Math is Magic?

Martin Gardner liked to write about card tricks done with math 'magic'. His first book "Mathematics, Magic and Mystery" was published by Dover in 1956. He also got the contract for his "Mathematical Games" column in Scientific American that year, and his first column was about flexagons. This greatly influenced me to experiment with twisted flexible circuits. Every time I wrote a letter to Gardner he would answer and often suggest a source for a question I asked him. I also wrote letters to Isaac Asimov and Buckminster Fuller and others and got answers. But Gardner's work was what I liked most, and his influence was felt by an untold number of people, young and old, interested in anything logical, mathematical, magical, or scientific. One observation about what mathematicians do is to reveal how a magic math trick works by proving it with known math. Thus, revealing math tricks using other revealed tricks or logic. Not sleight of hand but exact reasoning by proof of a theorem with exact reasoning, using postulates and axioms and previous proofs. Many proofs require new supporting theorems that must be proven before the proof is complete. A proposition by Pierre de Fermat around 1637 is known as Fermat's last theorem. It took over 300 years to prove and a very long proof by Andrew Wiles, involving math that did not exist in Fermat's time. In a case like this and much new math the magic cannot be fully revealed but an astute author like Gardner can reveal in a simple way how the basic techniques worked without causing the reader angst.

Mirrors and Observers

Are mathematicians magicians? One answer is yes and one is partly, and one is sometimes, and one is no, and all four are sometimes, are the answers. Here is a riddle: What is not necessarily created by an artist, needs no power source, and has many different pictures and movies to show? Well it might be kind of an obscure riddle and maybe not well posed but the answer is a mirror. When you look in it you see a picture of yourself. Move and sway, blink a little and you are watching a video of yourself watching you. Whenever there is light the mirror is active, showing a picture. It could be a still life or any motion observed by the mirror. In the distant past some royal rulers would not allow lay people to own mirrors. Often lay people were very suspicious of mirrors believing that the mirror could steal away some of their soul when they were not looking in it so they kept it blindfolded when not in use. Even today some people avoid looking in a mirror except when necessary. The person in the mirror is the person outside the mirror, (but not quite!) and there might be a bit of suspicion that the mirror person is critical of the real person and vice versa. Who is really looking at who? Does it mean we are born with a quantum theory instinct about entanglement of soul and light? Louis Carrol used mirrors with his masterworks "Alice in Wonderland" and "Through the Looking Glass". When I was in Africa, I started to take a picture of a lady making peanut butter. She immediately waved me away. My African friend said she may think the camera can steal a soul. In fact she probably just thought her appearance was not good that day.

Quantum Theory and Elementary Geometry

Mathematical magic long ago started with the need to find or stake out property lines and building perimeters and so forth. From these beginnings the Pythagorean triplet was born, and was later turned into the Pythagorean theorem by the Greeks. Thus the logical postulate of a point was born. Simple and utterly devoid of anything like magic. Yet modern Quantum theory says a point with no size cannot exist. It must at least have Planck size to exist. Also, it cannot be precisely located. In fact, it can and must be several places at once, existing as a standing wave or moving rapidly as a traveling wave, not a one-dimension mathematical line. Thus, points and lines have varying sizes, and probability attributes, and cannot be precisely located. Nature simply refuses to be roped down! But mathematicians love the lines and points. They do magical wonders in math. They cannot ever be tossed away in favor of natures system. Thus, there you have it, mathematical geometry was magical from its very beginnings. The same extends to infinitely thin planes, perfect polygons and polyhedrons, and all polymorphs. These things give us immense power because nature does not 'know' of them and instead tries in vain to find them with its main law that all must always move. That is not to say we cannot have Planck size moving points in math, but instead that perfect points are the anchors for Planck size things. How could we have a concept of size and location if we have no way to refer it to zero size and exact location? The same reasoning extends to Einstein's curved space. Without being able to refer it to flat space the curvature would not be so easily conceived. In fact physicists often use flat space to make curved space calculations simpler, similar to how you use a topo map to calculate how much dirt must be excavated to flatten a plot of land.

Euler and the Real World

Leonhard Euler was a bold math innovator and creator. He was not afraid to wrestle with and reveal new mathematics using untested techniques, often with no proof other than his method produced numbers that worked and were gems of mathematical substitution and manipulation. He wrote an important equation which was much later the starting inspiration for String theory. His polyhedral formula opened up, actually more correctly, created the field of topology. It is an amazingly simple formula relating the faces, edges and vertices of a polyhedron, $E = F + V - 2$. He proved this formula, and it is now recognized as a gem of mathematics because it created the science of topology. It is easy to count E, F and V on the 5 Platonic solids and see that this holds true. Is it true for all polyhedra? Yes, if they are convex, meaning that all dihedral edge angles are greater than 180 degrees, and all vertices point outward, and all faces are flat. For instance, a hexagon shaped antiprism could have the top and bottom 2 vertices pointing inward and touching each other making a single vertex. Thus, the formula is off by 1.

Euler took all this into account in proving his formula. In the real world you can have a box with one face missing to allow use as a container. The missing face can be called a null face, Nf. To make the formula work for Nf's we can include Nf = 1 in the formula as $E = F + NF + V - 2$. This works if two Nf share an edge by retaining the edge thus a skeletal polyhedron has zero F! Similarly, you could have a box with a lid and the formula still works if Nf is zero with the lid shut, and we make the lid a face when it is open. When the lid is shut Euler is polyhedral. When it is open, we have a lid face, Lf so we have 15E, 5F, 10V, 1Nf, 1Lf, and 15=5+10+1+1-2. It is not 'true Euler' but works anyway. Eulers proof took some effort because how do you know that some very odd polyhedron with a multitude of elements might not be Euler? Another question is, can two different polyhedrons have the same numbers of E, F, and V? That question is what creates an essential feature of topology. You can, indeed have two polyhedrons with the same E, F and V as a cube! This is not possible with a tetrahedron.

A magnetic and gravitic world

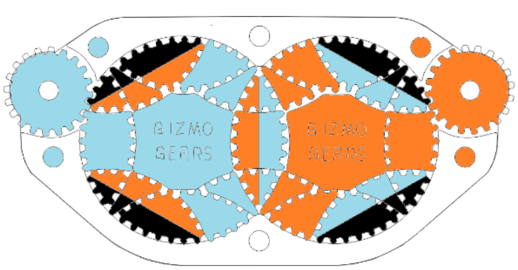
When I was about 6 or 7 years old my older brother showed me an old rusted V magnet he found in the yard. It was from the magneto of an old car. I was amazed, dumfounded how it could attract nails and bolts. From then on my interests were a battle between art and science. Gravity was also a battle for me. I thought it was on my side but found out otherwise after breaking my arm 4 times climbing trees. My dad got tired of the doctor bills. One time the doctor was drunk and my arm healed crooked. Doc put it against a bar and snapped it, then reset it correctly. The pain from the snap was sharp but quickly subsided. I had dreams about gravity. Floating high in the sky, gravity would suddenly return, then I would fall but always woke just at contact, a real nightmare. It began to seem that gravity must be respected. Evil Knievel would agree.

Two disk twisty puzzles

After college Hofstadter's column in Scientific American was about Rubik's Cube. That was my inspiration to invent Engel's Enigma, a name invented by Alexander Keewatin Dewdney who described it in his Scientific American column 1986. Finally, with a patent it was produced by Go Images. Royalties were minimal. Rubik was, and still is the great rage. It is easy to see why. The cube is magic, combining both a hidden mechanism and the mathematics of group theory. You hold a 'crystal' with 27 'atoms' in your hand with . The 3x3x3 cube is still the best selling twisty puzzle and has inspired contests, yearly events, magician stunts, and solving records of a few seconds. It inspires a constant stream of new designs. It would be great if you could afford to purchase choice new twisties. The Twisty Puzzle forum has collectors' inventories, puzzles for sale, new introductions and much more. You can start your own discussion by getting an account. A self-published 'Circle Puzzler's Manual' was written. It surprised me that twisty enthusiasts liked it. Jaap Scherphuis put it on his web site and I put it on mine. Meetings would be held. I attended one in California headed by Bram Cohen and Oskar Van Deventer. Tom Rodgers was there. CPM was discussed, and the words bandaged, and unbandaged were discussed. Bram wanted to know if a puzzle can always be unbandaged meaning that if you find a position where a cut is required to continue movement you make the cut to remove the 'bandage'. If not does some or all the puzzle pieces continue to get cut until they 'turn to dust', I believe, in Bram's words.

That meeting was partly my inspiration to design Battle Gears and Gizmo Gears twisty puzzles. Also it was my intention to add an extra challenge. This was financially a bad idea. Jaap produced a solution for Battle Gears on his website but had a difficult time with it. Jaap did not like it, since no simple algorithm worked, like with a cube. BG required a long careful analysis for several different situations, not a simple symmetrical set of a few repeated moves. It looks symmetrical but it is not. Very few were sold. Gizmo Gears was much worse, less than 5 or ten sold. My conjecture presented here for the first time is that it cannot be solved without reversing many or all the moves made to mix it up. It has dead ends where you **must reverse** and take a different path to make new moves. Here solved implies that every piece is returned to its starting position and orientation. What is the simplest coloring required to force a full solution?

Fractal Unbandaging Gizmo Gears



Bob Hearn, a master coder got wind of the unbandaging problem. He attempted to unbandage Battle Gears and later, someone found a solution. Oskar Van Deventer alerted me and so I sent the Gizmo Gears puzzle layout to **Andreas Nortmann** of the Twisty Puzzle Forum to have the coders try Gizmo Gears. This started a 'Gizmo Gears' on the Twisty Puzzle Forum. A short while later Bob revealed Gizmo becomes fractal but just past the critical radius. It became the first twisty puzzle to reveal the crystallographic mix-master of 2 disk fractals. Bob now sells this beautiful artwork printed on T-shirts on his Etsy store. The 'Gizmo Gears' forum went on for several years and has hundreds of posts with cool art by Bob, Brandon Enright, Jason Smith, and several others. The coders put great effort rendering the fractals, some taking a week or more of PC time. I did not attempt to do my own coding. My Kindle book "Incendiary Circles" uses, with credits, some figures and partial posts from the Gizmo forum reviewing the incredible work of the collaborators. I see it as the radii of the two disks increasing, like two single crest expanding waves, and surrounding waves all around also expanding to create an infinity of pieces. The fractals on the two circles are what the coder's work displays. Amazingly, Bob found Penrose tiles with n=5 rendering. Gizmo Gears is 3*4=12, so not surprising it was fractal. What is cool is that GG used the same lozenge design as a puzzle patented in the late 1800's. I just divided the lozenges using two symmetries 4 and 3 thus 3x4 or 12 when you attempt to unbandage. Bob came up with the idea of compound groups to describe bandaged twisty puzzles. He wrote this up along with several co-authors and published it on ArXiv, a site for preprinting new papers that may not have yet been peer reviewed by a journal. I downloaded and printed it. It is very well written and clear, with some beautiful color fractal renders. Bob has done presentations about these fractals with video renderings, one was at a G4G I was unable to attend.

Gardener's pulls back the curtain

Martin Gardener had a very simple writing style making you feel like you had entered a hidden world of treasures. In that regard his math writing style revealed the magic illusions behind math magic. Instead of describing the fearsome mental strain required to produce a proof he peeled back the layers hiding the inner secrets of much mysterious math. Math can be a difficult subject but if you are willing to research and master the current knowledge of a problem then you can really begin to contribute to, and possibly find a proof, and if not a proof, possibly contribute to what is known about the problem. You too, must pull back the curtain. Gardener could also pose a riddle that was so simple yet cofounding that even his solution left many people confused. For instance, he asked "When you look in a mirror why is your left and right reversed but not your up and down?" It is a psychological as well as a math kind of question. Gardener explains that it is due to our bilateral symmetry. For some people that does not satisfy. My own idea is to lean over sideways. Now your left and right is up and down reversed. What if your up and down really were reversed. You would have a hard time combing your hair but could easily see where you need to clean your toenails. You could examine your feet and knees without lifting them down much. You bend your head down and see your self looking up at you near the bottom of the mirror. It is a good thing our top and bottom are **not reversed**.

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