

The Statistics of the I Ching*

Jim Guinn

Professor of Physics and Astronomy

Georgia State University Perimeter College

The I Ching (pronounced “ee ching”), or Book of Changes, is one of the great classics of Chinese Confucian and Taoist philosophy. It is thought to be between two and three thousand years old. The main body of the text contains interpretations of hexagrams created through numerical processes that can be used as oracles or a source of wisdom. The method that the oracle reader (henceforth referred to as “reader”) uses to determine which hexagrams are to be used, is very mathematical and I believe to be an amazing combination of seemingly random choices to generate a series of hexagrams with over four thousand possible outcomes with predictable probabilities. There are two approaches for determining the hexagrams to be used, the traditional method using yarrow stalks, and another using coins. In this paper I will discuss the yarrow stalk method.

History: It seems that the development of the I Ching began with just two possible outcomes for the oracle. These were represented by a solid line



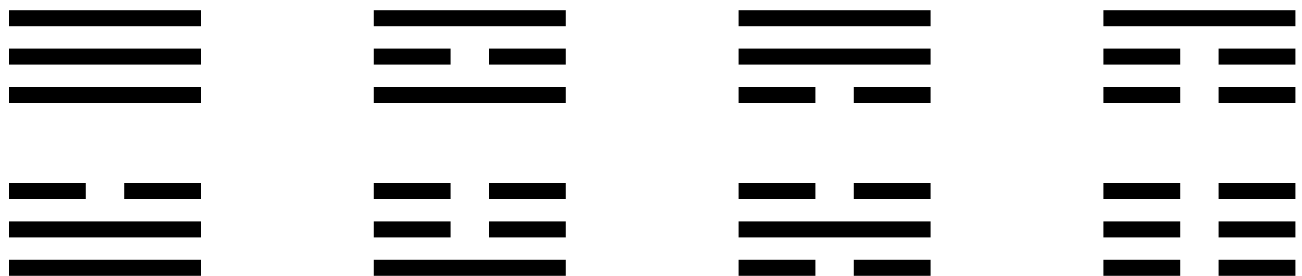
which represented yes or masculine (yang), and a broken line



which represented no or feminine (yin). As more variation in the oracle was needed, a second line was added, above the first, yielding four possibilities,



and then a third line above the first two



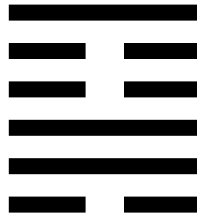
yielding eight trigrams. Each trigram has a name, an attribute, an image, and a family relation associated with it. For example,



* The information about the I Ching in this paper comes from “The I Ching or Book of Changes”, translated from Chinese into German by Richard Wilhelm, and translated from German into English by Cary F. Baynes, Bollingen Series XIX, Princeton University Press, 1950.

is named Sun, or the Gentle, is considered penetrating, is represented by wind or wood, and is the first daughter of the family. Two trigrams were then stacked, one atop the other, to form sixty-four possible hexagrams, and each hexagram had an interpretation depending on the trigram below and the trigram above.

For example,



is the hexagram Ku, with the trigram Sun below and the trigram Kên above. A further variation was added to include a second hexagram developed from the first hexagram. The first hexagram (first generation hexagram) represents the present state for the oracle requester and the second hexagram (second generation hexagram) represents the future state for the requester. The main body of the I Ching goes through the interpretation of each hexagram (as a first-generation or second-generation hexagram).

Each line of the hexagram was taken to have two forms. The solid masculine yang line could be either an old yang (old man) or a young yang (young man), and the broken feminine yin line could be either an old yin (old woman) or a young yin (young woman). Each type of line, solid or broken, in the first-generation hexagram could remain the same or change into a different line, in the second-generation hexagram depending on whether it is old or young in the first-generation hexagram. Old yang dies and is reborn as young yin, young yin grows up into old yin, old yin dies and is reborn as young yang, and young yang grows up into old yang. This meant that any hexagram in the present (first generation) could become any other hexagram in the future (second generation), yielding $64 \times 64 = 4096$ different possible oracular interpretations. The question was then which hexagrams to use for a requester's oracle.

The yarrow-stalk method is one of the two ways (the other is the coin method) of choosing the hexagrams for an individual. This method begins with fifty stalks, but one is put aside and is not used for the remainder of the procedure. (Although I have no evidence for this, I like to think that the fiftieth stalk started out as a suggestion to have one extra, in case a stalk gets broken or lost, and that over the years, the optional stalk turned into a required stalk.) The oracle requester then divides the forty-nine stalks into two groups in proportions of their choosing, one group placed on the left side and one placed on the right side. The oracle reader then does the following: one stalk from the right-side group is removed and placed between the oracle reader's little finger and ring finger of the left hand. The left-side group then has bundles of stalks removed four at a time and put aside until one, two, three, or four stalks remain. These are placed between the ring finger and middle finger of the oracle reader's left hand. The right-side group then has bundles of stalks removed four at a time until one, two, three, or four stalks remain; these are then placed between the middle finger and index finger of the oracle reader's left hand. The total number of stalks in the reader's left hand will now be either five or nine. Assuming a relatively equal separation of the initial group of forty-nine stalks, five stalks will remain with a probability of 0.75, and nine stalks will remain with a probability of 0.25. A remainder of five stalks count as three "points" while a remainder of nine stalks count as two "points". These five/nine stalks are then put aside and the remaining forty-

* The information about the I Ching in this paper comes from "The I Ching or Book of Changes", translated from Chinese into German by Richard Wilhelm, and translated from German into English by Cary F. Baynes, Bollingen Series XIX, Princeton University Press, 1950.

four/forty stalks (respectively) are divided by the oracle requester into two groups again and the previous procedure is repeated. Again, assuming relatively equal separation of the new group, the possible remainders now are eight, with a probability of 0.5, or four, with a probability of 0.5. The four stalks count as three “points” while the eight stalks again count as two “points”. The four/eight stalks are again set aside and the remaining thirty-two/thirty-six/forty stalks are used for a repetition of the procedure for a third time and final time. The possible final remainders are again eight, with a probability of 0.5, or four, with a probability of 0.5. Again, eight stalks count as two “points” and four stalks count as three “points”.

The point totals then determine the first line for the first-generation hexagram. The possible final point totals are six, with a probability of 0.3125, which forms the old yin line, a total of seven, with a probability of 0.4375, which forms the young yang line, a total of eight, with a probability of 0.1875, which forms the young yin line, and a total of nine, with a probability of 0.0625, which forms the old yang line. These are summarized in Table #1 below. Notice that the probability of forming a yang line (young yang + old yang = 0.4375 + 0.0625) is 0.5 and the probability of forming a yin line (young yin + old yin = 0.1875 + 0.3125) is 0.5. It is equally likely that a first-generation line is a yang line or yin line.

Table #1

Point Total	First Generation Line	Probability
6	Old Yin	0.3125 (= 5/16)
7	Young Yang	0.4375 (= 7/16)
8	Young Yin	0.1875 (= 3/16)
9	Old Yang	0.0625 (= 1/16)

For the second-generation line, old yin becomes young yang (yin → yang), young yang becomes old yang (yang → yang), young yin becomes old yin (yin → yin), and old yang becomes young yin (yang → yin). The probabilities for the second-generation lines are summarized below in Table #2.

Table #2

Point Total	Second Generation Line	Probability
6	Young Yang	0.3125 (= 5/16)
7	Old Yang	0.4375 (= 7/16)
8	Old Yin	0.1875 (= 3/16)
9	Young Yin	0.0625 (= 1/16)

Notice that for the second generation, the probability of having a yang line (young yang + old yang = 0.3125 + 0.4375) is 0.75, while the probability of having a yin line (young yin + old yin = 0.0625 + 0.1875) is 0.25. It is three times more likely for a second-generation line to be a yang line than a yin line.

The procedure described above is then done five more times, for a total of six lines, the first line appearing at the bottom of the hexagram, and the sixth line at the top of the hexagram. The hexagram formed with the first-generation lines represents the present state of the individual’s life, while the hexagram formed with the second-generation lines represents the future state of the individual’s life.

* The information about the I Ching in this paper comes from “The I Ching or Book of Changes”, translated from Chinese into German by Richard Wilhelm, and translated from German into English by Cary F. Baynes, Bollingen Series XIX, Princeton University Press, 1950.

With the probabilities given above, the most probable hexagram (probability ≈ 0.092) is one formed with three young yang lines, two old yin lines, one young yin line, and zero old yang lines. (This high probability is due in part to the probability of the individual lines, and in part to the large number of permutations of this arrangement.) This then is a first-generation hexagram with three yang lines and three yin lines. The most probable second-generation hexagram then has three old yang lines, two young yang lines, one old yin line, and zero young yin lines. This then is a second-generation hexagram with five yang lines and one yin line.

The highest probability of yang/yin lines, regardless of the age of the line, is also three yang and three yin, with a total probability of 0.3125.

The first-generation hexagram with the lowest probability is one formed with six old yang lines (probability $\approx 5.96 \times 10^{-8}$). The lowest probability of yang/yin lines, regardless of age, are either a hexagram with six yang or six yin lines (each of which have a probability ≈ 0.016).

I do not know whether the early developers of the yarrow-stalk method of choosing the hexagram lines understood the probabilities of the yarrow-stalk outcomes, but I believe that anyone who cast the oracle many times would eventually notice that some of the lines and hexagrams occurred more often than others. The early users of the yarrow stalks may have developed the method to yield, what was to them, an acceptable distribution of outcomes. Personally, I find the whole approach beautiful and fascinating. The procedure seems on the surface, to be rather complex, and yet yields a simple set of probabilities with some yin/yang symmetries and other asymmetries.

* The information about the I Ching in this paper comes from “The I Ching or Book of Changes”, translated from Chinese into German by Richard Wilhelm, and translated from German into English by Cary F. Baynes, Bollingen Series XIX, Princeton University Press, 1950.