INTRODUCTION

THE GENERALIZED OPERATOR A

Let x_1 and x_2 be members of the extended real line with $x_1 < x_2$. Let $X = (x_1,x_2) = \{x|x_1<x<x_2\}$. Let $C(X)$ be the set of real-valued continuous functions on X. Let $a,c \in C(X)$ with $a(x) > 0$ and $c(x) \geq 0$ for $x \in X$. Consider the classical operator B defined on $u \in \{u|u \in C(X) \text{ and } u'' \in C(X)\}$ by

(0.1) $$Bu = au'' - cu.$$

Now let $a_1, b_1, c_1 \in C(Y)$ where $Y = (y_1,y_2)$ and $a_1(y) > 0$ and $c_1(y) \geq 0$ for $y \in Y$. By making the change in independent variable given by

$$x(y) = \int_{y_0}^{y} \exp[-\int_{y_0}^{z} a_1^{-1}(t)b_1(t)dt]dz$$

where $y_0 \in Y$, the more general classical operator B^* defined by

(0.2) $$B^*v = a_1v'' + b_1v' - c_1v$$

can be put in the form (0.1) with $u(x) = v(y(x))$,

$$a(x) = a_1(y(x))\exp[-2\int_{y_0}^{y(x)} a_1^{-1}b_1dt] \text{ and } c(x) = c_1(y(x)).$$

Motivated partly by probabilistic considerations McKean has studied certain generalizations of (0.1) (see [12], [13]). His work is greatly influenced by Feller who formulated many of the underlying ideas and has made extensive contributions in the areas of generalized second order linear differential operators and their relationships to probability (see [5]-[9]). In this paper a generalization of (0.1) will be presented and some of its properties studied primarily from the point of view of analysis. This introduction includes the definition of the isomorphic permutation representations, therefore,

$$P(T,\varsigma;s_1,s_2,s_3,s_4) = (1/12)\cdot(s_1^4 + 8s_1s_3 + 3s_2^2) \tag{1.9}$$

<u>Example 8.</u> Let C_n be the <u>cyclic group</u> of order n, and let $N_n = \{1, 2, 3, \ldots, n\}$. The permutation group generated by the cyclic permutation $(1,2,3, \ldots ,n)$ is a faithful representation of C_n on N_n. The cycle index of this representation is the polynomial

$$P(C_n,N_n;s_1,s_2, \ldots ,s_n) = (1/n)\cdot\sum_{d|n} \phi(d)\ s_d^{\frac{n}{d}} \tag{1.10}$$

where the sum is taken over all the positive divisors d of n, and where ϕ denotes Euler's phi-function. For instance,

$$P(C_4,N_4;s_1,s_2,s_3,s_4) = (1/4)\cdot(s_1^4 + s_2^2 + 2s_4).$$

Let $N_u = \{1, 2, \ldots, u\}$ and $N'_v = \{1', 2', \ldots, v'\}$ be disjoint sets whose cardinalities u and v have least common multiple $[u,v] = n$. The permutation group generated by the permutation $(1,2, \ldots ,u)(1',2', \ldots ,v')$ is a faithful intransitive representation of C_n on $N_u \cup N'_v$. The cycle index of the representation $(C_n,N_u \cup N'_v)$ is the polynomial

$$P(C_n,N_u \cup N'_v;s_1,s_2, \ldots ,s_{u+v}) = (1/n)\cdot\sum_{d|n} \phi\left(\frac{n}{d}\right) s_{u/(u,d)}^{(u,d)}\cdot s_{v/(v,d)}^{(v,d)} \tag{1.11}$$

where (u,d) is the greatest common divisor of u and d.

<u>Example 9.</u> The <u>dihedral group</u> D_n of order 2n is the group of symmetries of a regular n-gon. If $V = \{v_1, v_2, \ldots, v_n\}$ is the set of vertices of a regular n-gon, then the permutation group generated by the two permutations $(v_1,v_2, \ldots ,v_n)$ and $(v_1)(v_2,v_n)(v_3,v_{n-1})(v_4,v_{n-2})\cdots$ is