The Game Turing Machine

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1 The Game

Turing Machine[1] is a guessing game produced by Scorpion Masqué[2]. Players compete to deduce a three digit code where the numbers range from 1-5. The winner being the person who can deduce the number with the fewest clues. The game is completely self-contained and everything needed to play is provided. The answers are provided by a series of cards with obfuscated values and the game does not require any form of app, although one can expand the list of possible games from the given twenty printed in the rulebook to millions by means of the companion website[3]. The way verification is accomplished is ingenious in its own right and I encourage people to watch the numerous explanatory videos on BoardGameGeek[4]. Clues are provided by means of Criteria cards of which there is a deck of forty-eight cards and each game specifies between 3 and 5 cards for that particular game. In addition, the game specifies a verification card which enables players to query whether a number they present matches one of the criteria on the card. The rulebook uses "verification card" and "verifier" more or less interchangeably. See Figure 1.

Note that the digits in the secret code are always expressed in the order: (blue, yellow, purple), therefore on the criteria cards a reference to "blue" is interchangeable with a statement about the "first digit", yellow, "second" and purple, "third".

Here is an example of one such criteria card:

$$B > P$$
$$B = P$$
$$B < P$$

At least one of these statements is true of the secret code. In this case it is also true that at most one of these criteria can be true as the three possibilities are mutually exclusive. However, this is not the case with all such cards. For example another criteria card states:

$$B = 1$$
$$Y = 1$$
$$P = 1$$

In this case, a given keyword can match more than one of the criteria. The code (1, 1, 1), for example, would match all three criteria. Each of the criteria cards lists a minimum of two and a maximum of nine criteria. One point that escapes many players on first examination, is that the verifier card performs validation for a single criterion on the criteria card. It does not say anything about the truth or falsity of any other criteria on that card.

Figure 1: The Turing Machine Game

2 Analysis

There are some assumptions one can make simply by looking at the choice of criterion cards independent of verifiers, which is public knowledge at the start of the game. The rules narrow down possibilities by the following implicit facts. In the discussion that follows all references to criteria cards and verification cards mean the cards spelled out as associated with a specific game.

1. The secret number must give the answer yes to at least one criterion per criteria card.
2. Only one number will give the answer "yes" to all of the verification cards, or stated a different way, there is a unique solution that gives the answer yes to the criteria tested by each verifier.
3. If you remove any of the criteria cards (and its verification card), the solution will no longer be unique.

The second property means that while the rulebook includes a table of correct answers for each of the twenty sample games, showing that a number gives a "true" answer to each card suffices to prove it is correct.

It follows from the above assertions that if you know which criterion on each criteria card is being tested by its associated verification card, you would be able to deduce the solution. You could, in that case, compute for yourself the yes/no answers given by each of these criteria to all one hundred twenty-five possible codes and the only number that passed every test would be the answer.

3 The Program

The authors were curious as to how much information was available to the players before the game began based on some of the implicit statements described above. First, we represented every one of the forty-eight criteria cards as a list of assertions expressed as a Python lambda function which was simply a function from the variables (b, y, p) to true/false. For example, the first criteria card corresponds to the expressions:

lambda b, y, p: $b > p$
lambda b, y, p: $b == p$
lambda b, y, p: $b < p$

We chose for our example the most complex puzzle that only used four criterion cards. The four criteria cards provided were:

Card A	$B + Y + P < 6$
	$B + Y + P = 6$
	$B + Y + P > 6$
Card B	$B = 1$
	$Y = 1$
	$P = 1$
Card C	$B < 4$ or $B = 4$ or $B > 4$
	$Y < 4$ or $Y = 4$ or $Y > 4$
	$P < 4$ or $P = 4$ or $P > 4$
Card D	$B < Y$ or $B = Y$ or $B > Y$
	$B < P$ or $B = P$ or $B > P$
	$Y < P$ or $Y = P$ or $Y > P$

An interesting property of this particular set of criteria cards is that the conditions are symmetrical with respect to the letter b, y, p. There are $729 = 3 \times 3 \times 9 \times 9$ possible combinations of criteria that could have been expressed by means of the provided verifiers. We then set out to further narrow the possibilities. For each of the 729 possible combinations, we tested all 125 possible codes to determine how many codes would satisfy them. Of the 729 combinations we immediately ruled out 489 combinations which have no solution whatsoever. Also 162 possible combinations were eliminated as they did not lead to a unique solution as these combinations yielded between 2 and 14 possibilities). This leaves 78 combinations which would result in a unique secret code. However, the combination of criteria uniquely defines the codeword, the opposite is not true. There can be different combinations yielding the same underlying code. Thus, 78 combinations of criteria to only 27 possible codes At this point, however, we have not used the property that every one of the criterion cards is necessary, meaning that if you consider possible solutions for any subset the solution is no longer unique. We computed the list again but in this next run we only counted those combinations or criteria such that they led to a unique code but if you were to eliminate one of the four criteria cards altogether (this eliminates either 3 or 9 criteria depending on which card is removed) we further required that there would be more than one solution. Adding this constraint narrowed the list of 27 to the following 9 possible solutions:

$$(1,2,3),\ (1,3,2)\ (2,1,3)\ (2,3,1)\ (3,1,2)\ (3,2,1)$$

$$(3,1,3)\ (1,3,3)\ (3,3,1)$$

Note that the possibilities comprise all the permutations of $(1,2,3)$ and $(1,3,3)$. This property will not be true in general and is a consequence of the fact that for this game the criteria cards operated symmetrically on the three digits.

4 Implications for Game Play

The above analysis was conducted with no knowledge of the assigned verification cards. The deductions were made solely on the basis of the information that is available to all players before the game begins. This fact simplifies the goal of each player, assuming they are capable of this much mental calculation, to deducing which code among the list of possibilities is expressed by this particular choice of verifiers. Each time one queries one of the verifiers, one receives a single bit of information which means that the smallest number of inquiries needed to deduce the code should be: $\lceil \log 2N \rceil$ where N is the length of this list, and so, for example, with bad luck one would need four guesses for this game.

In the actual game, things are slightly more complicated as the players have to divide their guesses among rounds and in a given round one can choose a single number to test one to three verification cards. An obvious next step for the program is to determine the most efficient strategy for narrowing down the possibilities. Recalling that one of the rules implicit in our analysis was that removing any of the criteria cards would remove the uniqueness property. However, looking at the nine possible solutions one notes that they all give the same answer for the third criteria card and there is no reason to test using this card.

5 Conclusions

The authors performed the above analysis on all twenty of the initial games presented in the rulebook and determined that for four of twenty games, applying the logic above, there was a unique solution, in other words a perfectly logical player could state the solution having made zero tests, In an additional five cases, there are only two possible solutions meaning that a solution can be determined in a single guess. Although it was not the intention from the start, it turns out that the game we chose as our first focus has the maximum number of possible solutions of the first twenty.

We are also quick to point out that the calculations performed are not feasible for a player to actually carry out and that this paper should not be construed as a criticism. On the contrary, it is a fascinating game with, as we have noted, layers upon layers.

References

[1] Turing Machine `https://www.scorpionmasque.com/en/turingmachine`. Acccessed on 2024-01-20.

[2] Scorpion Masqué `https://www.scorpionmasque.com/en`. Acccessed on 2024-01-20.

[3] Turing Machine.info `https://turingmachine.info/`. Acccessed on 2024-01-20.

[4] BoardgameGeek `https://boardgamegeek.com/boardgame/356123/turing-machine`. Acccessed on 2024-01-20.