From Art to Math

The Polymorphisms of the Polymorphic Elastegrity

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Introduction

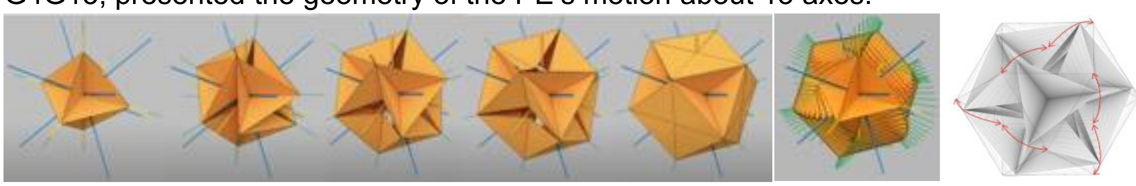
Summary of previous presentations

The Polymorphic Elastegrity (PE) is an art object created in 1982. Named elastegrity because it maintains integrity of form through elasticity,[1] aspects of it were reported at G4G12,[2] G4G13,[3] and G4G14.[4] At G4G15, we will take a closer look at the art processes, which account for its discovery and finding its diverse polymorphisms, which is why it is named "polymorphic."[5]

(a) (b) (c) (d) (e) (f)

Fig 1. (a) Polymorphic elastegrity, is composed with 8 asymmetrical tetrahedra "floating" on 12 pairs of elastically hinged right triangles; (b) Discovered monododecahedron, defined as having twelve congruent but not regular faces; (c) Brown University Mathematics Professor Thomas Banchoff (left) entered coordinates and discovered a path of monododecahedra; (d) at one end of the path rhombic faces are degenerate pentagons with one side of the pentagon equal to zero; (e) regular dodecahedron; (f) at the other end of the path is a cube with rectangle faces, degenerate pentagons with three one angle equal to 180º;

G4G13, presented the geometry of the PE's motion about 13 axes.



(a) (b) (c) (d) (e) (f) (g)

Fig 2. (a), (b), (.c), (d), & (e) 13 axes: 4 tetrahedral (blue) and 3 orthogonal (yellow) do not move; (f) 6 diameters (green), vertices move along ¼ of ellipses; 8 asymmetrical tetrahedra, each with three hinge triangles attached to it, rotate around the tetrahedral axes (blue) as they translate towards the center moving in sync. 4 tetrahedra rotate clockwise and 4 counterclockwise, expanding into a cuboctahedron or contracting into an octahedron; (c) at dihedral angles 90º the 12 vertices of the structure outline a regular icosahedron; (g) the 12 vertices of the structure move along ¼ of an ellipse.

G4G14, a tetradecahedron, a polyhedron with 14 faces, was outlined as the void between the Weaire-Phelan monododecahedron along the path described at G4G12. The Weaire-Phelan combination of two polyhedra approximates the minimum tension surface (bubbles).

. (a) (b) (c) (d) (e) (f)

Fig 3: G4G14 (a) Bubbles, minimum tension surface; (b) Weaire Phelan polyhedral approximation of bubbles with monododecahedron (yellow) and tetradecahedron (translucent blue); (c) PE folded into a WP monododecahedron supported by tetradecahedral skeleton; (d) 3D printed Weaire Phelan monododecahedra with tetradecahedral void clearly showing simple regularity of Weaire Phelan matrix; (e) "Fly-through" tetradecahedral void surrounded by WP monododecahedra; (f) Beijing Olympics "Water Cube" based on Weaire Phelan pattern sliced regular pattern at an angle to create interest.

Art skill-sets and the discovery process.

The Polymorphic Elastegrity resulted from what we termed in past G4G presentations dactylognosis (from *dactyl* = finger and *gnosis* = revealed knowledge.) As the fingertips crease, fold, and weave, it leads to discovery. It is the kind of exploration that sculptors, musicians, jugglers, and all artists engage in when creating. What is possible is found with no preconceptions or intention to simulate known forms. In previous G4G we mentioned that two Bauhaus design exercises provided the context for problem-solving that led to the invention of the PE. In G4G15, we will examine the exact art processes that led to problem-solving. *Techne (art in Greek),* where the word technology comes from, had its first meaning being “skillfully made.” We will examine how the skill set of *techne* (art) addresses the question: How did a class of structures that includes fifteen polymorphisms of the PE arose from two Bauhaus exercises?

Investigated initially in two art projects at the Yale School of Architecture, it was followed by exploring structure, symmetry, and material through the decade that followed. In architecture school, we quickly learn never to use subjective terms such as “beautiful” in design criticism. We are not interested in how a design looks. Instead, we learn to critically look at what we skillfully make to interpret and evaluate its potential for solving problems. As Picasso said in a 1956 interview, “I have a horror of people who speak about the beautiful. What is the beautiful? One must speak of problems in painting! Paintings are but research and experiment.”[6] This is even more true in architecture.

Two Art Bauhaus Exercises Give Rise to the Polymorphic Elastegrity[7]

Art Exercise 1: Creating Interest by Colliding A-B and A-B-C Sphere Closepacking

The 1971 assignment explored how to use regularity to create complexity in design. The premise was that design should always start by understanding simple rules that allow a process to create interest, exploiting the “tension” from colliding regularities. Design should never attempt to become interesting or beautiful. Instead, interest should arise by establishing regularity using the discovered simple rules and creating tension accommodating function.

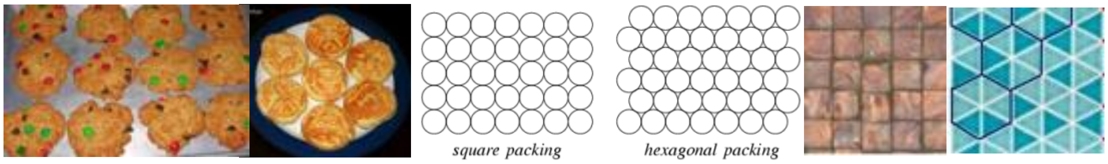
(a) (b) (c) (d) (e) (f)

Fig 4. Simple rules give rise to complexity: (a) The simple rules in knitting: yarn can only be placed over or under a needle; (b) & (c) generate an enormous variety of patterns through knitting; (d) a pentagonal interface mitigates the collision of square and triangular patterns; (e) San Francisco’s colliding grids and (f) Broadway Avenue slicing and colliding through Manhattan’s regular grid, were given as examples of creating hierarchy that supports commercial activity.

The assignment cited Anni Albers and referenced knitting as an example where simple rules give rise to interest in design. The simple rule of placing yarn over or under one or the other knitting needles leads to a large number of intricate patterns by varying the rhythm of the simple rules Fig. 4(a), (b), & (c). San Francisco’s colliding grids and Broadway Avenue slicing through Manhattan’s regular grid were given as examples of how disrupting established regularity gave rise to formal hierarchy, creating unique places that supported commercial activity Fig. 4(e) & (f). However, to interrupt regularity, one must first establish order. Interrupting regularities to adjust form to accommodate functions creates interest.

The assignment asked students to become familiar with the simple rules by which crystals close-pack in nature. First conjectured by Kepler, we were told that 100% of crystals in the periodic table crystalize in an A-B or A-B-C sphere close-packing pattern. We were to use these two equally dense, regular patterns found in nature for designing. Close-packing was first introduced as two-dimensional tiling. There are only two ways to tile with regular tiles leaving no gaps:

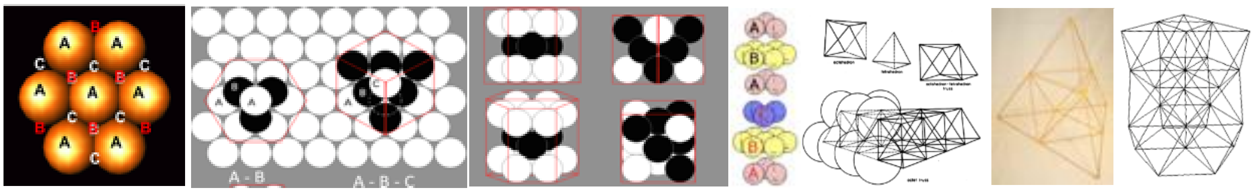
square or triangle/hexagonal, Fig 5(e) & (f). Staggering cookies in a pan allows the most or most closely packed cookies since the space between cookies is the smallest Fig 5(b). Connecting the centers of the close-packed cookies, with straight lines creates hexagonal/ trigonal tiling Fig 5(f).



(a) (b) (c) (d) (e) (f)

Fig 5 Close-packing in two dimensions: (a) compared to (b) and (c) to (d), more cookies fit in the pan staggered into a hexagonal packing because square packing leaves more space not covered by cookies.

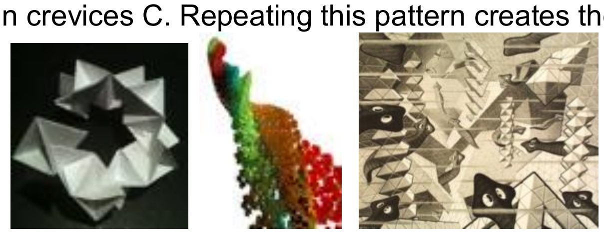
The two distinct ways of close-packing spheres were presented as an extension of close-packing in 2D by thinking of circles as sphere diameters Fig 5 (c) & (d).



(a) (b) (c) (d) (e) (f) (g) (h) (i)

Fig. 6 Sphere and crystal close-packing: (a) Each sphere A in the first layer is surrounded by 3 crevices B and 3 crevices C. Spheres placed in crevices B on the second layer touch each other and cantilever over crevices C, eliminating them from being used in the second layer. On the third layer, there is a choice to use the crevices over C or A of the first layer; (b), (d), & (g) placing them over crevice A results in repetition over every second layer forming a linear structure growing along an axis; (c), (e), & (h) placing over C results in repetition every third layer forming a structure growing around a center.

Each sphere touches six surrounding spheres, creating six crevices where a sphere can be placed on the level above Fig. 6(a). Name the spheres on the first layer, “A,” and those on the second layer, “B.” Of the six empty crevices surrounding a sphere on level A, only three can be used to place spheres on the second layer B because spheres cantilever over their surrounding crevices, thus eliminating them as locations for placing spheres. We can only utilize half of the available crevices on every level. On the third level, we have a choice to place spheres in crevices over spheres A on the first level, leaving the crevices C, which were not used on the second layer, unused. Proceeding like this, spheres close-pack in an A-B pattern, always leaving the crevices C unused, Fig 6(f-top). Alternatively, on the third layer, we can place spheres in crevices C. Repeating this pattern creates the A-B-C array equally dense as A-B Fig 6(f-bottom).



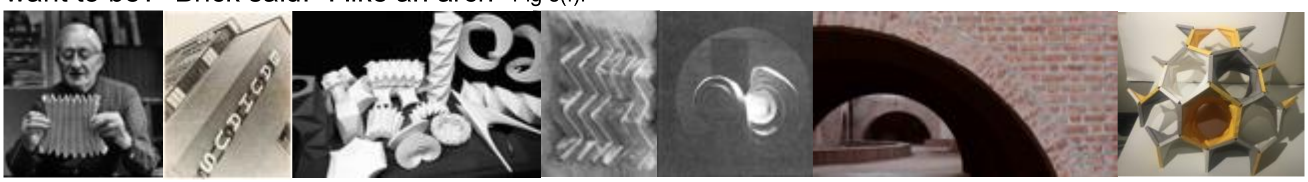
(a) (b) (c)

Fig. 7 (a) paper octahedra connected along faces and edges A-B-A-B-C recreates the 1971 helix; (b) 2014 replication of 1971 helix with branch helices; (g) M.C. Esher Flat Worms is an example of how regularity of the A-B-C close-packing creates pattern diversity.

Connecting sphere centers with straight lines turn sphere close-packing into a tetrahedral/ octahedral lattice Fig. 6(g). The assignment was to use applicator sticks to create designs using tetrahedral-octahedral close-packed lattices Fig 6(h), (I)). An experiment to see what happens when alternating A-B and A-B-C gave rise to a helix of tetrahedra and octahedra coiling around a pentagonal spatial axis. Along the length of the helical trajectory, grooves were formed to grow identical branch helices. In 2013, the 1971 finding was recreated with paper octahedra, Fig 7(a). In 2015, students alternating A-B and A-B-C electronically created a helix with identical helices branches, Fig 7(b). M.C. Escher’s flatworms, when first encountered, it was marveled for its design’s complexity. We wondered if this was a solution to the 1971 design problem. Upon close scrutiny, we realized Escher had only used the A-B-C close-packing array because all octahedra bordered octahedra along edges, never along faces Fig 7(c). If A-B had been introduced in the design, there would have been a rupture in the continuity of the close-packing space.

Bauhaus Exercise 2: Discovering the Properties of Materials to Create Form

In 1972, the second assignment was based on a well-known Bauhaus exercise to allow material and structure to dictate form Fig 8(a), (b), (c), (d) & (e). Josef Albers challenged his students to “investigate the intrinsic properties of materials, exploiting their structural possibilities and limitations.” Working with a wide range of materials–wire mesh, metal, paper, etc.–students were encouraged to examine the latent form possibilities of these substances. These exercises defined what Albers called “learning to see both statically and dynamically.”[8] In the spirit of materials dictating form, Louis Kahn, the legendary architect, asked Brick: “Brick, what do you want to be?” Brick said: “I like an arch” Fig 8(f).[9]



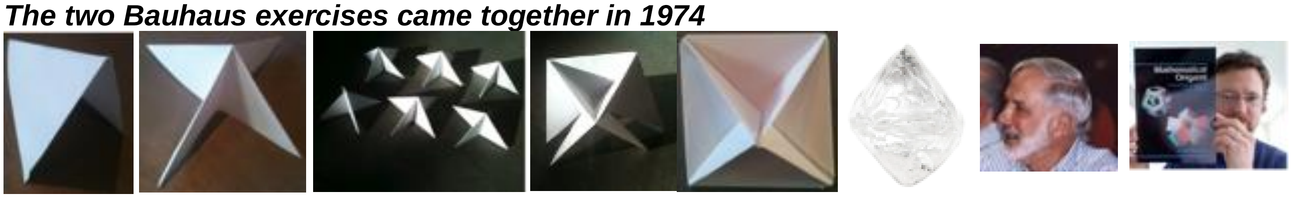
(a) (b) (c) (d) (e) (f) (g)

Fig 8 (a) Josef Gropius 1968 demonstrating Negative Poisson Ratio of zigzag fold expanding or contracting in two directions, invented in 1927-8 (b) Bauhaus German art school and design 1919-1933; (c) 1927-9 Gropius paper folding studio; (d) zigzag fold 1928; (e) saddle resulting from concentric creases 1928; (f) Brick to Luis Kahn: “I like an arch”; (g) Paper hyperbolic parabolas lattice (2015).

In that sense, the Bauhaus paper exercise is the opposite of origami, the elegant simulations of natural and geometric forms. Instead, the Bauhaus exercise explores material properties rather than simulating form with paper. It is precisely Frank Lloyd Wright’s idea of organic architecture centered around notions of a form’s function based on the interpretation of nature’s principles rather than the replication of nature’s appearance.[10]

In response to the 1972 assignment, window screen mesh was used to make hyperbolic paraboloids. Cutting the screen along the diagonals allows the squares to deform into rhombuses when bent, contributing to the curvature needed to form the hyperbolic paraboloids. I recognized the structure that emerged as a diamond lattice using the newly acquired knowledge of lattices. In 2015, students rediscovered the 1972 matrix of paraboloids by exploring the design potential of paper hyperbolic parabolas that came out of Alber’s 1927 studio. Students allowed material, structure, and symmetry to create form and arrived at the same structure created with wire mesh half a century earlier.

The two Bauhaus exercises came together in 1974



(a) b) (c) (d) (e) (f) (g) (h)

Fig 9 Creating a paper diamond: (a) square paper creased on the diagonal; (b) & (c) square creased along both diagonals; (d) 3 fragments of an octahedron woven; (e) a “*paper diamond*”; (f) octahedral diamond as found in nature; (g) Robert Neal was first in publishing this structure in 1968. His publisher named it *six-fold ornament;* (h) David Mitchel published it as *skeletal octahedron* in 1997.

Continued explorations of material, structure, and symmetry led to a 1974 experiment, creasing a diagonal on a square piece of paper, Fig 9(a). It created a surprisingly stable triangulated tetrahedral shape. A second diagonal was inserted to see what would happen, Fig 9(b), which increased both stability and strength. Since endless hours had been spent creating octahedra with applicator sticks, the pyramid with a square base was instantly recognized as a fragment of an octahedron. This raised the question of whether several such pyramids could form a whole octahedron. In a few minutes, a structure was woven with six pyramids and named “paper diamond.” Diamond because of its surprising hardness. Also, the six squares needed to weave it were reminiscent of carbon’s atomic number. Paper had said, “I like a diamond!”

David Mitchell, the author of Mathematical Origami, independently discovered the 6-square octahedral weaving in the 1980s. He named it “skeletal octahedron."[11] While researching his book, he identified artists who had independently created this paper structure and given it various names. Naming it *paper diamond* created the problem that crystals are not unitary structures but grow through self-assembly. This launched a multi-year exploration to find how to seed the growth of paper crystals into A-B and A-B-C lattices. The paper crystal was an opportunity to continue the first assignment using a faster method of assembling octahedral/ tetrahedral lattices than slow-drying Elmer’s glue on applicator sticks.

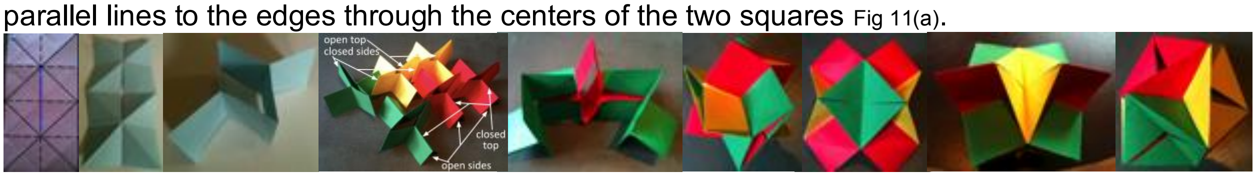
(a) (b) (c) (d) (e) (f) (g) (h) (I)

Fig 10 Two ways for growing paper crystals. I: (a) Square paper with cross slits; (b) Square paper folds into an octahedron; (c) and weaves into four cube corners outlining a tetrahedron; (d) little cubes are inserted into each other creating strong face-centered bonds; II: (e) A rectangle with a rotated square in its center flanged by two small rectangles; (f) the rotated square is folded so that the rectangles on either side are folded into squares; (g) the folded small squares are flipped into wings that can be inserted linking two octahedral units; (g) two units linked with inserted wings; (I) an octahedron with wings to grow the paper crystal in 6 directions.

Two ways to grow paper crystals were invented. The original paper octahedron was modified by cutting and folding the six vertices into 4 cube corners, which led to a square paper with cross slits weaving into a unit with four cube corners outlining a tetrahedron, Fig 10(a). The four-cube units are joined by inserting the little cubes into each other, forming strong face-centered bonds of both A-B and A-B-C paper crystals Fig 10(d). The second way found for growing paper crystals is along edges. A two-square rectangle, Fig 10(e), is folded into a pyramid with right triangle wings that can be inserted into each other, Fig 10(f) & (g), forming a malleable body-centered bond Fig 10(i).

A failed experiment led to the creation of the Polymorphic Elastegrity

Dactylognostic explorations attempted to improve growing paper crystals in terms of ease of assembly and stability. As we saw, one of these efforts described above required creating octahedra with wings Fig 10(g), so the wings could be inserted into each other to grow the crystal Fig 10(h). This required a two-square rectangle creased along the diagonals of each square and parallel lines to the edges through the centers of the two squares Fig 11(a).



(a) (b) (c) (d) (e) (f) (g) (h) (I)

Fig 11 A failed experiment to create a steadier paper crystal: (a) A 2-square rectangle with creases required to fold a pyramid with “wings”; (b) slit is torn from center to center of the two squares; (c) the rectangle with slit folded along long axis pushing the two ends together creates a cross of squares; (d) weave six crosses of little squares by placing one leg open on top over the leg with a closed ridge on top; (e) & (f) place legs open on top, over legs with a ridge on top; (f) result is three intersecting squares woven with a gaping open slit on top; (g) pushing the corner vertices towards the center creates a square closing the opening; (h) pushing the corner vertices further towards the center bisects the square and creates an elastic hinge; (I) When all 12 vertices have been flipped inside out, an icosahedron appears with 8 asymmetrical tetrahedra levitating on 12 elastic hinges. When the hinges expand to dihedral 180º, the structure becomes a cuboctahedron, and when they close to 0º, the structure contracts into an octahedron, at 90º the vertices outline a regular icosahedron.

In an experiment trying to improve paper crystal-growing, a slit was torn between the two centers of the squares, Fig 11(a) & (b). This experiment was also reported in G4G14; with more detail here, we aim to instruct how to create the PE at home. **Visit the video for the 6 yellow laser scored**

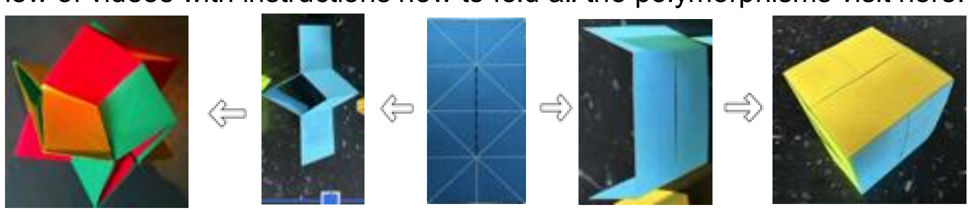
acetate in the gift exchange bag. If all else fails email epavlides@gmail.com for a zoom tutorial.

By folding the rectangle along its long axis into two and squeezing the two ends together, the slit opens and closes again perpendicular to the long axis forming a cross Fig 11(c). One axis of the cross is made with 4 little squares closed with a ridge on top and open on the sides; the other axis has 4 little squares open on top and closed on the sides Fig 11(c). Placing the squares open on top over the squares with a ridge on top, Fig 11(d) & (e), results in 3 intersecting squares Fig 11(l). However, this structure of 3 intersecting squares is not firm. It does not stay tight together as the slits gape ajar. With disappointment, this flaccid structure was discarded as a failed experiment, deemed too unstable to be of interest.

After forgetting the failed experiment, an identical structure was woven a few months later. Realizing this had been tried earlier and had failed, in an attempt to salvage wasted time, the slits were pried open Fig 11(g) & (h). Flipping the 12 outer vertices towards the center of the structure folds each of the 12 little squares into two elastically hinged triangles. 12 springs are formed supporting the 8 tetrahedra Fig 11(l). It stabilizes the entire structure into a chiral icosahedron. The 12 springs “float” the 8 irregular tetrahedra into a resilient structure that maintains its form in an elastic equilibrium. It deforms when a force is applied on one of the tetrahedral axes, but springs back pirouetting when released, returning to the original form.

Fifteen Polymorphisms followed the discovery of the lateral icosahedron

In past G4G meetings, various versions of the tree of shapes were reported. Below, we focus on the processes that led to unexpectedly discovering 3D objects, some known in mathematics, such as the five Platonic polyhedral, some novel. The multiplicity of forms this structure assumes through folding is surprising and the same form is created at various scales as you see below. For an overview of videos with instructions how to fold all the polymorphisms visit here.



(a) (b) (c) (d) (e)

Fig 12 (a) three intersecting squares; (b) cross of little squares; (c) the 2-square rectangle with the diagonals (d) C shape; (e) cube

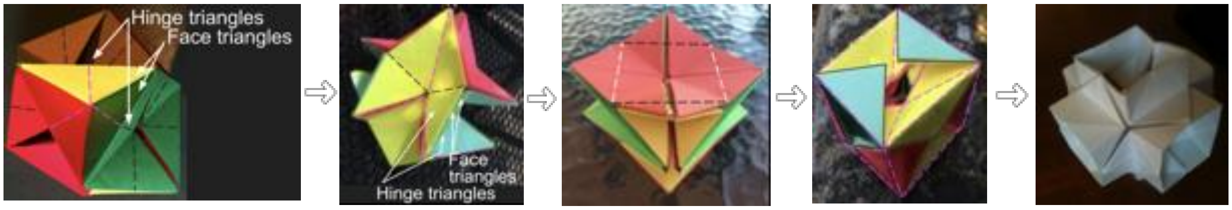
The journey of discovery starts with six 2-square rectangles with slits cut from center to center of the two squares Fig 12(c). These rectangles can be folded into crosses of little squares Fig 12(b) that can then be woven into 3 intersecting squares with gapping openings Fig 12(a) as we saw above or can also be folded into C shapes Fig 12(d) and can be woven into a cube with slits Fig 12(e).

(a) (b) (c) (d) (e)

Fig 13 (a) the Polymorphic Elasticity resulting from opening the slits of the 3 intersecting squares; (b) slit of 3 intersecting squares opened and closed forming the Polymorphic Elastegrity hinge; (c) 3 intersecting squares with gapping slits; (d) squares are folded along their diagonal in groups of 3 with tetrahedral symmetry; (e) tetrahedron.

In addition to bisecting the squares of the 3-intersecting squares 13(c) by opening the slits Fig 13(b) and pushing them down into an icosahedron Fig 13(a), they can be bisected by folding down in groups of three towards the same asymmetrical tetrahedron Fig 13(d) turning the entire structure into a regular tetrahedron Fig 13(a).

Icosahedral elastegrity=>***to 8 triradiational groups***=>***to square***=>***to cube***=>***to hypercube***



(a) (b) (c) (d) (e)

Fig 14 (a) Icosahedral Elastegrity, red dash line bisects tetrahedral faces, black dash line bisects hinge triangles; (b) 24 face right triangles bisected and squeezed between 24 bisected hinge right triangles form spokes of 8 triradiational groups of triangles. The structure is nicknamed "tumbleweed". Black dash lines are 24 newly created hinges; (c) two opposite spokes opened and flattened create a square; (d) folding corners with black dashes up 180º, corners with white dashes fold down 90º, creates a cube; (e) Unfolding triangles with pink creases under the flaps out and straightening the corners creates a hypercube representation in 3D.

Form-experimenting continued after creating the icosahedral elastegrity. Shortly after its discovery the right triangle faces were bisected along the red dashed lines while at the same time the three hinge triangles attached to the equilateral faces were bisected along the black dashed lines Fig 12(a). The simultaneous bisection was accomplished by crushing the rigid asymmetrical tetrahedra and flattening them into squares in three directions. Adjacent right triangle faces, and their attached hinge triangles were bisected and merged into four layers Fig 14(b). The new structure is nicknamed "tumbleweed".

Folding and merging the tetrahedral faces with their attached hinge triangles creates 24 new hinges, three for each crushed tetrahedron Fig 12(b black dashed lines). 24 former hypotenuse hinges were merged and bisected becoming top edges of the triradiational triangles Fig 12(b). The 12 hinges between pairs of hinge triangles now are hinges between the 8 triradiational groupings.

Further folding of the tumbleweed gives rise to numerous new objects, including a square, a cube, a hypercube, and an object symmetrical to a twelve-strut tensegrity.

Square Opening and flattening two opposite spokes of the tumbleweed creates a square Fig 12(c). The discovery that the structure can flatten into a square came before the "tumbleweed" was discovered. During the initial visit with Professor Banchoff, who had been recommended as someone who may be interested in examining the Polymorphic Elastegrity, he skillfully squeezed the icosahedron into a square. This was before he became involved with the project in a later year when he discovered a path of monododecahedra presented at G4G12.

At that meeting, Professor Banchoff's exquisite line quality when sketching demonstrated he possessed high artistic competence. After retiring, Professor Banchoff took a drawing class at the Providence Art Club, and as a member, he was allowed to take advanced art classes. Initially uncertain if he qualified for a class open only to artists, he was pleasantly surprised that he possessed advanced artistic skills, confirming the evaluation based on the line quality of his sketch. The discovery that the Polymorphic Elastegrity flattened into a square, just like his 3D hypercube model, was dactylognostically, not computationally derived. Some STEM researchers with no formal art training have acquired advanced artistic skills through alternative means.

Cube Folding 180º the corners of the square along the black dashed lines into the center of the square and 90º the white dashed lines, the tumbleweed turns into a cube Fig 14(d). The 12 edges are hinges allowing opposite faces to rotate in either direction collapsing into a flat square. Moving symmetrically in two directions is referred to as jitterbugging.

Hypercube Folding the triangular flaps up 90º from the faces of the cube and unfolding the triangles under the flaps and straightening them creates a hypercube Fig 14(e).

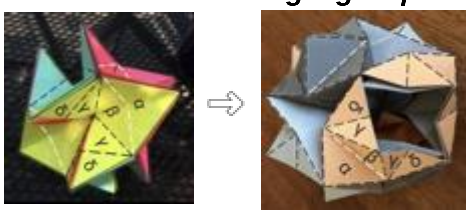
8 Triradiational groups of triangles=>to monododecahedron=>path of monododedahedra

(a) (b) (c)

Fig 16 (a) tumbleweed with 12 spokes, each spoke made of two right triangles; (b) flattening the two triangles of each spoke into a square and folding down 51.83…º along the dashed lines from center to triradiational center of the tumbleweed creates a monododecahedron when the dihedral angle of the flaps to the cube face θ = ~ 38.17… as presented in G4G12; (c) Further folding creates a monododecahedral path that includes the vertices of the regular dodecahedron shown above.

As reported in G4G12, with Professor Banchoff, we discovered a monododecahedron path.

8 triradiational triangle groups=> *to object with 12-bar tensegrity symmetry*

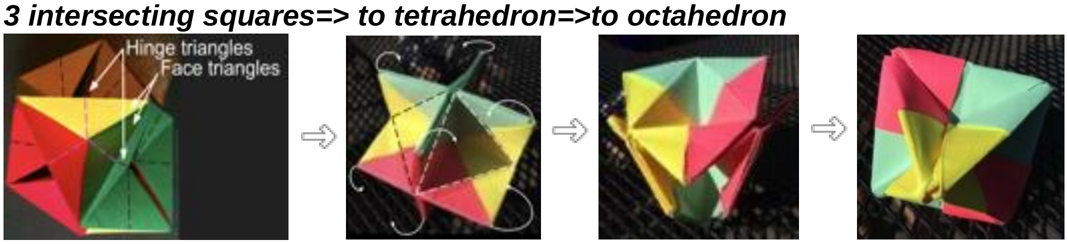


(a) (b)

Fig 17 (a) tumbleweed with 12 spokes, each spoke made of two right triangles bisected with white lines, black lines link the center of the structure to the center of the triradiation; (b) flattening the spokes and pulling triangles α, β, γ, δ up opens the slit into four vertices outlining a tetrahedron; (b) object symmetrical to a 12-bar tensegrity.

The 8 triradiational groups of triangles can turn into an object with the symmetry of a twelve-bar tensegrity by pulling up the merged triangles as shown Fig 17(a) & (b).

3 intersecting squares=> to tetrahedron=>to octahedron



(a) (b) (c) (d)

Fig 18 (a) & (b) 3-Intersecting Squares are created by bisecting the four hinge triangles around each slit and squeezing them together between four bisected face triangles; (b) &(c) squares are folded with dihedral angle 54.83…º along their diagonal of the squares in groups of 3 with tetrahedral symmetry; (c) tetrahedron; (d) by pushing in the 4 corners of the square 180º gives rise to an octahedron.

We saw above crashing the rigid asymmetrical tetrahedra by bisecting the 48 triangles of the structure with elastic hinges to create 8 triradiational groups of triangles. The same creases may be used to push four bisected triangles back-to-back around each slit to create 3 intersecting squares Fig 18(b). The faces of the bisected tetrahedral right triangles squeeze in between the bisected hinge triangles. They create the 12 little squares that compose the 3 Intersecting Squares Fig 18(b). Further folding of the 3-interesting squares gives rise to a tetrahedron and an octahedron.

Tetrahedron: Fold the 12 squares along their diagonals in 4 groups of three, with white dashed diagonal indicating folding together and black dashed diagonal indicating folding apart. At a dihedral angle of 54.83…º, the folded triangles form a tetrahedron Fig 18(c).

Folding the triangles of the tetrahedron at 180º from their original place in the 3-Intersecting Squares creates an octahedron Fig 18(d).

3 intersecting squares=>to tetrahedron=>to octahedron

(a) (b) (c)

Fig 19 (a) 3-Intersecting Squares; (b) tetrahedron; (c) octahedron

3-Intersecting Squares created when we first wove the 6 crosses of little squares; (b) folding along the diagonals of the square as we did in Fig18(b) creates a tetrahedron; (c) folding the flaps down 180º from the 3 intersecting squares as we did in Fig18(b) creates an octahedron.

Cube=>to dodecahedron=>to rhombicosahedron

(a) (b) (c) (d)

Fig 20 (a) Cube with slits on the faces; (b) the vertices outline a regular dodecahedron; (c) rhombicosidodecahedron, (d) rhombicosidodecahedron adorning the office of John Conway hanging from the ceiling on the extreme left of the photo.[12]

Pushing in the corners of the original cube with slits woven with six C-shaped elements, Fig 12(e) lifts the midpoints as the 6 slits open creating 12 vertices. The 20 vertices consisting of the 12 midpoint vertices when the dihedral angles of the hinge triangles are lifted to 90º combined with the 8 corners of the cube outline a regular dodecahedron Fig 20(b). The prescored diagonals make it possible to assemble the cube with the slits open. The resulting dodecahedron was delicate and required glue to stabilize. Additional folds were added to increase friction to firm it up, Fig 20(c). Inspecting the vertices of the resulting spherical structure, we discovered they outlined 12 pentagons, 20 triangles, and 30 squares. Later, we found out that the structure we had created by weaving 6 folded rectangles with slits as an art project was first described by Archimedes and called it ρομβοεικοσιδωδεκάεδρο. Still interesting to mathematicians, we see a rhombicosahedron hanging adorning John Conway's office.

Conclusion

We left the mathematical proofs out because of space constraints, but also as a puzzle-opportunity for the mathematically inclined. Given that all triangles are right triangles, they are derived through bisection and have well-defined dihedral angles, one can provide geometric and computational proofs of the mathematical objects suggested by the physical objects.

This article provides an example of how art skill sets could facilitate inquiry in STEM areas. Discoveries resulted from curiosity about how materials would behave when certain physical actions were attempted. The processes of discovery were the ones used by sculptors to create form. Carefully examining the resulting structures revealed interesting mathematics, suggested useful applications in engineering,[13] and provided explanatory properties for biology.[14]

How important are artistic skills in the education of a scientist or an engineer? Were Einstein's violin playing and Feynman's bongo drumming a coincidence or materially connected to their scientific contributions? It is possible that the work of artists can contribute to advances in engineering and science?[15] At least one Nobel Prize depended on expertise in water coloring: Fleming's discovery of penicillin. An art experiment that went awry requiring visual interpretation.[16]

TREE OF FORMS VIDEOS

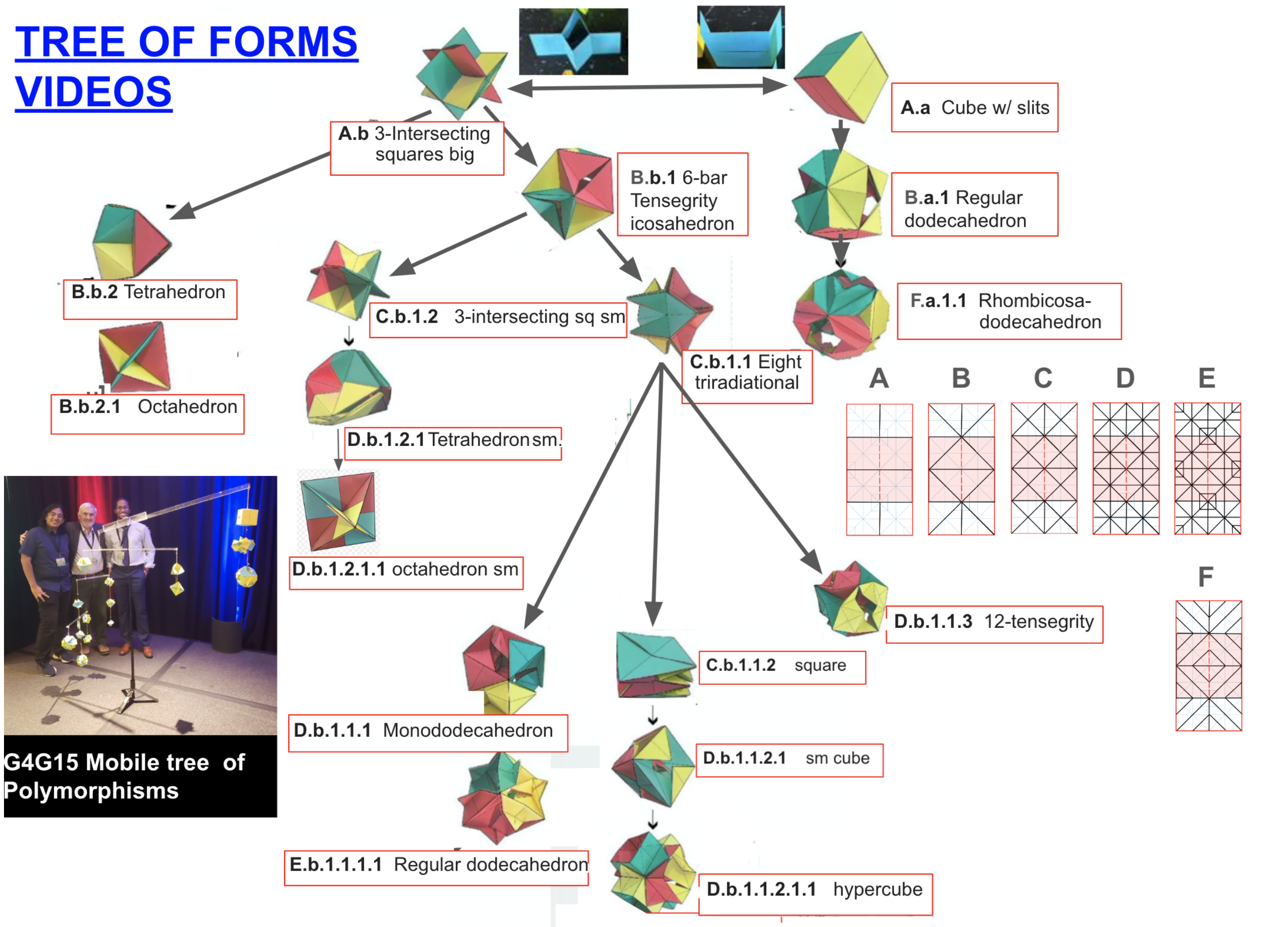


Fig 21 Tree showing derivation of forms through further folding. Instructions on how to fold them can be found here.

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[1] The chiral icosahedron has the same symmetry as a 6-bar tensegrity and initially referred to as tensegrity. The term elastegrity was suggested when inventor of tensegrities Kenneth Snelson pointed out the term had been misapplied to structures with moment connections such as geodesic domes, which did not maintain integrity of shape due to tension. Elastegrity more accurately describes structures that maintains integrity of shape due to elasticity.

[2] E. Pavlides, T. Banchoff *Chiral Icosahedral Hinge Elastegrity's Shape-shifting* G4G12, 2016.

[3] E. Pavlides, P. Fauci *Chiral Icosahedral Hinge Elastegrity's Geometry of Motion*, G4G13, 2018.

[4] Eleftherios Pavlides, Thomas Banchoff, Alba Malaga, Chelsy Luis, Kenneth Mendez, Ryan Kim, Daanish Qureshi, *Tetradecahedron as Palimpsest of the Monododecahedral 1-Parameter Family of the Polymorphic Elastegrity*, G4G14.

[5] At G4G12, it was termed “chiral icosahedral hinge elastegrity. In 2020 an editor simplified it to “Pavlides Elastegrity”, by analogy to the Hoberman Sphere and the Rubik’s Cube. We concluded polymorphic is more descriptive.

[6] Rachel Hooper, *Picasso Black and White*, March 11, 2013.

[7] The exercises were assigned by sculptor and beloved Yale Professor Kent Bloomer.

[8] F. Horstman, Preliminary Course, and the Matière *Josef Albers Papers*, Josef & Anni Albers Foundation, Bethan, CT.

[9] W. Lesser. *You Say to Brick: The Life of Louis Kahn*, Farrar, Straus and Giroux, 2017.

[10] Frank Lloyd Wright: From Within Outward Audioguide (NY: Antenna Audio, Inc. & the S. R. Guggenheim Found, 2009)

[11] Mitchell, D. *Mathematical Origami*, Tarquin, 1997. Mitchell named it “Skeletal Octahedron”. In correspondence, he identified Bob Neale as the earliest independent creator, who published it in 1968 as "Sixfold Ornament."

[12] John Horton Conway in his office at Princeton University in 1993. Photo credit Dith Pran/The New York Times

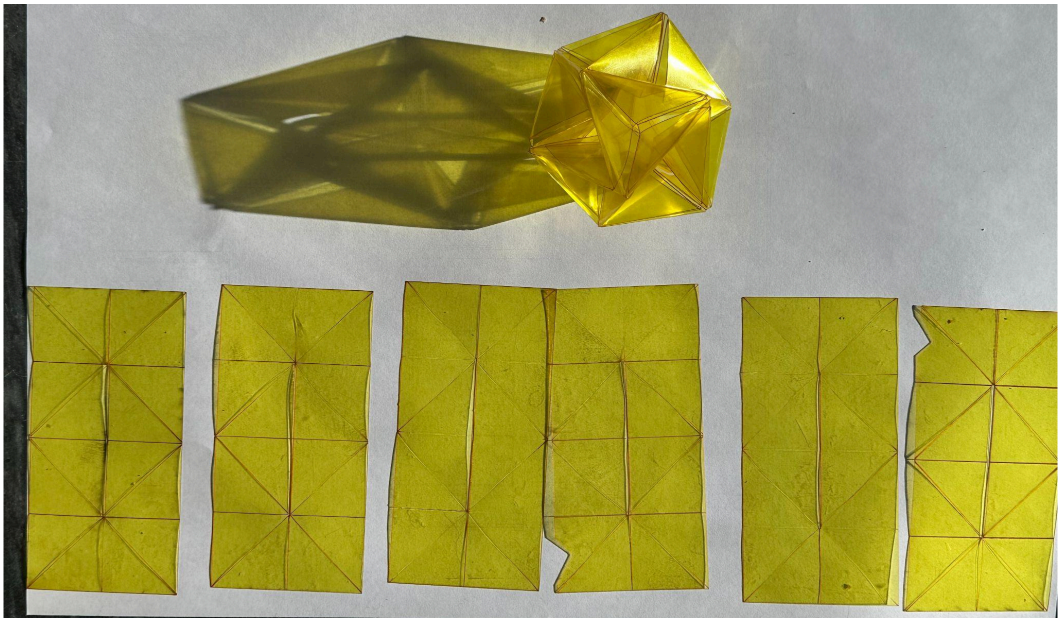
[13] RI NASA funded students with summer stipends to work on the Polymorphic Elastegrity 2019, 2020, 2021, & 2022. The PA has -1 Negative Poisson’s Ratio w/ 36 hinges contracting when pressured. It has very large specific energy absorption. Made with 48 right triangles, has tiny mass and enormous energy absorption and dissipation as 36 elastic hinges contract and compress air as the volume of each cell contracts exponentially.

[14] There is an extensive literature on tensegrity as the basis of all biological structure at all size scales in numerous species including wood, for example D. Ingber Tensegrity as the architecture of life IASS, Boston, MA, 2018, pp. 1-4. and L. Brownell The “architecture of life,” described by computer modeling, Wyss Institute, 2018.
The Polymorphic Elastegrity augments the tensegrity theory at NSF Mechanobiology Symposia 2019 and 2021.

[15] J. L. Aragón et al. Turbulent Luminance in Impassioned van Gogh Paintings. J Math Imaging Vis 30, 275–283 (2008).

[16] Rob Dunn, Painting With Penicillin: Alexander Fleming’s Germ Art, Smithsonian Magazine, July 11, 2010.

Creating Polymorphic Elastegrity (PE) Science Puzzle [click for animations]



Video on how to assemble the 6 element into jumpy polymorphic elastegrity **click here**

Or you may prefer **this video**

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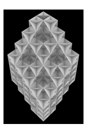
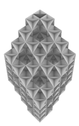
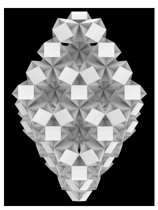
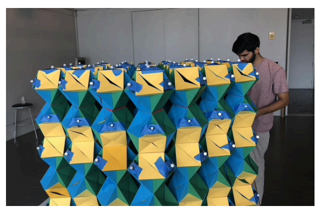
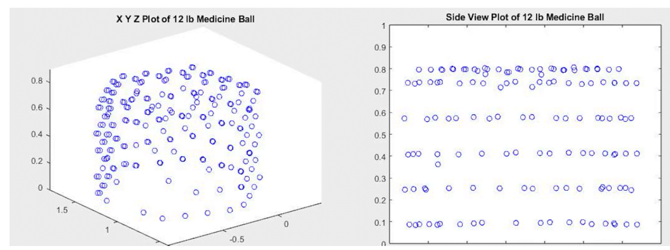
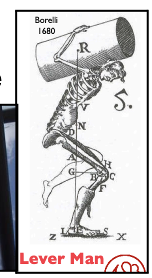
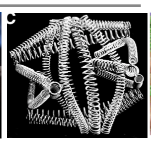
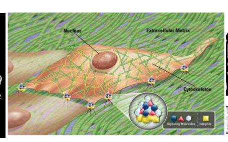
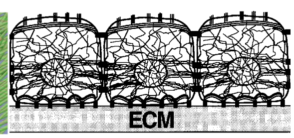






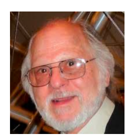
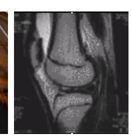
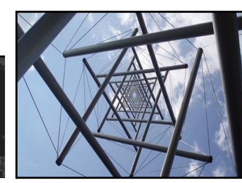
a. b. d. e. f.

a. Matrix contracted; b. pulsating; c. expanded; e. drop sphere on185 units + motion sensors; f. 6660 hinges contracting simultaneously.



Engineering NASA-funded the PE in hopes of developing impact mitigation technology that is highly auxetic, with Poisson's ratio -1 along 4 axes, enabling it to resist impact by migrating mechanical deformation outside the contact zone without adding reinforced additives. That feature is crucial for protection (e.g., armor, automobile fender, planetary landers.)

a. b. c. d. e. f. g . h .

a. Ingber; b. tensegrity w/springs; c. cytoskeleton; d.ExraCelularMatrix. e. Levin; f. bones do not touch; g. tensegrity tower; h. Borelli's Lever Man

Biology: addressing objections to the tensegrity conjecture and augmenting it:

Ingber, Wyss Institute director conjectured that the cytoskeleton is a tensegrity, not balloon w/molasses.

Levin, an orthopedic surgeon, conjectured the musculoskeletal is a tensegrity, not levers as per Borelli.

The PE addresses persisting objections to the tensegrity theory that persist even after 40 years:

(1) The 36 hinges per unit allow stability and pulsing in sync; tensegrity exhibits asynchronous pulsing;
(2) Explains pumping biofluids by exponential expansion and contraction of the PE's interior volume;
(3) Has simple assembly through folding a membrane by contrast to tensegrity that requires two materials & scaffolding;
(4) Conformation by folding a single membrane accommodates shape-shifting vital to biological function;
(5) Can be created easily at all size scales as smaller unit matrices can create larger units;
(6) Explains extreme auxeticity in biology found experimentally in certain directions;

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