Spotted It! – Strategies to Win Dobble

Gathering 4 Gardner Conference (G4G15)
Atlanta, Georgia from February 21–25, 2024

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***Abstract*— Dobble or Spot It! is a game created by Blue Orange Games for two up to eight players. In the game the players have to find symbols in common between cards. The mathematics of the game is surprisingly interesting and several papers have been published on it. However, the works concentrate on the creation of dobble decks. This paper describes a psychological effect while playing and a statistical strategy to win the game.**

I. Introduction

Dobble or Spot It! Is a game created by Blue Orange Games for two up to eight players. In the game the players have to find symbols in common between cards. The game contains a deck of 55 cards with eight symbols on each card. Each player receives a pile with the same number of cards. One card remains in the middle of the table. Now each player compares the top card of their individual pile with the card in the middle of the table. As soon as they find a matching symbol, they may discard the card, which now becomes the new card in the middle of the table. The first player to discard all cards wins the round.

The game is particularly interesting because there is no chance. There is no dice and since all pairs of cards have exactly the same number of matching symbols, which is one, so there are no better or worse cards. In fact, at first glance it seems difficult to develop strategies for the game. Rather, it seems as if it is primarily about a talent or ability to find such pairs quickly.

The idea of the game goes back to the 19th century. In 1850, Thomas Penyngton Kirkmanan Anglican clergyman, submitted a puzzle to *The Ladies and Gentleman's Diary*, an annual recreational mathematics magazine that took content from both amateurs and professional mathematicians. The question read, *Fifteen young ladies in a school walk out three abreast for seven days in succession: it is required to arrange them daily, so that no two shall walk twice abreast.* Dobble is a variation on this Kirkman's Schoolgirl problem.[1]

This paper will show that there are some psychological aspects that have an influence on quickly finding the right matching symbol. Thus, the game is not completely independent of luck. On the other hand, there is at least one theoretical strategy to gain advantage in the game: counting cards.

II. Dobble Mathematics

A complete Dobble game would have to consist of 57 cards and in this work all calculations will be made on a deck of 57 cards. At first, the number 57 seems quite unusual as it corresponds to $3 \cdot 19$ and the connection is initially unclear. The reasoning is as follows: First, any Dobble card is designed: it contains 8 symbols. each card in the game must match exactly one of these symbols. The rest of the deck is then divided into eight rows corresponding to the symbols of the first card. Suppose the first symbol on the first card corresponds to an armadillo. Since all cards in the first row must contain an armadillo, they must each contain 7 other unique symbols.

If a row contains k cards, then there are $8+7(k-1)$ different symbols in total (8 for the first card, and 7 new ones for each additional card in the row). For $m = 8$, that would be 57 symbols. However, the number m must also equal 8, as every card that is not in the armadillo row must share exactly one symbol with every card in this row. However, there are exactly 8 symbols on each card, so such a card can only match 8 cards in the armadillo row. The largest possible deck for cards with 8 symbols therefore consists of 57 cards.

In the US, there exists a variation of the game called Spot It! Junior, which only has 6 symbols per card. Consequently, the game consists of $6 + 5(6 - 1) = 31$ cards [2]. Interestingly, Heemstra showed in 2014 that it is impossible to create a Dobble for $k = 7$ with 43 cards [3]. Games with 11 and 12 symbols per card have been calculated, whether a game with 13 symbols per card is possible is still unknown [4]. Another strange fact about Dobble is, that geometry can be used, too, to answer these questions. In fact, Dobble has almost exactly the same structure as a geometrical object called a finite projective plane. This connection was described by Donna Dietz in 2013 [5]. However, there are also Dobble games that are not a finite projective plane [6]. A very nice mathematical description of Dobble is provided by the thesis of Christian Kathrein from the University of Graz [7].

In fact, the games on the market contain two cards less than a complete Dobble. A comment by Guillaume Gille-Naves, a former member of the development team for the French version of Dobble, explains that the game has 55 cards (instead of 57) for marketing purposes and to ensure that the rules of the game could be printed on extra cards without exceeding the manufacture limitation of 60 cards [8].

III. Evaluatung Dobble Games

To find starting strategies for Dobble games, I let two of my children play Dobble against each other. I wrote down each symbol that was called out loud. The Hungarian version of Dobble with 57 cards was used for the games, which only contains animals as symbols. I noted down a total of $k = 10$ games so that I could do some statistics. The rows of noted symbols are not the same length, as the game ends as soon as one of the children wins.

Table I shows the called out symbols of an example game. This game ended with the victory of one of the children after the 38th symbol was called out

TABLE I

NOTED SYMBOLS OF AN EXAMPLE GAME

No.	Symbol	No.	Symbol
1	Panda	20	Sheep
2	**Frog**	21	Hedgehog
3	**Frog**	22	Icebear
4	Zebra	23	**Scorpion**
5	Kangaroo	24	**Scorpion**
6	Camel	25	**Scorpion**
7	Donkey	26	Camel
8	Jellyfish	27	Raccoon
9	Seahorse	28	Eagle
10	Lion	29	Owl
11	Grasshopper	30	Goldfish
12	Horse	31	Sloth
13	Seahorse	32	Icebear
14	Chicken	33	**Dolphin**
15	Sloth	34	**Dolphin**
16	Crocodile	35	Pelican
17	**Bat**	36	Turtle
18	**Bat**	37	Buffalo
19	**Bat**	38	Pidgeon

A Dobble play ended after an average of 45.8 symbols called. It is immediately noticeable that in the example game it occurs several times that the same symbols are called out several times in direct succession and thus form tuples in the table (highlighted in bold here).

We consider a sequence of three cards. If the same symbol is named twice, this is called an *event*. If there are exactly two symbols that are called one after the other and are followed by another symbol, then this is a *double*. Accordingly, we call three identical symbols in a row a *triple* (This corresponds to two consecutive events.) and four symbols in a row a *quadruple* (This corresponds even to three consecutive events.). Table II shows the statistics for the events, doubles etc. in the ten games observed.

For each game the *Events per Sequences of 3 Cards* was calculated by dividing the *No. of events* by the *Total Symbols Called* -1. The last entry in the table is the rate of events in the total $n = 448$ $(458-10)$ inspected pairs of called symbols.

We want to investigate how high the expected value is for such events in a game. For the beginning of the game, however, it is clear how to calculate the probability of an event. Let's

TABLE II

STATISTICS OF THE TEN OBSERVED GAMES

Game	Total Symbols Called	No. of Doubles	No. of Triples	No. of Quadruples	No. of of Events	Events per Sequences of 3 Cards
1	46	4	0	0	4	0.089
2	48	7	0	0	7	0.149
3	48	5	0	0	5	0.106
4	45	2	0	0	2	0.045
5	38	2	2	0	6	0.162
6	43	6	1	0	8	0.190
7	45	6	0	0	6	0.136
8	41	3	2	0	7	0.175
9	53	6	0	0	6	0.115
10	51	6	0	3	15	0.300
Total	458	47	5	3	66	0.147

assume that the first match of the first hand card and the card in the middle was the armadillo. There are eight cards in the game showing the armadillo, two are used up. There are 55 cards left in the deck, so the chance is $p = 6/55$, about 0.109, that the next card is one with armadillo. However, this is not the probability of a double since we have to ensure that the third symbol called is not the armadillo.

In the same way we can calculate the probability of calling three times the same symbol in a sequence of four cards. In our example there are 54 cards left in the pack and only five armadillo cards. The probability is therefore $6/55 \cdot 5/54 = 0.0099$. Accordingly, the probability of calling four times the same symbol in a sequence of five cards is $6/55 \cdot 5/54 \cdot 4/53 = 0.00073$. Extending this idea up to sequences of eight cards we have the probability of $6/55 \cdot 5/54 \cdot 4/53 \cdot 3/52 \cdot 2/51 \cdot 1/50 = 1.72 \cdot 10^{-6}$ for a septuple.

There are other interesting questions.

- After how many turns will a game be over?
  If only one player at a time finds the matching symbol, the game ends after $n = 28$ moves (in a two-player game). If the players are equally good up to their very last call, the game ends after $2n - 1 = 55$ moves. What is the distribution of the random number M of moves of a game, $28 \leq M \leq 55$? Assuming that $q > 1 - q$ is the probability that a player names the symbol before his opponent, then the distribution of the random number M can be calculated as follows
  $$P(M = n+k) = \binom{n+k-1}{k} \left(q^n(1-q)^k + (1-q)^n q^k\right)$$
  for $k = 0, 1, \ldots, n-1$. Based at the observed average, the most likely value is $q = 0.607$.
  If one assumes that on average both players can name the matching symbols in the same time, i.e. $q = 1/2$, then the mathematical expectation of the number of moves until a player wins would be $\mathrm{E}M = \mu_M = 50.05$. The standard deviation is $\sigma_M = 22.72$. This means that the average value of 45.8 for the game length calculated from the 10 observed games is within the associated 95%-confidence interval $\mu_M \pm 1.96 \cdot \sigma_M/\sqrt{10} = 50 \pm 14$.
- How many different symbols are mentioned during a game?

As there are a total of 57 different symbols in the game, a maximum of 55 different symbols can be named (one different symbol in each turn). As each symbol can be named a maximum of seven times in a game, at least 28 divided by seven, this is four different symbols that must be named. Table III shows the number of different symbols that were called in the ten observed games. The question asked is analogous to the question: what is the expected value of the number of different numbers drawn in r rounds of roulette? If symbols were actually randomly determined 55 times in a row from the existing 57 symbols by drawing and replacing them, then the average would be

$$57 \cdot \left(1 - (1 - 1/57)^{55}\right) = 35.5$$

different symbols drawn. However, since on average only 45.8 symbols were mentioned, only 31.7 symbols would be expected

TABLE III
NUMBER OF DIFFERENT SYMBOLS CALLED IN THE TEN OBSERVED GAMES

Game	Total Symbols Called	Different Symbols Called
1	46	34
2	48	33
3	48	37
4	45	35
5	38	29
6	43	28
7	45	31
8	41	30
9	53	32
10	51	29
Means	45.8	31.6

But can we assume that the probability of events occurring changes over the course of the game? The cards are evenly distributed and the symbols on the cards are also evenly distributed. The cards are drawn evenly without putting them back.

Since the probability $p = 6/55$ for an event calculated for the beginning of the game does not change over the entire game, one should expect to see $(m-2)p$ events in a game with m moves. The observed games show a slight accumulation of events compared to the calculated probability p.

The author assumes that the explanation for this lies in a psychological effect. There are many indications in research that what we humans see depends on what we hear, i.e. that the senses influence each other [9], [10]. This means that when I hear a certain symbol pronounced, such as armadillo, my visual cortex is conditioned to recognize armadillos and I will detect an armadillo much faster than if the symbol has not just been named.

Now a player can be lucky enough to have one of his cards in hand show a symbol just mentioned more often than the opponent's deck does. However, this psychological effect then gives him an advantage in the game. This means that the game is not completely free of luck.

IV. Statistical Evidence

The results of the ten observed games are already sufficient to reject the hypothesis of a purely random sequence of symbols and thus to prove that repeating symbols are actually recognized more often.

As the population of our statistical model, we consider all (equally probable) sequences of three different cards. As already explained, the probability that two identical animals are named (event A) is $p = P(A) = 6/55$. The random number X of events in n inspections is a binomially distributed random variable with mathematical expectation $\mathrm{E}(X) = n \cdot p$ and variance $\mathrm{Var}(X) = n{\cdot}p{\cdot}(1{-}p) = \sigma^2$. Thus, the number X of events occurring in the total number $n = 448$ of inspections in the observed ten games has the expectation $\mathrm{E}(X) = 448 \cdot 6/55 = 48.9$ with $\mathrm{Var}(X) = \sigma^2 = 448 \cdot 6/55 \cdot 49/55 = 43.5$ or $\sigma = 6,6$. The corresponding confidence interval to the significance level of $\alpha = 0.05$ is $np{\pm}1.96\sigma$. With a probability of more than 95% X must therefore satisfy $35 \leq X \leq 62$. Because this obviously did not occur in the series of games observed, as can been seen from Table II, $X = 66$, it can be assumed that there is actually a higher event rate.

This can be explained by a modified calculation model for the event probability p'. In order to do this, sequences of four consecutive cards are considered. As before, the first two cards determine the event symbol (armadillo), while the following two cards belong to the two players. Assuming that the player who has the armadillo on his card always says this prior to the other player, then the value of p increases almost twice. More precisely, the probability that neither player can continue with the armadillo is $1 - p' = 49/55 \cdot 48/55$. That means $p' = 1 - 49/55 \cdot 48/55 = 0.208$.

If the player who can continue with armadillo is only able to use this advantage with probability λ and name the matching symbol prior to his opponent, the event probability scales linearly in λ,

$$p(\lambda) = p + \lambda(p' - p).$$

Based on this and the estimation $p(\lambda) = 0.147$ from Table II we get the most probable value of λ as $\hat{\lambda} = 0.386$.

V. Dobble Strategies

At first glance, there are no real strategies for winning a game of Dobble. The game seems to depend on individual talent. However, it is possible to improve the chances of winning, at least statistically, through strategy.

Let us assume that the human brain has to compare the symbols sequentially, and there is at least some evidence that this is a common strategy [11]. This means that I choose a symbol from my hand card, for example the armadillo, and look for it on the card in the middle. If I don't find a match, I choose the next symbol, check whether there is a match, and continue like this.

As a direct result of the above-mentioned probabilities, it is an advantage to first compare symbols that have not yet been mentioned or that have not yet or rarely been seen on previous cards (However, it is very difficult to count all the symbols.

It can be assumed that all 57 symbols are seen at least once during a game. The player would therefore have to count 57 values in his head at the same time and would hardly have time to do so.). Since it is very difficult oder not possible to quickly record and count which symbols can be seen on all cards, we concentrate on the symbols that have been named by a player.

Since it is still difficult for the human brain to count how many times a maximum of 57 symbols have already been named (In the previous chapter, we showed that there are 31.6 symbols on average.), let's concentrate on the symbols that are actually named together. According to the statistics in the previous chapter, this occurs in most games, and the average of these events is 6.6 times.

A single symbol can be called a maximum of seven times in a game. However, it is not irrelevant whether the mentions are connected, i.e. whether they occur directly one after the other, or whether they are not connected, i.e. whether a different symbol is mentioned between each mention. In the case of related entries, the maximum is seven.

This means that according to the psychological effect that we humans see what we hear. The human mind automatically checks whether the symbol we have just heard leads to another match. Whether this is the case or not is a matter of luck. However, if this is not the case, symbols that have not yet been heard as doubles, triples or even quadruples should first be searched for sequentially on the other card. Since at least two and a maximum of nine of these events occurred in the games examined, it is not too difficult to memorize these events. For the symbols that have not yet been mentioned in an event, the chance that there will be a match is increased.

It is interesting to note that the proposed strategy does not work for computers. A computer always needs eight comparisons to find a match between the two cards. This means that when two computers play Dobble against each other, the question of whether one of them wins depends only on the order of the symbols on the cards. In each round, one of the two computers is lucky that its symbol is higher up in the list of eight comparisons.

With the proposed strategy, however, the computer gains nothing, but loses time, as it requires an extended comparison with the stored events. It does not save any comparison, but has to compare more. with humans, however, it is different, the comparison with the memorized symbols takes much less time than searching for a symbol on the other card. This is where the strategy makes sense.

Nevertheless, it would be interesting to have two PCs play Dobble against each other. A lot could certainly be learned from these games and the statistics could be improved.

VI. Conclusion

It's obvious that the math behind the game Dobble is exciting. However, most of the papers published so far relate to the creation of Dobble sets. I am not aware of any paper that deals with the gameplay itself.

The game is especially interesting at first sight because luck does not seem to be a component of the game. However, in this paper we have shown that the game is not completely free of luck. This is due to the effect anchored in the human psyche that we recognize a symbol that we hear more quickly because the visual and auditory cortex are strongly connected. This effect could already be statistically verified, on a small sample.

On the other hand, this work has proposed a strategy for Dobble games, the counting of cards. It does not have a great influence on the course of the game, but slightly improves the statistical chances of finding a pair first.

Dobble is a fascinating game that even very young children can learn well. The game is also well suited to introducing pupils or students to mathematical problems. In the past, it has already been used in language lessons [12], [13], but it could definitely be used in math courses.

In this sense – Let's play Dobble!

References

[1] Linda Rodriguez McRobbie: *The Mind-Bending Math Behind Spot It!, the Beloved Family Card Game*, In: *Smithsonian Magaziner*, 2020-03-01.

[2] Tom Clark: *Spot It! Mathematical Structure in a Children's Game*, In: *MTCircular*, vol. 10, 2017.

[3] Marcus Heemstra: *The Mathematics of Spot It*, In: *The Journal of Undergraduate Research*, vol. 12(7), pp. 74–86, 2014.

[4] Aleisa Dornbierer-Schat: *Can You Spot It?*, In: *Dordt Voice*, vol. 61(1), pp. 22–24, 2017.

[5] Donna A. Dietz: *Spot it (R) Solitaire.*, In: *arXiv preprint*, arXiv:1301.7058, 2013.

[6] Bianca Gouthier, Daniele Gouthier: *On the existence of "Spot It!" decks that are not projective planes.*, In: *arXiv preprint*, arXiv:2201.09100, 2022.

[7] Christian Kathrein: *Ein Einblick in die Mathematik hinter dem Kartenspiel Dobble.*, *Diplomarbeit an der Universitt Graz*, 2021. (in German)

[8] Deepu Sengupta: *A Mathematical Analysis of Spot It!*, 2016.

[9] Casey O'Callaghan: *Seeing What You Hear: Cross-Modal Illusions and Perception.*, In: *Philosophical Issues*, vol. 18(1), pp. 316–338, 2008.

[10] Jamal R. Williams, Yuri A. Markov, Natalia A. Tiurina, Viola S. Strmer: *What You See Is What You Hear: Sounds Alter the Contents of Visual Perception.*, In: *Psychological science*, vol. 33(12), pp. 2109–2122, 2022.

[11] Herbert A. Simon: *The human mind: The symbolic level.*, In: *Proceedings of the American Philosophical Society*, vol. 137(4), pp. 638–647, 1993.

[12] Helena Levy, Adriana Hanulìkovà: *Spot It and Learn It! Word Learning in Virtual Peer?Group Interactions Using a Novel Paradigm for School?Aged Children.*, In: *Language Learning*, vol. 73(1), pp. 197–230, 2023.

[13] Suci Ayu Kurniah, P. Rusliana ,Erna Fitriani Pratiwi: *Enhancing Students Vocabulary Mastery using Spot It! Card Game in Distance Learning.*, In: *IDEAS: Journal on English Language Teaching and Learning, Linguistics and Literature*, vol. 8(2), pp. 483–491, 2020.