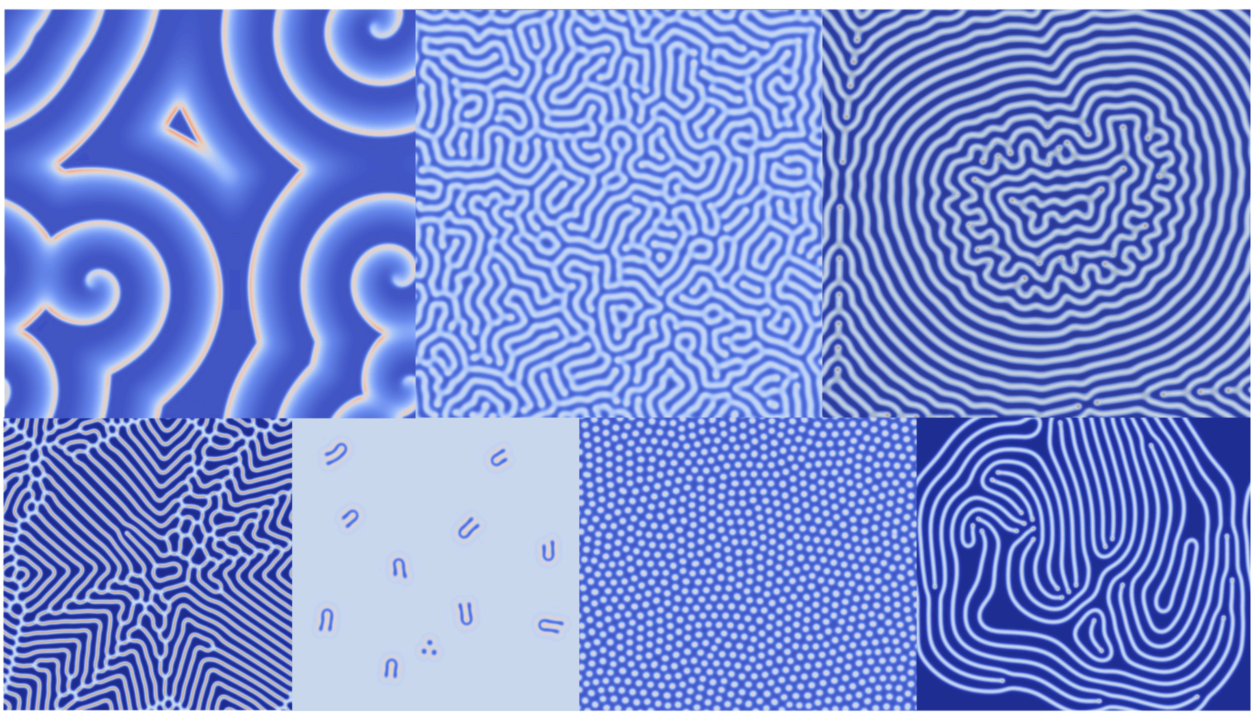
Another approach is to construct a function with fine tuned symmetric properties and use the function as a tool for domain coloring. It is difficult to construct such functions and it is hard to control its properties.

We are offering a general approach to make seamless patterns using dynamic (ordinary differential equations or partial differential equations) on an orbifold. An orbifold can be cut and flattened. The flattened orbifold will tile the whole space. If the pattern is seamless on the orbifold the tiled pattern will be seamless as well. So the problem of making a seamless symmetric pattern is reduced to creating a seamless pattern on the orbifold. This problem can be successfully solved using appropriate boundary conditions. Such patterns can be animated in real time in web browsers using javascript and WebGL.

Animated results of such an approach are presented in youtube videos.

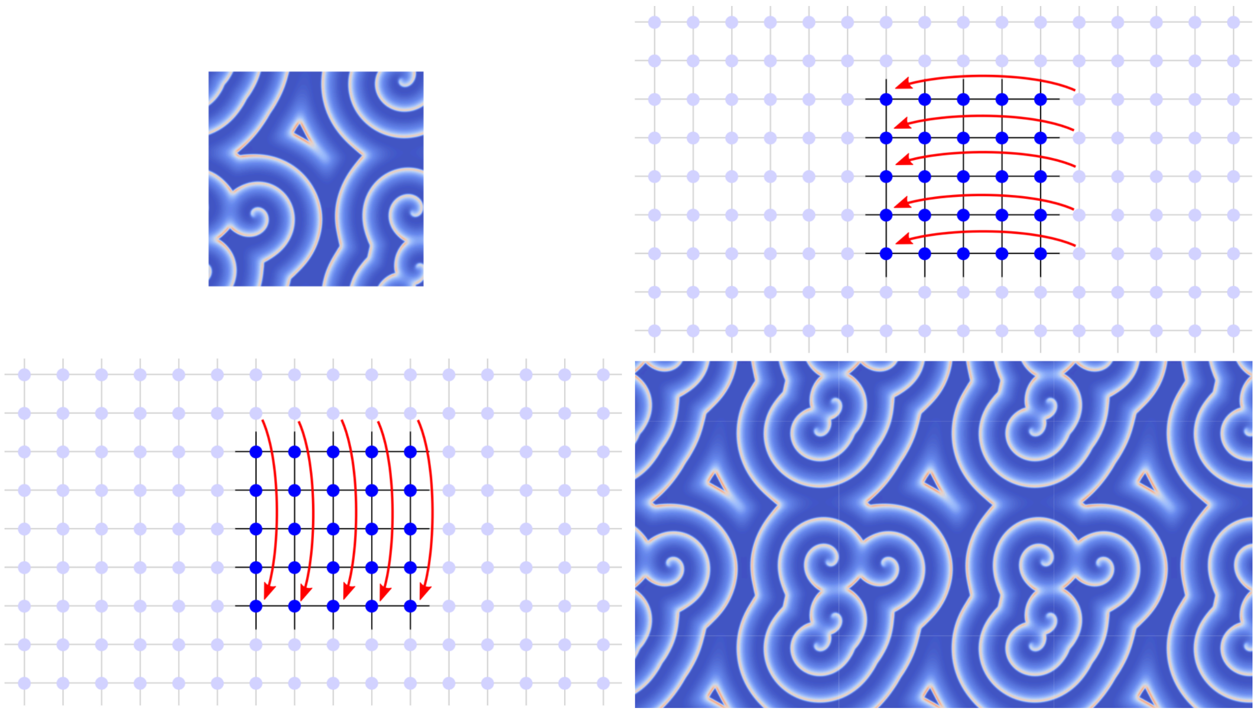
Gray-Scott Reaction diffusion

Gray-Scott reaction diffusion equation is a system of two nonlinear Partial Differential Equations (PDE) for a concentration of two different chemicals. The general behavior of its solution depends on two parameters: “kill” and “feed”. The equation is being solved numerically on a regular spatial grid patch. Starting from some initial values the solution evolves into a recognizable time-dependent pattern. The results of the simulation are visualized by mapping the concentration of one chemical to shades of color. The sequence of images for each time step can be displayed in real time during the simulation or can be exported into a video stream. Few static examples of generated patterns for different parameters are shown below.



Regular simulation on a 2-torus manifold

The solution of PDE depends not only on initial values but also on boundary conditions - constraints on the solution on the boundary of the system. In order to minimize the effect of the boundary the Periodic Boundary Conditions (PBC) are normally used. PBC requires that the values on opposite sides of the rectangular grid patch are equal. This effectively allows us to imitate the behavior of an infinite system using a grid of finite size. If the solution on the finite grid patch is periodically extended to the whole plane, the result will also be a solution of the PDE on the whole plane and the solution will be continuous and smooth across the boundaries of the finite grid patch. The simulation on the finite grid patch using Periodic Boundary Conditions is equivalent to simulation on the finite two dimensional manifold 2-torus.



Symmetric Simulation on Orbifold

The 2-torus is one of the 17 orbifolds which correspond to two dimensional wallpaper symmetries. Tiling the plane with images of the finite patch creates a pattern in the plane with symmetry **(O)** (in orbifold notations). Can we obtain other symmetries using appropriate replacement instead of Periodic Boundary Conditions? The cut and flattened orbifold is just some region of a plane with a boundary. In principle we can try to carefully fit a grid into the orbifold directly and perform the simulation on that grid and try to obey the corresponding boundary condition which reflects the way the orbifold was cut. This however would be a rather tedious procedure which requires dealing with details of a specific shape of the orbifold.

Instead we are using a simple rectangular grid which is larger than the flattened orbifold and completely covers it. The periodic boundary conditions are replaced with symmetrization. We call the rather universal and simple procedure **Sym**metric **Sim**ulation (**SymSim**). The idea of SymSim is to replace each regular simulation steps with three simple substeps:

a: symmetrization

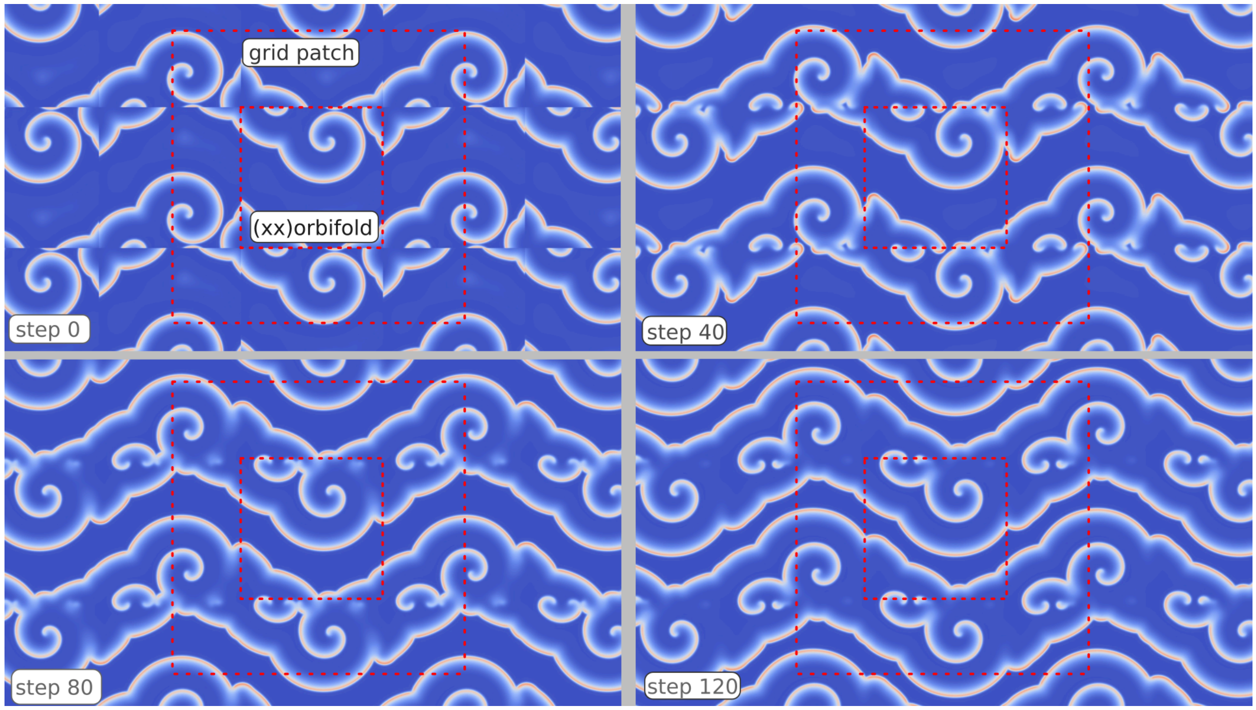
b: simulation

c: symmetric visualization

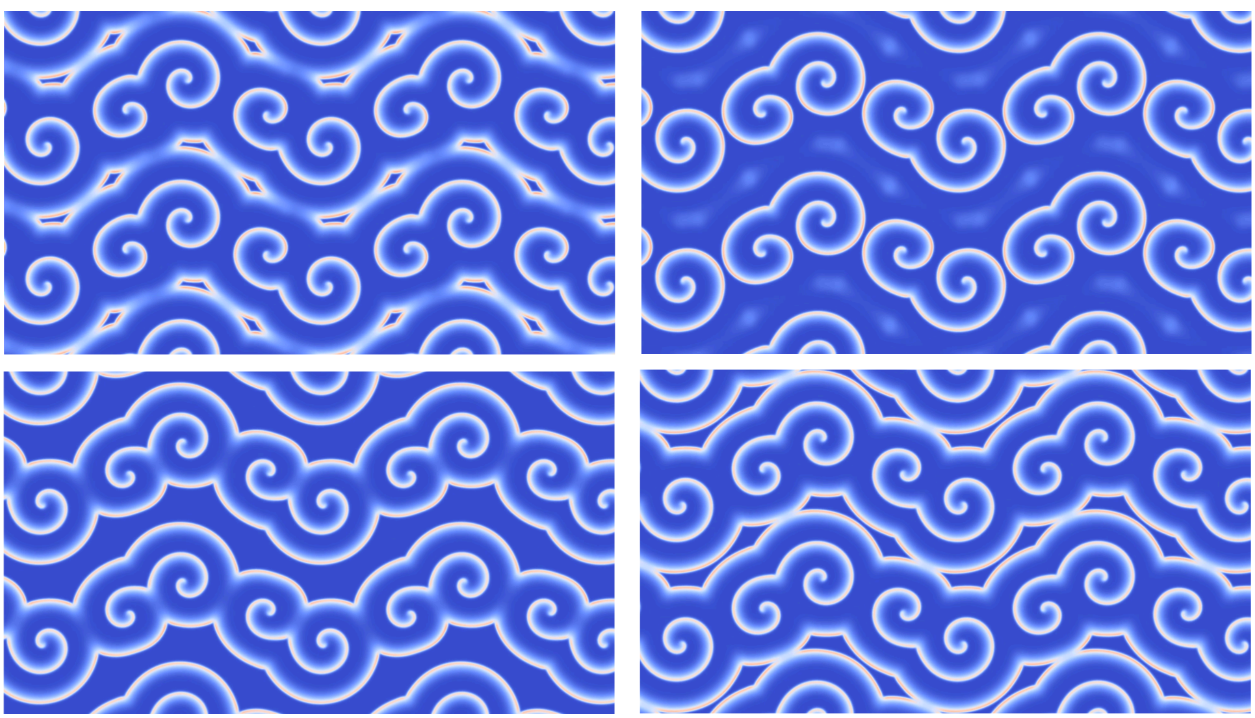
During the symmetrization step we replace the values at all the grid points which are outside of the flattened orbifold with the values from the corresponding points inside of the orbifold. It can be very efficiently performed using reverse pixel lookup procedure. The regular simulation step is performed as usual on the rectangular grid. The symmetric visualization step is once again using reverse pixel lookup procedure to map coordinates of each pixel of the plane into a point inside of the flattened orbifold and map its value into shade of color.

The symmetrization step effectively takes care of all the potentially complicated boundary conditions on the orbifold. The symmetric visualization does the extension of the finite image of the flattened orbifold into the whole plane.

Below are a few steps from simulation on orbifold **(xx)** . We start from initial values which were generated during simulation on a grid patch with periodic boundary conditions used in the previous image. The initial values were continuous functions on the 2-torus but are not continuous functions on the orbifold **(xx).** Therefore after the first symmetrization step we see strong discontinuities in the pattern. However after a few simulation steps those discontinuities disappear and the pattern of spiral waves becomes smooth.



Below are a few frames from the video of the previous spiral waves pattern taken after the waves become completely smooth and the initial discontinuities are completely forgotten.



SymSim properties

- SymSim can be used in a wide range of geometries: Euclidean, Hyperbolic, Spherical, Inversive. The simulation grid is simple. The symmetrization works.
- SymSim can be used in any number of dimensions.
- The solutions obtained using the SymSim procedure are true solutions of the original equations. The effect of the symmetrization procedure is to select a symmetrical solution from the wider class of arbitrary solutions.
- SymSim computations add very little cost to the original simulation. In practice the symmetrization step can be used only once per 1000 simulation steps.
- SymSim was initially implemented for the Gray-Scott Reaction Diffusion equation. At the moment it works also for the Complex Ginzburg-Landau equation (superconductivity phase transitions). The Navier–Stokes equations (fluid dynamics) support is under development.
- SymSim is implemented as an online application using HTML, JavaScript and WebGL2. It runs on any platform which supports these technologies (desktop, tablet, cellphone). It is available at http://bulatov.org/symsim

SymSim examples

There are numerous SymSim videos on youtube.

Gray-Scott reaction diffusion videos

https://www.youtube.com/playlist?list=PLcayjHq4OzLYKBc8TSTKAEAAyPn6NGTUZ

Complex Ginzburg Landau equation videos

https://www.youtube.com/playlist?list=PLcayjHq4OzLY0GvQ4_uGUXA3YDrMPRKTo

More information is available at http://bulatov.org/symsim