Four prophets and three sages

Riddle and solution by Ivo Fagundes David de Oliveira and Yogev Shpilman, 14th of December 2023

A long time ago four prophets spoke. Ever since, no more prophets have ever nor will ever come to be under the sun. Each prophet came to the land after the other had passed away, and each knew and agreed with their predecessors: The second knew and agreed with everything the first said, the third knew and agreed with everything the first two said, and finally, the fourth knew and agreed with everything the first three said.

The sacred writings testify that:

One prophet is known to have said "there will be, or has been, at least one false prophecy uttered by a false prophet", another prophesied that "there will be, or has been, at least one true prophecy uttered by a true prophet", yet another prophesied "there will be, or has been, at least one true prophecy uttered by a false prophet", and finally one that prophesied that "there will be, or has been, at least one false prophecy uttered by a true prophet".

Each prophecy was prophesied by a different prophet. However, the order in which these prophecies were uttered remains unknown to us.

It is known that a true prophet only says true things and never agrees with false prophecies. It is also known that a false prophet may or may not prophesy something true; a false prophet is false either because he prophesied a false prophecy, or because he agreed with a false prophecy, or both.

Now it came to pass that three wise men, who knew the order in which the prophets came, strongly disagreed on which prophecies were fulfilled and which weren't. Thankfully, in the first great council, chaired by the three wise men, after heated debates and long meditative moments of prayer, they finally decided to resolve the conflict by simple majority voting: they placed votes for each of the four prophecies simultaneously, deeming each one to be either true or false. To their surprise, the majority outcome of all of the votes completely aligned with the opinion of the eldest of the wise men, and from that moment onward he was considered the greatest among equals.

Question:	In which	order	did these	prophecies	come?	Which	prophecies	were
true and wh	nich were	e false'	?					

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SOLUTION:

We label the prophecies as follows:

Prophecy FF There will be or has been, at least one false prophecy

uttered by a false prophet.

Prophecy FT There will be or has been, at least one **false prophecy**

uttered by a **true prophet**.

Prophecy TF There will be or has been, at least one **true prophecy**

uttered by a false prophet.

Prophecy TT There will be or has been, at least one **true prophecy**

uttered by a true prophet.

It is possible to immediately notice that prophecy **FT must be false**, as no true prophet would prophesy a false prophecy. This means that prophecy **FF must be true**, as prophecy FT is a false prophecy uttered by a false prophet. Because the three sages strongly disagreed, **the truthfulness of prophecies TT and TF must remain ambiguous**. We also notice the following:

- A. If prophecy FF came after FT, that makes the prophet of FF a false prophet (because he agreed with a false prophecy that was uttered before him), which means prophecy TF must be true. That wouldn't leave room for three sages to disagree, and we thus conclude **prophecy FT came after prophecy FF** (but maybe not right after).
- B. If FF was the first prophecy, its prophet must have been a true prophet, which means prophecy TT must be true. That wouldn't leave room for three sages to disagree, and thus **prophecy FF wasn't first**.
- C. If TF was the first prophecy, then prophecies TF and TT are either both true or both false. That wouldn't leave room for three sages to disagree, and thus **prophecy TF wasn't first**.

Thus, from A, B and C we conclude that **prophecy TT was the first** under the sun. Now, prophecy TF could be 2nd, 3rd, or 4th. Let's explore these options:

- D. If TF came 2nd under the sun, then its truthfulness is opposite to that of TT, which wouldn't leave room for three sages to disagree. (We consider the order: TT, TF, FF, FT. If TT is false, then TF must be true as FF is a true prophecy by a false prophet. If TT and TF are true, then the first three prophecies were uttered by true prophets. Prophecy FT is as always false, which means there was no true prophecy by a false prophet, in contradiction to the assumption that TF is true. Thus if TT is true in that considered order, TF must be false).
- E. If TF came 3rd and TT was true, there would be a paradox between prophecy TF and itself (We consider the order: TT, FF, TF, FT. If TT is a true prophet, then so is FF. That means, for TF to be false it must be uttered by a true prophet, which is a contradiction, and for it to be true it must be uttered by a false prophet but since there was no false prophet before it, it also cannot be true), which means TT must be false and the sages would agree on it unanimously.

These facts uniquely determine the position of Prophecy TF to be the fourth and thus the full order is: $TT \rightarrow FF \rightarrow FT \rightarrow TF$

Prophecies TT/TF could be true/true, true/false, or false/true (but not false/false as if TT is a false prophet, so do the other prophets, which makes FF a true prophecy which makes TF a true prophecy). After the sages vote for these options, we conclude that prophecies TT, TF, and FF were fulfilled, while prophecy FT wasn't.