eigenvector, namely $Q\,S^{(x)}$, where $S^{(x)}$ is a column vector with $S^{(x)}{}_i = x^i$, and this eigenvector is positive if x is real and ≥ 1.

The Martin exit boundary of a Markov chain consists of all normalized, minimal, non-negative regular functions, see [2]; since the only regular functions for a slowly-spreading chain are the constants, the boundary consists of a single point.

The boundary becomes more interesting if we consider the space-time chain, i.e. the chain whose states are pairs (i,n) where i is a state of the original chain and n is a time and the pair can be reached from the starting pair $(0,0)$. If $P_{ii} \geq \epsilon > 0$ for all i then the Martin boundary is a set of functions h_x ; x is any real number in the closed interval $[x_o,\infty]$ where x_o is a certain positive number that depends on P. The function h_x are: for x finite, $h_x(i,n) = x^{-n}\left[QS^{(x)}\right]_i$ and $h_\infty(i,n) = \delta_{in}/P^{(i)}_{oi}$, where δ_{in} is the Kronecker delta. For $i \leq n$, which is after all just where h is defined, the pointwise limit of $h_x(i,n)$ as $x \to \infty$ is $h_\infty(i,n)$, so the notation h_∞ is reasonable.

A representation is available for any dual of a slowly spreading chain. A dual chain is associated with a positive superregular measure α and its transition probabilities are $\hat{P}_{ij} = \alpha_j\, P_{ji}/\alpha_i$. Let D be a diagonal matrix with $D_{ii} = 1/\alpha_i$.

conditions which are the same at both ends of the interval, $p(x) = q(\pi - x)$ has the same spectrum as q. If $q \notin L_g$, then $p \neq q$.

3. WHEN ARE TWO SEQUENCES COMPLEMENTARY SPECTRA?

For example, given sequences $\{s_n\}$, $\{t_n\}$; if there exists $\{\lambda_n\}$, $\{\mu_n\}$ which are the complementary spectra of a problem with known radius δ [as in Theorem 3], and if $||\lambda - s||^2 + ||\mu - t||^2 < \delta^2$, then $\{s_n\}$, $\{t_n\}$ are complementary spectra for some p, with these boundary conditions. More generally:

<u>THEOREM 4</u>: ([4], p.81)

Let two complementary sets of boundary conditions be given. Two sequences $\{s_n\}$, $\{t_n\}$ are complementary spectra for an equation $y'' + (\lambda - q)y = 0$ with these boundary conditions IF AND ONLY IF there exists a finite chain of functions $0 = q_o, q_1, \ldots, q_k = q$ with radii δ_i [as in Theorem 3] and complementary spectra $\{\lambda_n^i\}$, $\{\mu_n^i\}$ for the given boundary conditions, respectively, for $i = 0, 1, \ldots, k-1$; for which $|s_n - \lambda_n^i| < |s_n - \lambda_n^{i-1}|$, $|t_n - \mu_n^i| < |t_n - \mu_n^{i-1}|$ and $||\lambda^i - \lambda^{i-1}||^2 + ||\mu^i - \mu^{i-1}||^2 < \delta_{i-1}^2$ for $i = 1, 2, \ldots, k$, where $\lambda_n^k = s_n$ and $\mu_n^k = t_n$ for all n.