Patterns in a Botanical Garden

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Abstract: *Mathematics can help illuminate and enrich our understanding of the many patterns found in a botanical garden. Instances of tilings, spirals, fractals, Fibonacci numbers, spherical geometry, and many types of symmetry, all among Martin Gardner's favorite topics, appear among the plants and structures at the Santa Fe Botanical Garden.*

When we look at the world around us, we don't usually think about mathematics, or even notice math that may be right in front of our eyes. Yet an eye for math can greatly enrich our appreciation and understanding of what we see.

The Santa Fe Botanical Garden is a wonderful place for exploration with mathematics in mind, from the bilateral symmetry of leaves to branching fractal forms and Fibonacci numbers embedded in spiral patterns.

Counting and Measuring

Most people associate the term mathematics with numbers and, indeed, numbers do play a role in mathematics. At the same time, we encounter numbers in all sorts of ways in everyday life.

Let's start by characterizing the Santa Fe Botanical Garden, noting how we use numbers as key parts of these descriptions.

The Garden sits about 7,200 feet above sea level, near the southern end of the Rocky Mountains, which were formed 80 million to 55 million years ago.

Left: The Sangre de Cristo Mountains near Santa Fe represent the southernmost subrange of the Rocky Mountains.

The Garden gets about 9 to 13 inches of precipitation (rain and snow) annually, putting the area in the climate category of semi-arid steppe.

Now covering nearly 9 acres, this botanical garden is relatively new; its oldest section opened to the public in 2013.

Note how numbers help us describe, measure, and understand what we experience or encounter.

But there's much more to mathematics than just numbers and counting (and arithmetic). More broadly, we can think of mathematics as the study (or science) of patterns, though the patterns may themselves involve numbers. At the same time, patterns play an important role in botany, especially for identifying and characterizing plants.

Spheres and Symmetry

The leaves of a beaked yucca (Yucca rostrata) grow in a distinctive spherical shape. In effect, the plant looks the same from any direction, displaying spherical symmetry. For a sphere, the distance from its center to any point on the surface is the same.

*Left: Beaked yucca (*Yucca rostrata*) has a roughly spherical shape).*

Here's an interesting botanical question: How does this species of yucca achieve its spherical shape? What "rules" do its cells follow so that each leaf ends up roughly the same length?

In studying patterns, one key concept is that of symmetry. Take a look at the individual leaves of an agave plant.

*Left: Havard's agave (*Agave havardiana*).*

You'll notice that the left side of each leaf is just about identical to the right side. These leaves have mirror (or bilateral) symmetry: one side is a reflection of the other.

The leaves of many plants, large or small, have the same left-right symmetry.

Reflection is arguably the simplest type of symmetry. More generally, an object has some form of symmetry when, after a flip, slide, or turn, the object looks the same as it did originally.

Four Edges

The Garden's Rose and Lavender Walk features a wide variety of roses and several types of lavender (Lavandula).

Feel the stem of a lavender plant. You'll notice that the stem is not rounded but has edges. Indeed, the stem has (roughly) a square cross section.

The square stem is a characteristic of plants in the mint family (Lamiaceae). This family includes not only mint and lavender but also basil, rosemary, sage, thyme, salvia, and others.

*Left: Garden sage (*Salvia officinalis*) is a member of the mint family and has a square stem.*

Five Petals

The number 5 comes up repeatedly when you examine members of the rose family of plants (Rosaceae). The flowers of these plants typically have five sepals and five petals.

Wild roses have just five petals, as do a few varieties of cultivated roses such as 'Golden Wings.' The sweetbriar rose (*Rosa eglanteria*) is another example of a rose with five petals found in the Garden. However, most cultivated roses, which are bred for their appearance, have many more petals, generally multiples of five (though they still have just five sepals).

The fruit trees in the Orchard Garden are all members of the Rosaceae family. In springtime, the apple, apricot, cherry, plum, peach, and pear trees all produce blossoms with five petals.

The number 5 can also come up in surprising ways. Cut across an apple to reveal its core, and you'll find a five-pointed star shape in the center.

Cactus Spirals

If you look closely at a cactus, you can often detect distinctive patterns (though the spines may sometimes hide the underlying pattern), particularly spirals and helices. Note, for example, the way in which the spines and ridges on a cane cholla (Cylindropuntia spinosior) create a helical (spiral) pattern.

*Left: Cane cholla (*Cylindropuntia spinosior*) helix*.

The helical pattern is even more evident in the woody skeleton that serves as the framework for a cholla cactus.

Left: The woody skeleton of a tree cholla shows a helical pattern, as seen in the offset slits of the limb.

Similarly, observe how the leaves of an agave appear to grow in a spiral fashion. The leaves are not lined up like the spokes of a wheel.

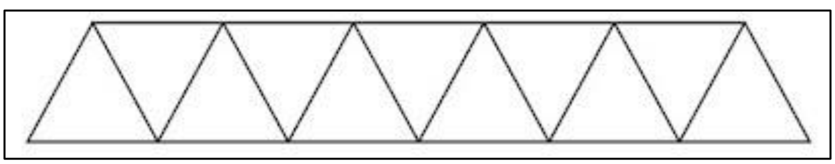
An agave's spiral growth pattern is also evident when a stalk (inset) forms at the end of the plant's life.

You may also notice a resemblance between an agave stalk and the young shoot of an asparagus plant. It turns out asparagus, agave, and yucca are genetically related and all belong to the Asparagaceae family.

Triangles, Squares, and Symmetry

Kearny's Gap Bridge is a recycled structure, originally built in 1913 for a highway near Las Vegas, New Mexico, and installed at the Garden in 2011 to connect the two sides of the Arroyo de los Pinos.

The most important geometric element is the use of equilateral triangles, characteristic of what is called a Warren truss, named for British engineer James Warren, who patented the weight-saving design in 1846.



A truss is a framework supporting a structure. A Warren truss consists of a pair of longitudinal (horizontal) girders joined only by angled cross-members (struts), forming alternately inverted equilateral triangle-shaped spaces along its length.

It's a particularly efficient design in which the individual pieces are subject only to tension or compression forces. There is no bending or twisting. This configuration combines strength with economy of materials and can therefore be relatively light.

Look at the pattern of struts along the "railing." This is an example of translational symmetry. Shifting the pattern to the left or right leaves the pattern the same.

Left: Behind the railing is another geometric feature: a protective fence in the form of a square grid.

In general, the repeated patterns of a symmetrical design make it easier for engineers to calculate and predict how a structure will behave under various conditions. They are characteristic of a wide range of human-built structures.

Counting Petals

During seasons when flowers are in bloom, it can be rewarding to examine the blossoms of individual plants, paying close attention to the number of petals characteristic of a given type of blossom.

*A chocolate daisy (*Berlandiera lyrata*) blossom appears to have eight "petals."*

Certain numbers come up over and over again: 3, 5, 8, 13, 21, 34. We don't often find flowers with four, seven, or nine petals, though they do exist. For example, sundrop (*Oenothera hartwegii*) blossoms have four petals.

The larger numbers are generally characteristic of daisies, asters, and sunflowers, all belonging to the Asteraceae family. However, in this case, each "petal" is actually an individual flower, known as a ray floret. And these florets are associated with the spirals of seeds in the plant's central disk.

The 'Arizona Sun' blanket flower (Gaillardia x grandiflora 'Arizona Sun') belongs to the Asteraceae family of plants. This particular example has 34 petal-like ray florets.

The numbers 3, 5, 8, 13, 21, and 34 all belong to a numerical sequence named for the 13th-century Italian mathematician Leonardo of Pisa (also known as Fibonacci). Each consecutive number is the sum of the two numbers that precede it. Thus, 1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8, 5 + 8 = 13, 8 + 13 = 21, 13 + 21 = 34, and so on.

Is it just a coincidence that the number of flower petals is more often than not a Fibonacci number, or does it point to something deeper—a pattern—about the way plants grow? That's a question that's been pondered for centuries.

Perhaps the statistics are skewed. For example, the number of flower petals can be characteristic of large families of plants. The flowers of plants in the rose family (Rosaceae), which includes many fruit trees such as apple, peach, and cherry and shrubs such as fernbush, serviceberry, and mountain mahogany, typically have five petals. So, we are likely to find the number 5 come up again and again when counting petals in the Garden.

Fibonacci numbers also come up in other ways. Take a look at the bottom of a pine cone. Pine cones have rows of diamond-shaped markings, or scales, which spiral around both clockwise and counterclockwise. If you count the number of these spirals, you are likely to find 5, 8, 13, or 21.

Left: The overlapping scales of a pine cone produce intriguing spiral patterns.

You find similar spirals among the seeds at the center of sunflowers and in the helical patterns that many cacti and succulents such as agave feature.

Left: The number of ray florets (above) displayed by a sunflower is often a Fibonacci number, as is the number of clockwise and counterclockwise spirals of seeds at a sunflower's center.

The patterns are intriguing (though sometimes difficult to discern and count), and mathematicians, physicists, and other scientists have, over the years, proposed various sets of "rules" that might govern how plants grow and produce the patterns observed in nature. One such set of rules, for example, leads to an efficient three-dimensional packing of "cells." It's a growth pattern that results in the optimal spacing of scales or seeds to reduce crowding, and applies to the helixes of cacti, the spiraling leaves of an agave, the scales of a pine cone, or the seeds and ray florets of members of the aster family.

Branches and Patches

Examine the leaf of a bigtooth maple (*Acer saccharum*).

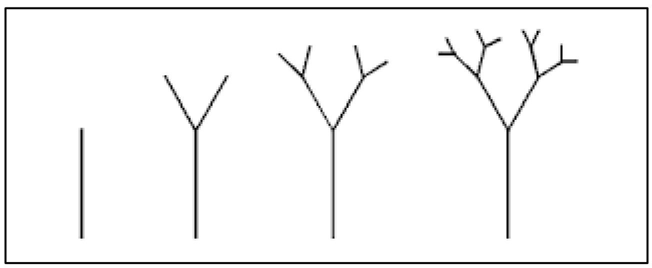
You'll notice that the left side of the leaf is just about identical to the right side. These maple leaves have mirror (or bilateral) symmetry: one side is a reflection of the other.

But there's another pattern on display. If you look closely, you will also see a network of veins: a main vein that branches into smaller veins, and these veins in turn branch into smaller veins, and so on.

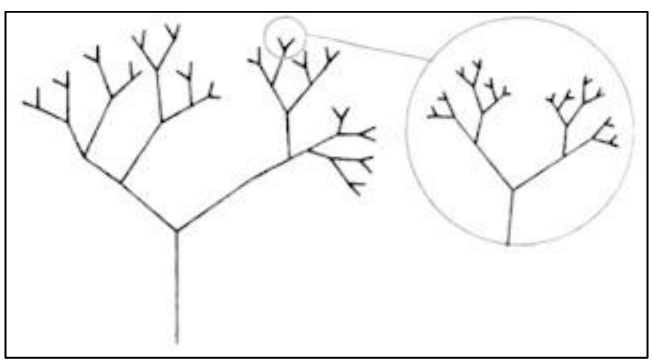
Such branching structures are characteristic of many natural forms. Cypress and juniper trees, for example, have fronds that show this type of pattern.

Left: The fronds of an Arizona cypress (Cupressus arizonica) have a distinctive branching structure.

In many cases, the branches look (at least roughly) like miniature versions of the overall structure. Such patterns are said to be self-similar. Mathematicians can create self-similar forms simply by repeating the same geometric structure on smaller and smaller scales to create an object known as a fractal. Each part is made up of scaled-down versions of the whole shape.



This example illustrates the first few steps in creating a simple geometric branching structure that has a self-similar, or fractal, pattern.



A magnified portion of a fractal looks like the overall structure.

The notion of self-similarity can also apply in other ways to natural forms. Just as a tree's limbs and twigs often have the same branching pattern seen near its trunk, clouds keep their distinctive wispiness whether viewed distantly from the ground or close up from an airplane window.

Left: The edge of a cloud may have many indentations, and those indentations when examined closely reveal smaller indentations, and so on.

Take a look at a raw stone surface. Do you see any straight lines, circles, triangles?

Instead, you may see some large hollows and ridges, and when you look closely, you see smaller hollows and ridges within these features, and so on. So there is a kind of pattern, even if the features are irregular.

Left: The patchiness of lichen growth on a stone surface has a fractal quality.

In general, in nature, you often see patterns in which shapes repeat themselves on different scales within the same object. So clouds, mountains (rocks), and trees wear their irregularity in an unexpectedly orderly fashion. In all these examples, zooming in for a closer view doesn't smooth out the irregularities. Objects tend to show the same degree of roughness at different levels of magnification or scale.

The characteristic furrows and ridges of Ponderosa pine (Pinus ponderosa) *bark have a self-similar, or fractal, quality*.

Where else might you find fractal patterns? Try a grocery-store produce department, where you'll see striking fractal patterns in such vegetables as cauliflower and Romanesco broccoli.

Left: This image looks like a fern, but the self-similar, or fractal, form on display was actually generated point by point by a computer following a simple set of rules.

Although the Garden doesn't have any ferns, it does have fernbush (Chamaebatiaria millefollum). Its leaves have roughly the same branching pattern displayed by fern fronds.

Left: Fernbush (Chamaebatiaria millefollum) *leaves display a branching structure similar to that of a fern.*

Patterns

There are many other patterns to observe in the Garden. For example, you could study and catalog the arrangements of leaves on plant stems (phyllotaxis).

Studying patterns is an opportunity to observe, hypothesize, experiment, discover, and create. By understanding regularities based on the data we gather, we can predict what comes next, estimate if the same pattern will occur when variables are altered, and begin to extend the pattern.

In the broadest sense, mathematics is the study of patterns—numerical, geometric, abstract. We see patterns all around us, in a botanical garden and just about anywhere else, and math is a wonderful tool for helping us to describe, understand, and appreciate what we are seeing.

See also "DC Math Trek" and "Where's the Math?"