Latin Erdős

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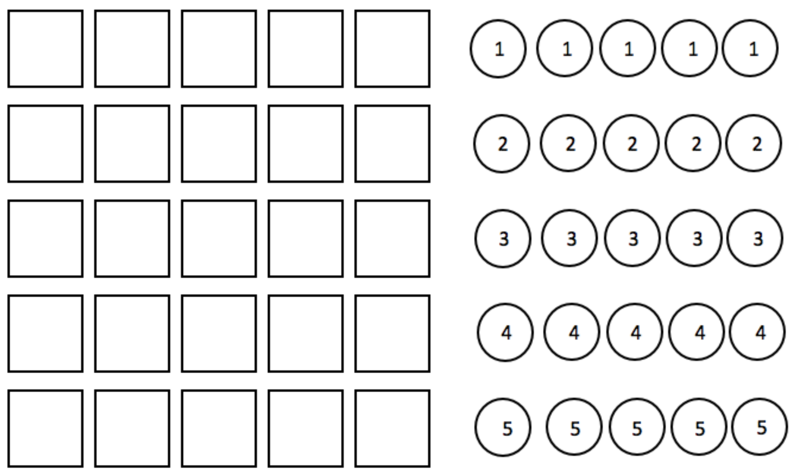
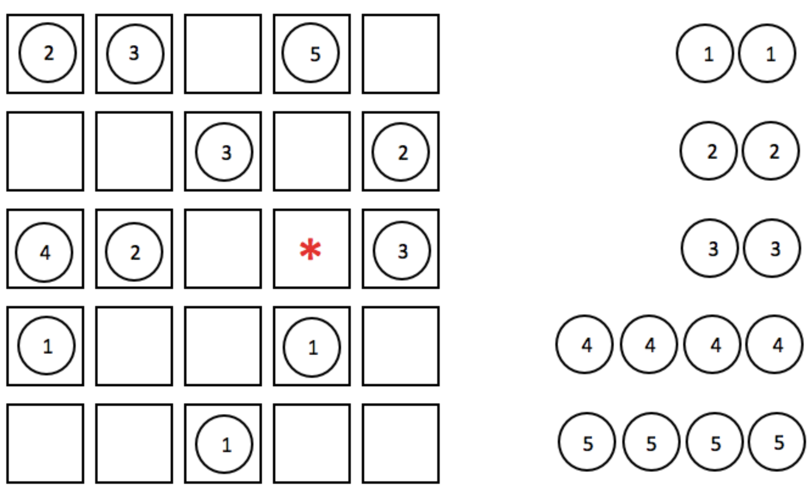
In this game each move consists in placing a disc with a number in a free square, provided such a placement does not give rise to any repetition of numbers in any row or column.



Example:



In the marked cell no number can be played.

Two players alternate. The first that, on his turn, is not able to play, for not having any legal move at his disposal, looses.
It can happen that the board gets completely filled with numbered pieces. In this case the winner is the player that conquered more columns.

A column is conquered by the player that, on his move, first gets an increasing sequence of length 3 in that column. In this context, increasing means that the numbers get larger from the player to his adversary.
Example:

B

4
2
3
*
*

A

If player A places 1 in one of the squares marked with *, he wins that column (sequence 1-3-4); if the player B places in one of those cells the number 5, he wins the column (sequence 2-3-5).
It can happen that a column contains one increasing sequence of length 3 for each player, as in the example

4
2
3
1
5

(1-2-4, 2-3-5)

thus, it is important to know who got his first, or whose turn it was when they both got it (whoever makes the move owns the column).
This game can be played in three ways:
I (Beginner) – With random factor. In this version the pieces should have their faces turned down and the players should turn one in each turn;
II (Expert) – Complete information. The pieces show their faces at all times and each player, on his turn, chooses freely the piece to place in the board.
III (Gambler) – Mixed. The pieces, with their faces down, are randomly divided by the players (12 for one, 13 for the other). The player with 13 pieces starts.

Notes

1) A square filled with numbers according to our rules is called a *Latin Square*. Latin squares were first studied by the Swiss mathematician Leonhard Euler in the 18^{th} century.

2) In Version II the first player can be sure of always having a legal move at his disposal. He should start by placing 3 in the central cell and, after that, when his adversary plays *x*, he should play *6-x* in the cell that is symmetric with respect to the central square.

This strategy does not guarantee more columns won at the end…

3) To be sure that each filled column is won by one of the players, we must rely on a mathematical theorem of Erdős and Szekeres from 1935!

http://www.luduscience.pt/erdos.html